NCERT EXEMPLAR PROBLEMS - SOLUTIONS

10

MATHEMATICS

With Exclusive Important Questions of
Diksha Platform 2020

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Published by: Agrawal Group Of Publications (AGP)

Edition: 2020

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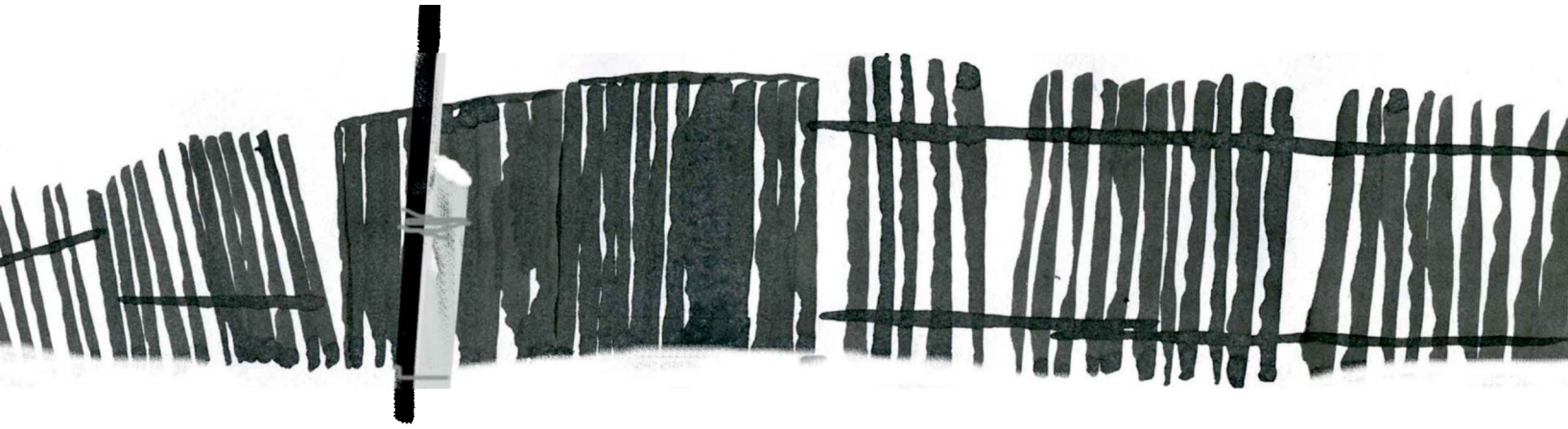

Hello!

NCERT has developed Exemplar Problems in Mathematics for Class X with the objective to cover the fundamentals of all Class X topics. NCERT exemplar questions are conceptual in nature and cover quite a large percentage of problems asked in any board examination competitive and entrance examinations.

This Educart Exemplar book is prepared by our country's top experts, to give students a single reference book throughout the year to clear doubts. It also serves the purpose of revision, in the last moments before Board examinations, especially, as a major part of paper comes from this book statistically.

Below are the new features for this year:

Diksha VidyaDaan
NCERT recommended Diksha 2.0 platform (launched February 2020) questions (important ones) are solved at the end of each chapter.

Alternative Solutions
In certain questions, more than one style of solving the question under the guidelines of NCERT is provided. This is followed by 'Tricks applied' section to learn about the method used..

Time Stamp
Questions that have appeared in previous board examinations are identified, so that students use that extra data to prioritise important questions.

We sincerely hope that the contents of this book will prove immensely beneficial to all students. We wish you the best of luck.

Managing Editor

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1 Real Numbers

EXERCISE 1.1

Choose the correct option from the given four options in the following questions:

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers, then the HCF (a, b) is:

(A) xy (B) xy^2
(C) x^3y^3 (D) x^2y^2

Ans. (B)

Explanation:

Given that $a = x^3y^2 = x \times x \times x \times y \times y$

and $b = xy^3 = x \times y \times y \times y$

$\Rightarrow$ HCF of a and b = HCF (x^3y^2, xy^3)

$= x \times y \times y = xy^2$

We know that HCF is the product of the smallest power of each common prime factor involved in the numbers.

2. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$ where a and b are prime numbers, then the LCM (p, q) is:

(A) ab (B) a^2b^2
(C) a^3b^2 (D) a^3b^3

Ans. (C)

Explanation: Given that

$p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

$\Rightarrow$ LCM of p and q = LCM (ab^2, a^3b)

$= a \times b \times b \times a \times a = a^3b^2$

We know that LCM is the product of the greatest power of each Prime factor of the numbers.

3. The product of a non–zero rational and an irrational number is:

(A) always irrational
(B) always rational
(C) rational or irrational
(D) one

Ans. (A)

Explanation: Product of a non–zero rational and an irrational number is always irrational.

For example:

$\frac{7}{9}$ is rational and $\sqrt{2}$ is irrational numbers.

Their product is an irrational number.

$\frac{7}{9} \times \sqrt{2} = \frac{7\sqrt{2}}{9}$, which is an irrational number.

4. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is:

(A) 10 (B) 100
(C) 504 (D) 2520

Ans. (D)

Explanation: As we require the least number, the problem is based on finding the LCM.

Factors of 1 to 10 numbers are as follows:

$1 = 1$
$2 = 1 \times 2$
$3 = 1 \times 3$
$4 = 1 \times 2 \times 2$
$5 = 1 \times 5$
$6 = 1 \times 2 \times 3$
$7 = 1 \times 7$
$8 = 1 \times 2 \times 2 \times 2$
$9 = 1 \times 3 \times 3$
$10 = 1 \times 2 \times 5$

LCM of number 1 to 10

$=$ LCM $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2520$

5. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(A) one decimal place
(B) two decimal places
(C) three decimal places
(D) four decimal places

[CBSE 2017, 13]

Ans. (D)

Explanation:

2	1250
5	625
5	125
5	25
5	5
	1

Simplifying the given fraction:

$$\frac{14587}{1250} = \frac{14587}{5^4 \times 2}$$

$$\Rightarrow \quad = \frac{14587}{5^4 \times 2} \times \frac{2^3}{2^3}$$

$$= \frac{116696}{5^4 \times 2^4} = \frac{116696}{10^4}$$

$$= 11.6696$$

Number is 11.6696

Hence, the given rational number will terminate after four decimal places.

EXERCISE 1.2

1. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75, what is the HCF of (525, 3000)? Justify your answer.

Ans. HCF = 75

Since 3, 5, 15, 25 and 75 are the only common factors of 525 and 3000, 75 is the HCF of 525 and 3000.

Justification: Using Euclid's Division Algorithm:

Given positive integers a and b, there exist unique integers q and r, satisfying,

$$b = a \times q + r \qquad [0 \le r < a]$$

[Using dividend = divisor × quotient + remainder]

$$3000 = 525 \times 5 + 375$$

$$525 = 375 \times 1 + 150$$

$$375 = 150 \times 2 + 75$$

$$150 = 75 \times 2 + 0$$

So, the highest common factor among 3, 5, 15, 25 and 75 of 525 and 3000 is 75.

2. Explain why 3 × 5 × 7 + 7 is a composite number. [CBSE 2017]

Ans.

1 is neither prime nor composite.

If a number greater than 1 is not prime, it is a composite number.

Further, a prime number is divisible by 1 and the number itself.

Now,

$$3 \times 5 \times 7 + 7 = 105 + 7$$

$$= 112$$

$$= 7 \times 16$$

As 112 is divisible by 7 and 16, it is a composite number.

Trick Applied

Prime numbers have only two factors: 1 and the number itself. Also, according to the definition of composite numbers, any number with more than 2 factors is said to be composite.

3. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Ans. No.

We know that:

"The HCF of any two numbers must be a factor of the LCM of those numbers."

So, two numbers cannot have their HCF 18 and LCM 380, as 18 does not divide 380.

4. Without actually performing long division, find if $\frac{987}{10500}$ will have terminating or non–terminating (repeating) decimal expansion. Give reasons for your answer. [CBSE 2010, 09]

Ans. Yes, it will have a terminating decimal expansion.

Simplified denominator has factor in the form of $2^m \times 5^n$.

So, this is a terminating decimal.

3	987
7	329
	47

5	10500
3	2100
7	700
5	100
5	20
2	4
	2

Now, $\frac{987}{10500} = \frac{3 \times 7 \times 47}{2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7}$

$$= \frac{47}{5^3 2^2} \times \frac{2}{2} = \frac{94}{5^3 2^3}$$

$$= \frac{94}{1000} = 0.094$$

And we know, if p/q is a rational number, such that the prime factorization of q is of the form $2^m \times 5^n$ where n and m are non–negative integers, then x has a decimal expansion which terminates.

Hence, it terminates.

5. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form $\frac{p}{q}$? Give reasons. [CBSE 2013]

Ans.

As 327.7081 is a terminating decimal number, the denominator of the rational number must be of the form $2^m \times 5^n$.

Thus, $327.7081 = \frac{3277081}{10000}$

$$= \frac{3277081}{10^4}$$

$$= \frac{3277081}{2^4 \times 5^4} = \frac{p}{q}$$

So, the prime factors of q are 2 and 5.

Here, q is of the form $2^m \times 5^n$, where m and n are natural numbers. The prime factors of p and q will be either 2 or 5 or both.

EXERCISE 1.3

1. Prove that $\sqrt{3}+\sqrt{5}$ is irrational. [CBSE 2020, 18, 15, 13, 10, 09]

Ans.

Let us suppose that $\sqrt{3}+\sqrt{5}$ is rational.

Then $\sqrt{3}+\sqrt{5} = a$, where a is a rational number.

Therefore, $\sqrt{3} = a - \sqrt{5}$

Squaring both sides, we get

$$(\sqrt{3})^2 = (a - \sqrt{5})^2$$

$$\Rightarrow \quad 3 = a^2 + 5 - 2\sqrt{5}a$$

[Using $(a-b)^2 = a^2 + b^2 - 2ab$]

$$\Rightarrow \quad 2a\sqrt{5} = 2 + a^2$$

$$\Rightarrow \quad \sqrt{5} = \frac{2+a^2}{2a}$$

which is a contradiction, as the right hand side is a rational number while the left hand side $\sqrt{5}$ is irrational.

Hence, our assumption is wrong and $\sqrt{3}+\sqrt{5}$ is irrational.

2. Show that 12^n cannot end with the digit 0 or 5 for any natural number n. [CBSE 2020, 17, 15]

Ans.

We know that if any number ends with the digit 0 or 5, it is always divisible by 5.

If 12^n ends with the digit zero or 5, it must be divisible by 5.

This is possible only if prime factorisation of 12^n contains the prime number 5.

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$\Rightarrow \quad 12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$$

Since its prime factorisation does not contain 5, hence, 12^n cannot end with the digit 0 or 5 for any natural number n.

3. On a morning walk, three people step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk, so that each can covers the same distance in complete steps? [CBSE 2015]

Ans.

We know that the LCM is the product of the greatest power of each Prime factor of the numbers.

We have to find the LCM of 40, 42 and 45 to get the required minimum distance.

For this, we find prime factorisation,

$$40 = 2 \times 2 \times 2 \times 5$$

$$42 = 2 \times 3 \times 7$$

$$45 = 3 \times 3 \times 5$$

$$\text{LCM}(40, 42, 45) = 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 7$$

$$= 2520$$

Hence, each person should walk a minimum distance of 2520 cm, so that each of them can cover the same distance in complete steps.

4. Write the denominator of rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Then, write its decimal expansion without actual division.

Ans.

Denominator of the rational number $\frac{257}{5000}$ is 5000.

Now, $5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

$$= (2)^3 \times (5)^4$$

which is of the type $2^m \times 5^n$,

where $m = 3$ and $n = 4$ are non–negative integers.

Simplifying the given fraction:

$$\frac{257}{5000} = \frac{257}{5^4 \times 2^3}$$

$$\Rightarrow \quad = \frac{257}{5^4 \times 2^3} \times \frac{2}{2}$$

$$= \frac{514}{5^4 \times 2^4} = \frac{514}{10^4}$$

$$= 0.0514$$

So, 0.0514 is the required decimal expansion of the rational number $\frac{257}{5000}$.

5. Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p and q are primes.

Ans.

Let us suppose that $\sqrt{p}+\sqrt{q}$ is rational.

Let $\sqrt{p}+\sqrt{q} = a$, where a is a rational number,

$\Rightarrow \quad \sqrt{p} = a - \sqrt{q}$

On squaring both sides, we get

$\Rightarrow \quad p = a^2 + q - 2a\sqrt{q}$

[Using $(a - b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow \quad \sqrt{q} = \frac{a^2 + q - p}{2a}$

Therefore, the above statement is a contradiction as the right hand side is a rational number,

while the left hand side $\sqrt{q}$ is irrational, since p and q are prime numbers.

So, our assumption is wrong. Hence, $\sqrt{p}+\sqrt{q}$ is irrational.

Note: Q. 10 follows from Q. 14.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. Are the smallest prime and the smallest composite numbers co-prime ? Justify.

Ans. No.

We know that,

Smallest prime number is 2 and smallest composite number is 4.

HCF of (2, 4) = 2

Since, there is a common factor 2.

So, they are not co-prime.

2. Write the exponent of 3 in the prime factorization of 144.

Ans.

Prime factorization of $144 = 2^4 \times 3^2$

So, exponent of 3 = 2.

3. If HCF of two numbers is 1, the numbers are called relatively ..... or ..... .

(A) Prime, co-prime (B) Composite, prime

(C) Both (A) and (B) (D) None of the above

Ans. (A)

Explanation: Prime numbers are those numbers which have only two factors *i.e.,* 1 and itself. Example, 3, 5, 11 etc.

Co-prime numbers: Two numbers that have only 1 as a common factor.

Example,

35 and 39

$35 = 1 \times 5 \times 7$

$39 = 1 \times 3 \times 13$

Here, common factor is 1.

4. The product of the LCM and HCF of two natural numbers is 24. The difference of two numbers is 2. Find the numbers.

Ans.

Let the natural numbers be p and q.

According to question,

$p \times q = 24$ ...(i)

and $p - q = 2$

$p = 2 + q$ ...(ii)

From (i) and (ii)

$(q + 2) \times q = 24$

$q^2 + 2q - 24 = 0$

$q^2 + 6q - 4q - 24 = 0$

$(q + 6)(q - 4) = 0$

$q = -6, 4$

$q = 4$

[Since –6 is not a natural number]

So, the numbers are 4, 6.

5. Two alarm clocks ring their alarms at regular intervals of 72 seconds and 50 seconds if they first beep together at 12 noon, at what time will they beep again for the first time?

Ans.

Here, we need to find the LCM of 72 and 50.

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$50 = 2 \times 5 \times 5$

LCM of 72 and 50 $= 2^3 \times 3^2 \times 5^2 = 1800$

So, 1800 sec = 30 min

Hence, alarm clocks will beep again for the first time at 12.30 pm.

6. Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively, if they start tolling together, after what time will they next toll together?

Ans.

The required time is the LCM of 12, 15 and 18.

$12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

$18 = 2 \times 3 \times 3$

$\text{LCM} = 2^2 \times 3^2 \times 5 = 180$

So, next time the bells will ring together after 180 minutes or 3 hours.

7. Find the two numbers which on multiplication with $\sqrt{360}$ gives a rational number. Are these numbers rational or irrational?

Ans.

$$\sqrt{360} = \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$$

$$= 6\sqrt{10}$$

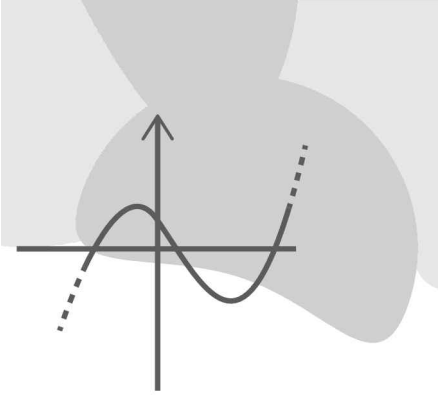
If we multiply $6\sqrt{10}$ with $\sqrt{10}$ and 1.

We get,

$$6\sqrt{10} \times \sqrt{10} \times 1 = 60$$

Hence, numbers are $\sqrt{10}$ and 1.

Where, 1 is a rational number and $\sqrt{10}$ is an irrational number.



2 Polynomials

EXERCISE 2.1

Choose the correct option from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is:

(A) $\frac{4}{3}$ (B) $-\frac{4}{3}$

(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

[CBSE 2018, 15, 13, 10, 09]

Ans. (A)

Explanation: We know that if α is a zero of a quadratic polynomial $p(x) = ax^2 + bx + c$,

then $p(\alpha) = 0$.

Given that one of the zeroes of the quadratic polynomial say $p(x) = (k-1)x^2 + kx + 1$ is -3.

So, $p(-3) = 0$

$\Rightarrow (k-1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow (k-1)(9) - 3k + 1 = 0$

$\Rightarrow 9k - 9 - 3k + 1 = 0$

$\Rightarrow 9k - 3k - 8 = 0$

$\Rightarrow 6k = 8$

$\Rightarrow k = \frac{4}{3}$

2. A quadratic polynomial, whose zeroes are -3 and 4, is:

(A) $x^2 - x + 12$ (B) $x^2 + x + 12$

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$ (D) $2x^2 + 2x - 24$

[CBSE 2020, 15, 13, 12]

Ans. (C)

Explanation: A quadratic polynomial in x is given by

$p(x) = k\{x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})\}$,

where k is a constant.

Sum of the zeroes $= -3 + 4 = 1$

and product of the zeroes $= (-3) \times 4 = -12$.

The required polynomial, therefore, is

$p(x) = k\{x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})\}$,

$= k\{x^2 - (1)x + (-12)\}$

$= k\{x^2 - x - 12\}$

$= \frac{x^2}{2} - \frac{x}{2} - 6$ $\left\{\text{Taking } k = \frac{1}{2}\right\}$

Hence, the quadratic polynomial, whose zeroes are -3 and 4 is $\frac{x^2}{2} - \frac{x}{2} - 6$.

3. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then:

(A) $a = -7, b = -1$ (B) $a = 5, b = -1$

(C) $a = 2, b = -6$ (D) $a = 0, b = -6$

[CBSE 2016]

Ans. (D)

Explanation: Let $p(x) = x^2 + (a+1)x + b$.

We know that if α is one of the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then $p(\alpha) = 0$.

It is given that 2 and -3 are the zeroes of the given quadratic polynomial.

Therefore, $p(2) = 0$ and $p(-3) = 0$

$p(2) = (2)^2 + (a+1)(2) + b = 0$

$\Rightarrow 4 + 2a + 2 + b = 0$

$\Rightarrow 2a + b + 6 = 0$ ...(i)

Also, $p(-3) = (-3)^2 + (a+1)(-3) + b = 0$

$\Rightarrow 9 - 3a - 3 + b = 0$

$\Rightarrow -3a + b + 6 = 0$ ...(ii)

From (i) and (ii), we get

$2a + b + 6 = -3a + b + 6$

$\Rightarrow 5a = 0$

$\Rightarrow a = 0$

Putting the value of 'a' in (i), we have

$2(0) + b + 6 = 0$

$\Rightarrow b = -6$

Hence, if the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then the required values of a and b are $a = 0$ and $b = -6$.

Alternate Method:

It is given that 2 and -3 are the zeroes of the given quadratic polynomial.

Sum of the zeroes = 2 + (–3) = –1 ...(i)

Product of the zeroes = 2(–3) = –6 ...(ii)

The equation of a quadratic polynomial is given by

$p(x) = k\{x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})\}$,

where k is a constant.

Let $p(x) = x^2 + (a + 1)x + b$.

Comparing the two equations we get:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$\Rightarrow$ sum of the zeroes = $-(a + 1)$

$\Rightarrow$ $-1 = -a - 1$ [Using (i)]

$\Rightarrow$ $-1 + 1 = -a$

$\Rightarrow$ $-a = 0$

$\Rightarrow$ $a = 0$

Product of the zeroes = constant term ÷ coefficient of x^2

$\Rightarrow$ Product of the zeroes = b

$-6 = b$ [Using (ii)]

$\Rightarrow$ $b = -6$

Hence, if the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then the required values of a and b are $a = 0$ and $b = -6$.

4. The number of polynomials having zeroes as –2 and 5 is:

(A) 1 (B) 2

(C) 3 (D) more than 3

Ans. (D)

Explanation: A quadratic polynomial is given by

$p(x) = k\{x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})\}$,

where k is a constant.

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

and product of the zeroes = constant term ÷ coefficient of x^2

Sum of the zeroes = – 2 + 5 = 3

and product of the zeroes = (–2)5 = –10

A quadratic polynomial is given by

$= k\{x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})\}$

which becomes,

$= k\{x^2 - 3x - 10\}$

where k is any real number.

Thus, we can say that $kx^2 - 3kx - 10k$ will also have –2 and 5 as their zeroes.

As k can take any real value, there can be infinite polynomials having –2 and 5 as their zeroes.

Hence, the required number of polynomials are infinite i.e., more than 3.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is:

(A) $-\frac{c}{a}$ (B) $\frac{c}{a}$

(C) 0 (D) $-\frac{b}{a}$ **[CBSE 2012]**

Ans. (B)

Explanation: Let $p(x) = ax^3 + bx^2 + cx + d$.

It is given that one of the zeroes of the cubic polynomial $p(x)$ is zero.

Let α, β and γ be the zeroes of the polynomial

$p(x) = ax^3 + bx^2 + cx + d$

And let $\alpha = 0$ [Given]

We know that:

Sum of the product of two zeroes at a time = coefficient of x ÷ coefficient of x^3 i.e.,

Sum of the product of two zeroes at a time = $\frac{c}{a}$

$\Rightarrow$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\Rightarrow$ $0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a}$ [Using $\alpha = 0$]

$\Rightarrow$ $0 + \beta\gamma + 0 = \frac{c}{a}$

$\Rightarrow$ $\beta\gamma = \frac{c}{a}$

Hence, product of the other two zeroes is $\frac{c}{a}$.

6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is:

(A) $b - a + 1$ (B) $b - a - 1$

(C) $a - b + 1$ (D) $a - b - 1$

[CBSE 2012]

Ans. (A)

Explanation: Let $p(x) = x^3 + ax^2 + bx + c$

let α, β and γ be the zeroes of the polynomial

$p(x) = x^3 + ax^2 + bx + c$ and

And $\alpha = -1$ [Given]

We know that if α is one of the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then $p(\alpha) = 0$.

$\Rightarrow$ $p(\alpha) = p(-1) = 0$

$\Rightarrow$ $(-1)^3 + (-1)^2a + (-1)b + c = 0$

$\Rightarrow$ $-1 + a - b + c = 0$

$\Rightarrow$ $c = 1 - a + b$ ...(i)

We know that:

Product of the zeroes = – constant term ÷ coefficient of x^3

i.e., Product of zeroes = $-\frac{c}{1}$

$\Rightarrow$ $\alpha\beta\gamma = -c$

$\Rightarrow$ $(-1)\beta\gamma = -c$ [Using $\alpha = -1$]

$\Rightarrow$ $\beta\gamma = c$

$\Rightarrow$ $\beta\gamma = 1 - a + b$

[From equation (i)]

Hence, product of the other two zeroes is $1 - a + b$ or $b - a + 1$.

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are:

(A) both positive

(B) both negative

(C) one positive and one negative

(D) both equal

Ans. (B)

Explanation: Let $p(x) = x^2 + 99x + 127$

Then zeroes of the polynomial are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-99 \pm \sqrt{(99)^2 - 4(127)}}{2}$$

$$x = \frac{-99 \pm \sqrt{9801 - 508}}{2}$$

$$x = \frac{-99 \pm 96.4}{2}$$

$$x = \left(-\frac{2.6}{2}, -\frac{195.4}{2}\right)$$

$$x = (-1.3, -97.7)$$

Both the zeroes are negative.

Hence, the zeroes are both negative.

8. The zeroes of the quadratic polynomial $x^2 + kx + k$, where $k \neq 0$,

(A) cannot both be positive

(B) cannot both be negative

(C) are always unequal

(D) are always equal

Ans. (A)

Explanation: Let $p(x) = x^2 + kx + k$ where $k \neq 0$.

On comparing $p(x)$ with $ax^2 + bx + c$, we get

$a = 1$, $b = k$ and $c = k$.

Let α and β be the zeroes of the polynomial $p(x)$.

We know that:

Sum of the zeroes

$$\alpha + \beta = -\frac{b}{a}$$

$$\Rightarrow \quad \alpha + \beta = -\frac{k}{1} = -k \qquad \text{...(i)}$$

And product of the zeroes

$$\alpha\beta = \frac{c}{a}$$

$$\Rightarrow \quad \alpha\beta = \frac{k}{1} = k \qquad \text{...(ii)}$$

Case 1: *k* is negative

If k is negative,

$\alpha\beta$ [from equation (ii)] is negative.

It means α and β are of the opposite sign.

$\Rightarrow$ Both the zeroes are of the opposite signs.

Case 2: *k* is positive

If k is positive,

$\alpha\beta$ (from equation (ii)) is positive but $\alpha + \beta$ is negative.

If the product of the two numbers is positive, then either both are negative or both are positive. But the sum of these numbers is negative, so the numbers must be negative.

$\Rightarrow$ Both the zeroes are negative.

Hence, in both the cases, both the zeroes cannot be positive.

Alternate Solution:

Let $p(x) = x^2 + kx + k$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-k \pm \sqrt{k^2 - 4k}}{2}$$

$$x = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

$\Rightarrow$ $k(k - 4) > 0$

$\Rightarrow$ $k \in (-\infty, 0) \cup (4, \infty)$

Here, k lies in two intervals; therefore, we need to consider both the intervals separately.

Case 1:

When $k \in (-\infty, 0)$

i.e., $k < 0$

We know that in a quadratic equation

$$p(x) = ax^2 + bx + c,$$

if either $a > 0$, $c < 0$ or $a < 0$, $c > 0$, then the zeroes of the polynomial are of the opposite signs.

Here, $a = 1 > 0$, $b = k < 0$ and $c = k < 0$.

$\Rightarrow$ Both the zeroes are of the opposite signs.

Case 2:

When $k \in (4, \infty)$

i.e., $k > 0$

We know, in a quadratic polynomial, if the coefficient of the terms are of the same sign, then the zeroes of the polynomial are negative.

i.e., if either $a > 0$, $b > 0$ and $c > 0$ or $a < 0$, $b < 0$ and $c < 0$, then both the zeroes are negative.

Here, $a = 1 > 0$, $b = k > 0$ and $c = k > 0$.

$\Rightarrow$ Both the zeroes are negative.

Hence, in both cases, both the zeroes cannot be positive.

9. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$ are equal, then:

(A) *c* and *a* have opposite signs

(B) *c* and *b* have opposite signs

(C) *c* and *a* have the same sign

(D) *c* and *b* have the same sign

Ans. (C)

Explanation: Given that the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, where $c \neq 0$, are equal.

The zeroes of a quadratic polynomial are equal when the discriminant is equal to 0

i.e., $D = 0$

$b^2 - 4ac = 0$

$\Rightarrow \quad 4ac = b^2$

$\Rightarrow \quad ac = \frac{b^2}{4} > 0$ [square of any positive or negative number is positive]

$\Rightarrow \quad ac > 0$

Therefore, for $ac > 0$, a and c must have the same sign

i.e., either $a > 0$ and $c > 0$ or $a < 0$ and $c < 0$.

Alternate Solution:

Given that the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, where $c \neq 0$, are equal.

Let α and β be the zeroes of the polynomial $p(x)$.

If α and β are equal, these must have the same sign (both positive or both negative).

$\Rightarrow \quad \alpha\beta > 0$

Product of zeroes

$\alpha\beta = \frac{c}{a}$

$\Rightarrow \quad \frac{c}{a} > 0$ [Using $\alpha\beta > 0$]

As $\frac{c}{a} > 0$, which is only possible when a and c have the same signs, so they α and β have the same sign.

10. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it:

(A) **has no linear term and the constant term is negative.**

(B) **has no linear term and the constant term is positive.**

(C) **can have a linear term but the constant term is negative.**

(D) **can have a linear term but the constant term is positive.**

Ans. (A)

Explanation: Let

$p(x) = x^2 + ax + b$.

And let α be one of the zeroes, and

$-\alpha$ is the other zero of the polynomial $p(x)$.

[Given]

Product of the zeroes = constant term ÷ coefficient of x^2

Product of the zeroes $= \frac{b}{1}$

$\alpha(-\alpha) = b$

$-\alpha^2 = b$ i.e., $b < 0$

That is, the constant term is negative.

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

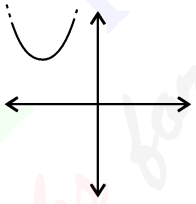
$\alpha - \alpha = -\frac{a}{1}$

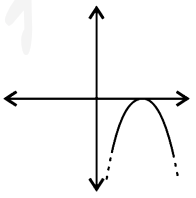
$0 = -a$

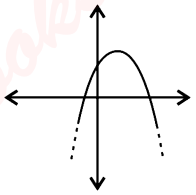
$\Rightarrow \quad a = 0$

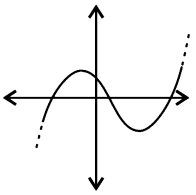
Hence, it has no linear term and the constant term is negative.

11. Which of the following is not the graph of a quadratic polynomial?

(A) 

(B) 

(C) 

(D) 

Ans. (D)

Explanation: From the given options only option D has more than two roots so it cannot be graph of quadratic polynomial.

For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding polynomial $ax^2 + bx + c$, has one of the two shapes: either open upwards like $\cup$ (parabolic shape) or open downwards like $\cap$ (parabolic shape), depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (D) cannot be possible.

Alternate Solution:

Also, the curve of a quadratic polynomial crosses the x-axis at most two points but in option (D), the curve crosses the x–axis at three points, so it does not represent a quadratic polynomial.

Hence, (D) is not the graph of a quadratic polynomial.

EXERCISE 2.2

1. Answer the following and justify:

(i) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Ans.

(i) No, the quadratic polynomial $x^2 + kx + k$ cannot have equal zeroes for some odd integer $k > 1$.

Explanation: Let $p(x) = x^2 + kx + k$.

If $p(x)$ has equal zeroes, then it's discriminant should be zero.

$$D = b^2 - 4ac = 0 \quad \text{...(i)}$$

On comparing $p(x)$ with $ax^2 + bx + c$, we get

$a = 1$, $b = k$ and $c = k$

$\Rightarrow (k)^2 - 4(1)(k) = 0$ [Using equation (i)]

$\Rightarrow k(k - 4) = 0$

$\Rightarrow k = 0, 4$

So, the quadratic polynomial $p(x)$ has equal zeroes only at $k = 0, 4$.

But $k > 1$, so $k = 4$ which is even (not odd number), so the given statement is not correct.

2. Are the following statements true or false? Justify your answers.

(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

(ii) If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.

(iii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coeffcients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non–negative.

(vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $\frac{1}{2}$.

Ans.

(i) False.

Explanation: The given quadratic polynomial is $ax^2 + bx + c$.

Let α, β be the zeroes of the polynomial

$$p(x) = ax^2 + bx + c$$

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

It is given that $\alpha > 0$, $\beta > 0$

$\Rightarrow \alpha + \beta > 0$

$\Rightarrow \alpha + \beta = -\frac{b}{a} > 0$

For $-\frac{b}{a} > 0$, b and a must have opposite signs.

Product of the zeroes = constant term ÷ coefficient of x^2

It is given that $\alpha > 0$, $\beta > 0$

$\Rightarrow \alpha\beta > 0$

$\Rightarrow \alpha\beta = \frac{c}{a} > 0$

For $\frac{c}{a} > 0$, c and a must have same signs.

Case 1: when $a > 0$

$\Rightarrow a > 0$, $b < 0$ and $c > 0$

Case 2: when $a < 0$

$\Rightarrow a < 0$, $b > 0$ and $c < 0$

Hence, the coefficients have different signs.

(ii) False.

Explanation: The quadratic polynomial cuts the x-axis at most at two points i.e., it can either touch the x-axis at one point or intersect at two points or doesn't touch the axis at all.

When two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.

Hence, the polynomial intersecting the x-axis at only one point can be a quadratic polynomial.

(iii) True.

Explanation: If the graph of a polynomial intersects the x-axis at exactly two points, then it may or may not be a quadratic polynomial. If a polynomial of a degree more than two has two real roots and the other roots are imaginary, then the graph of the polynomial will intersect at two points on the x-axis.

For example, $x^4 - 1$ is a polynomial intersecting the x-axis at exactly two points and it is not a quadratic polynomial.

(iv) True.

Explanation: Let α, β and γ be the zeroes of the polynomial $p(x) = ax^3 + bx^2 + cx + d$, where it is given that

$$\alpha = \beta = 0$$

Clearly $a \neq 0$

Product of all zeroes = – (constant term) ÷ coefficient of x^3

$$\alpha\beta\gamma = -\frac{d}{a}$$

$$\Rightarrow \quad 0 = -\frac{d}{a}$$

$$\Rightarrow \quad d = 0 \text{ [no constant term]}$$

Therefore, the polynomial is $p(x) = ax^3 + bx^2 + cx$ as $d = 0$.

Sum of the products of two zeroes at a time = coefficient of x ÷ coefficient of x^3

$$\Rightarrow \quad \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\Rightarrow \quad 0 + (0)\gamma + (0)\gamma = \frac{c}{a}$$

$$\Rightarrow \quad 0 = \frac{c}{a}$$

$$\Rightarrow \quad c = 0 \quad \text{[no linear term]}$$

Therefore, the polynomial is $p(x) = ax^3 + bx^2$.

Hence, the polynomial does not have a linear and a constant term.

Alternate method:

Let α, β and γ be the zeroes of the cubic polynomial.

It is given that two of the zeroes have value 0.

Let $\beta = \gamma = 0$

and $p(x) = (x - \alpha)(x - \beta)(x - \gamma)$

$= (x - \alpha)(x - 0)(x - 0)$

$= x^2(x - \alpha)$

$= x^3 - \alpha x^2$

which clearly does not have a linear and constant term.

(v) True.

Explanation: Let α, β and γ be the zeroes of the polynomial

$p(x) = ax^3 + bx^2 + cx + d$,

where $\alpha, \beta, \gamma < 0$.

Product of all zeroes = – (constant term) ÷ coefficient of x^3

As $\alpha, \beta, \gamma < 0$

$\Rightarrow \quad \alpha\beta\gamma < 0$

$$\alpha\beta\gamma = -\frac{d}{a} < 0$$

$$\Rightarrow \quad \frac{d}{a} > 0$$

$\Rightarrow d$ and a have the same signs.

Sum of the products of two zeroes at a time = coefficient of x ÷ coefficient of x^3

As $\alpha, \beta, \gamma < 0$

$\Rightarrow \alpha\beta > 0, \beta\gamma > 0$ and $\alpha\gamma > 0$

$\Rightarrow \quad \alpha\beta + \beta\gamma + \alpha\gamma > 0$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} > 0$$

$\Rightarrow c$ and a have the same signs.

Sum of the zeroes = – (coefficient of x^2) ÷ coefficient of x^3

As $\alpha, \beta, \gamma < 0$

$\Rightarrow \quad \alpha + \beta + \gamma < 0$

$$\alpha + \beta + \gamma = -\frac{b}{a} < 0$$

$$\Rightarrow \quad \frac{b}{a} > 0$$

$\Rightarrow b$ and a have the same sign.

$\Rightarrow a, b, c$ and d have the same sign.

(vi) False.

Explanation: Let α, β and γ be the zeroes of the polynomial $p(x) = x^3 + ax^2 - bx + c$, where α, β, γ are positive i.e. $\alpha, \beta, \gamma > 0$.

Product of all zeroes = – (constant term) ÷ coefficient of x^3

As $\alpha, \beta, \gamma > 0$

$\Rightarrow \quad \alpha\beta\gamma > 0$

But $\alpha\beta\gamma = -\frac{c}{1}$

$\Rightarrow \quad -c > 0$

$\Rightarrow \quad c < 0$

Sum of the products of two zeroes at a time = coefficient of x ÷ coefficient of x^3

$$= -\frac{b}{1}$$

It is given that

$\alpha, \beta, \gamma > 0$

$\Rightarrow \alpha\beta > 0, \beta\gamma > 0$ and $\alpha\gamma > 0$

$\Rightarrow \quad \alpha\beta + \beta\gamma + \alpha\gamma > 0$

$\alpha\beta + \beta\gamma + \alpha\gamma = -b > 0$

$\Rightarrow \quad b < 0$

Sum of the zeroes = – (coefficient of x^2) ÷ coefficient of $x^3 = -a$

$\alpha, \beta, \gamma > 0$

$\Rightarrow \quad \alpha + \beta + \gamma > 0$

$\alpha + \beta + \gamma = -a > 0$

$\Rightarrow \quad a < 0$

$\Rightarrow$ All the coeffcients a, b and c are negative.

Therefore, the given statement i.e., at least one of a, b and c is non-negative, is false.

(vii) False.

Explanation: A quadratic polynomial $kx^2 + x + k$ has equal roots when its discriminant is equal to zero.

i.e., $D = 0$

$b^2 - 4ac = 0$

$b^2 = 4ac$

$(1)^2 = 4(k)(k)$

$1 = 4k^2$

$k^2 = \frac{1}{4}$

$k = \pm\frac{1}{2}$

Hence, $k = \frac{1}{2}$ is not the only value for which the polynomial has equal roots.

EXERCISE 2.3

Find the zeroes of the following polynomials by the factorisation method and verify the relations between the zeroes and the coefficients of the polynomials: [CBSE 2020, 12]

1. $4x^2 - 3x - 1$

Ans.

Let $f(x) = 4x^2 - 3x - 1$

$= 4x^2 - (4x - x) - 1$

$= 4x^2 - 4x + x - 1$

$= 4x(x - 1) + 1(x - 1)$

$= (x - 1)(4x + 1)$

The zeroes of $f(x)$ are given by $f(x) = 0$

So, the value of $4x^2 - 3x - 1$ is zero when $x - 1 = 0$ or $4x + 1 = 0$

i.e., when $x = 1$ or $x = -\frac{1}{4}$

$\Rightarrow x = 1, -\frac{1}{4}$

So, the zeroes of $4x^2 - 3x - 1$ are 1 and $-\frac{1}{4}$.

Verification:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$\alpha + \beta = -\frac{b}{a}$

$\Rightarrow 1 - \frac{1}{4} = -\frac{-3}{4}$

$\Rightarrow \frac{3}{4} = \frac{3}{4}$

Product of the zeroes = constant term ÷ coefficient of x^2

$\alpha\beta = \frac{c}{a}$

$\Rightarrow 1\left(-\frac{1}{4}\right) = -\frac{1}{4}$

$\Rightarrow -\frac{1}{4} = -\frac{1}{4}$

Hence, verified.

2. $3x^2 + 4x - 4$

Ans.

Let $f(x) = 3x^2 + 4x - 4$

$= 3x^2 + (6x - 2x) - 4$

$= 3x^2 + 6x - 2x - 4$

$= 3x(x + 2) - 2(x + 2)$

$= (x + 2)(3x - 2)$

The zeroes of $f(x)$ are given by $f(x) = 0$

So, the value of $x^2 + 4x - 4$ is zero when $x + 2 = 0$ or $3x - 2 = 0$

i.e., when $x = -2$ or $x = \frac{2}{3}$

$\Rightarrow x = -2, \frac{2}{3}$

Verification:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$\alpha + \beta = -\frac{b}{a}$

$-2 + \frac{2}{3} = -\frac{4}{3}$

$-\frac{4}{3} = -\frac{4}{3}$

Product of the zeroes = constant term ÷ coefficient of x^2

$\alpha\beta = \frac{c}{a}$

Product of the zeroes = $(-2)\left(\frac{2}{3}\right) = -\frac{4}{3}$

Hence, verified.

3. $5t^2 + 12t + 7$

Ans.

Let $f(t) = 5t^2 + 12t + 7$

$= 5t^2 + (5t + 7t) + 7$

$= 5t^2 + 5t + 7t + 7$

$= 5t(t + 1) + 7(t + 1)$

$= (t + 1)(5t + 7)$

$= (t + 1)(5t + 7)$

The zeroes of $f(t)$ are given by $f(t) = 0$.

So, the value of $5t^2 + 12t + 7$ is zero when $t + 1 = 0$ or $5t + 7 = 0$

i.e., when $t = -1$ or $t = -\frac{7}{5}$

$\Rightarrow t = -1$ or $-\frac{7}{5}$

Verification:

Sum of the zeroes = – (coefficient of t) ÷ coefficient of t^2

$\alpha + \beta = -\frac{b}{a}$

$(-1) + \left(-\frac{7}{5}\right) = -\frac{12}{5}$

$-\frac{12}{5} = -\frac{12}{5}$

Product of the zeroes = constant term ÷ coefficient of t^2

$\alpha\beta = \frac{c}{a}$

$(-1)\left(-\frac{7}{5}\right) = +\frac{7}{5}$

$+\frac{7}{5} = +\frac{7}{5}$

Hence, verified.

4. $t^3 - 2t^2 - 15t$

Ans.

Let $f(t) = t^3 - 2t^2 - 15t$

Taking t common

$t(t^2 - 2t - 15) = t\{t^2 - (-3t + 5t) - 15\}$

$= t(t^2 + 3t - 5t - 15)$

$= t\{t(t + 3) - 5(t + 3)\}$

$= t(t + 3)(t - 5)$

The zeroes of $f(t)$ are given by $f(t) = 0$.

So, the value of $t^3 - 2t^2 - 15t$ is zero when $t = 0$ or $t + 3 = 0$ or $t - 5 = 0$

$\Rightarrow \quad t = -3, 0, 5$

Verification:

Sum of the zeroes = – (coefficient of t^2) ÷ coefficient of t^3

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$(0) + (-3) + (5) = -\left(-\frac{2}{1}\right)$$

$\Rightarrow \quad 2 = 2$

Sum of the products of two zeroes at a time = coefficient of t ÷ coefficient of t^3

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$(0)(-3) + (-3)(5) + (0)(5) = -\frac{15}{1}$$

$\Rightarrow \quad -15 = -15$

Product of all the zeroes = – (constant term) ÷ coefficient of t^3

$$\alpha\beta\gamma = -\frac{d}{a}$$

$$(0)(-3)(5) = 0$$

$$0 = 0$$

Hence, verified.

5. $2x^2 + \frac{7}{2}x + \frac{3}{4}$

Ans.

Let $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$

$= 8x^2 + 14x + 3$

(Multiplying the given equation by 4)

$= 8x^2 + (12x + 2x) + 3$

$= 8x^2 + 12x + 2x + 3$

$= 4x(2x + 3) + 1(2x + 3)$

$= (2x + 3)(4x + 1)$

The zeroes of $f(x)$ are given by $f(x) = 0$.

So, the value of $2x^2 + \frac{7}{2}x + \frac{3}{4}$ is zero when $x = -\frac{3}{2}$ or $x = -\frac{1}{4}$

$\Rightarrow \quad x = -\frac{3}{2}, -\frac{1}{4}$

Verification:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$$\alpha + \beta = -\frac{b}{a}$$

$$\left(-\frac{3}{2}\right) + \left(-\frac{1}{4}\right) = -\frac{7}{4}$$

$$-\frac{7}{4} = -\frac{7}{4}$$

Product of the zeroes = constant term ÷ coefficient of x^2

$$\alpha\beta = \frac{c}{a}$$

$$\left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right) = \frac{3}{4} \div \frac{2}{1}$$

$$\frac{3}{8} = \frac{3}{8}$$

Hence, verified.

6. $4x^2 + 5\sqrt{2}x - 3$

Ans.

Let $f(x) = 4x^2 + 5\sqrt{2}x - 3$

$= 4x^2 + (6\sqrt{2}x - \sqrt{2}x) - 3$

$= 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3)$

$= (\sqrt{2}x + 3)(2\sqrt{2} - 1)$

The zeroes of $f(x)$ are given by $f(x) = 0$.

So, the value is zero when $4x^2 + 5\sqrt{2}x - 3 = 0$, when $x = -\frac{3}{\sqrt{2}}$ or $\frac{1}{2\sqrt{2}}$.

Verification:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$$\alpha + \beta = -\frac{b}{a}$$

$$\left(-\frac{3}{\sqrt{2}}\right) + \left(+\frac{1}{2\sqrt{2}}\right) = \frac{-5\sqrt{2}}{4}$$

$$\frac{-5}{2\sqrt{2}} = \frac{-5}{2\sqrt{2}}$$

Product of the zeroes = constant term ÷ coefficient of x^2

$$\alpha\beta = \frac{c}{a}$$

$$\left(-\frac{3}{\sqrt{2}}\right) \times \left(+\frac{1}{2\sqrt{2}}\right) = -\frac{3}{4}$$

$$-\frac{3}{4} = -\frac{3}{4}$$

Hence, verified.

7. $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

Ans.

Let $f(s) = 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

$= 2s^2 - s - 2\sqrt{2}s + \sqrt{2}$

$= s(2s - 1) - \sqrt{2}(2s - 1)$

$= (2s - 1)(s - \sqrt{2})$

The zeroes of $f(s)$ are given by $f(s) = 0$

So, the value is zero when $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2} = 0$

i.e., when $s = \frac{1}{2}$ or $\sqrt{2}$

$\Rightarrow \quad s = \frac{1}{2}, \sqrt{2}$

Verification:

Sum of the zeroes = – (coefficient of s) ÷ coefficient of s^2

$$\alpha + \beta = -\frac{b}{a}$$

$$\frac{1}{2} + \sqrt{2} = -\frac{-(1+2\sqrt{2})}{2}$$

$$+\frac{(1+2\sqrt{2})}{2} = +\frac{(1+2\sqrt{2})}{2}$$

Product of the zeroes = constant term ÷ coefficient of s^2

$$\alpha\beta = \frac{c}{a}$$

$$\frac{1}{2} \times \sqrt{2} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Hence, verified.

8. $v^2 + 4\sqrt{3}v - 15$

Ans.

Let $f(v) = v^2 + 4\sqrt{3}v - 15$

$= v^2 + (5\sqrt{3}v - \sqrt{3}v) - 15$

$= v^2 + 5\sqrt{3}v - \sqrt{3}v - 15$

$= v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3})$

$= (v + 5\sqrt{3})(v - \sqrt{3})$

The zeroes of $f(v)$ are given by $f(v) = 0$.

So, the value is zero when $v^2 + 4\sqrt{3}v - 15 =$ zero

i.e., when $v = -5\sqrt{3}$ or $v = \sqrt{3}$

$\Rightarrow \quad v = -5\sqrt{3}, \sqrt{3}$

Verification:

Sum of the zeroes = – (coefficient of v) ÷ coefficient of v^2

$$\alpha + \beta = -\frac{b}{a}$$

$$-5\sqrt{3} + \sqrt{3} = -4\sqrt{3}$$

$$-4\sqrt{3} = -4\sqrt{3}$$

Product of the zeroes = constant term ÷ coefficient of v^2

$$\alpha\beta = \frac{c}{a}$$

$$(-5\sqrt{3})(\sqrt{3}) = -15$$

$$-15 = -15$$

Hence, verified.

9. $y^2 + \frac{3\sqrt{5}}{2}y - 5$

Ans.

Let $f(y) = y^2 + \frac{3\sqrt{5}}{2}y - 5$

$= 2y^2 + 3\sqrt{5}y - 10$

$= 2y^2 + (4\sqrt{5}y - \sqrt{5}y) - 10$

$= 2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5})$

$= (y + 2\sqrt{5})(2y - \sqrt{5})$

The zeroes of $f(y)$ are given by $f(y) = 0$

So, the value of $y^2 + \frac{3\sqrt{5}}{2}y - 5$ is zero when

$$y = -2\sqrt{5} \text{ or } y = \frac{\sqrt{5}}{2}$$

$$\Rightarrow \quad y = -2\sqrt{5}, \frac{\sqrt{5}}{2}$$

Verification:

Sum of the zeroes = – (coefficient of y) ÷ coefficient of y^2

$$\alpha + \beta = -\frac{b}{a}$$

$$-2\sqrt{5} + \frac{\sqrt{5}}{2} = -\frac{3\sqrt{5}}{2}$$

$$-\frac{3\sqrt{5}}{2} = -\frac{3\sqrt{5}}{2}$$

Product of the zeroes = constant term ÷ coefficient of y^2

$$\alpha\beta = \frac{c}{a}$$

$$(-2\sqrt{5})\left(\frac{\sqrt{5}}{2}\right) = -5$$

$$-5 = -5$$

Hence, verified.

10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Ans.

Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= 21y^2 - 11y - 2$

$= 21y^2 + (3y - 14y) - 2$

$= 21y^2 + 3y - 14y - 2$

$= 3y(7y + 1) - 2(7y + 1)$

$= (7y + 1)(3y - 2)$

The zeroes of $f(y)$ are given by $f(y) = 0$

So, the value of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ is zero when

$y = -\frac{1}{7}$ or $y = \frac{2}{3}$

$\Rightarrow y = -\frac{1}{7}, \frac{2}{3}$

Verification:

Sum of the zeroes = – (coefficient of y) ÷ coefficient of y^2

$$\alpha + \beta = -\frac{b}{a}$$

$$-\frac{1}{7} + \frac{2}{3} = -\frac{-11}{21}$$

$$\frac{11}{21} = \frac{11}{21}$$

Product of the zeroes = constant term ÷ coefficient of y^2

$$\alpha\beta = \frac{c}{a}$$

$$\left(-\frac{1}{7}\right)\left(\frac{2}{3}\right) = \frac{-\frac{2}{3}}{7}$$

$$-\frac{2}{21} = -\frac{2}{21}$$

Hence, verified.

EXERCISE 2.4

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorization. [CBSE 2016, 10, 09]

(i) $-\frac{8}{3}, \frac{4}{3}$ **(ii)** $\frac{21}{8}, \frac{5}{16}$

(iii) $-2\sqrt{3}, -9$ **(iv)** $-\frac{3}{2\sqrt{5}}, -\frac{1}{2}$

Ans.

(i) It is given that:

Sum of the zeroes $= -\frac{8}{3}$

Product of the zeroes $= \frac{4}{3}$

Required polynomial

$$p(x) = x^2 - (\text{sum of the zeroes}) + (\text{product of the zeroes})$$
$$= x^2 + \frac{8x}{3} + \frac{4}{3}$$
$$= 3x^2 + 8x + 4$$

Using factorization method:

By splitting the middle term

$3x^2 + 8x + 4$

$$= 3x^2 + (6x + 2x) + 4$$
$$= 3x^2 + 6x + 2x + 4$$
$$= 3x(x + 2) + 2(x + 2)$$
$$= (x + 2)(3x + 2)$$

$\Rightarrow \quad x = -2, -\frac{2}{3}$

Hence, the zeroes of $p(x)$ are -2 and $-\frac{2}{3}$.

(ii) Given that:

Sum of the zeroes $= \frac{21}{8}$

Product of the zeroes $= \frac{5}{16}$

Required polynomial

$$p(x) = x^2 - (\text{sum of the zeroes}) + (\text{product of the zeroes})$$
$$= x^2 - \frac{21x}{8} + \frac{5}{16}$$
$$= \frac{1}{16}(16x^2 - 42x + 5)$$

Using factorization method:

By splitting the middle term

$$= \frac{1}{16}(16x^2 - 42x + 5)$$
$$= \frac{1}{16}\{16x^2 - (40x + 2x) + 5\}$$
$$= \frac{1}{16}\{16x^2 - 40x - 2x + 5\}$$
$$= \frac{1}{16}\{8x(2x - 5) - 1(2x - 5)\}$$
$$= \frac{1}{16}\{(2x - 5)(8x - 1)\}$$

$\Rightarrow \quad x = \frac{1}{8}, \frac{5}{2}$

Hence, the zeroes of $p(x)$ are $\frac{1}{8}$ and $\frac{5}{2}$.

(iii) Given that:

Sum of the zeroes $= -2\sqrt{3}$

Product of the zeroes $= -9$

Required polynomial

$$p(x) = x^2 - (\text{sum of the zeroes}) + (\text{product of the zeroes})$$
$$= x^2 + 2\sqrt{3}x - 9$$

Using factorization method:

By splitting the middle term

$$= x^2 + 2\sqrt{3}x - 9$$
$$= x^2 + 3\sqrt{3}x - \sqrt{3}x - 9$$
$$= x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3})$$
$$= (x + 3\sqrt{3})(x - \sqrt{3})$$

$\Rightarrow \quad x = -3\sqrt{3}, \sqrt{3}$

Hence, the zeroes of $p(x)$ are $-3\sqrt{3}, \sqrt{3}$.

(iv) Given that:

Sum of the zeroes $= -\frac{3}{2\sqrt{5}}$

Product of the zeroes $= -\frac{1}{2}$

Required polynomial

$$p(x) = x^2 - (\text{sum of the zeroes}) + (\text{product of the zeroes})$$
$$= x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$$
$$= \frac{1}{2\sqrt{5}}\{2\sqrt{5}x^2 + 3x - \sqrt{5}\}$$

Using factorization method,

By splitting the middle term

$$= \frac{1}{2\sqrt{5}}\{2\sqrt{5}x^2 + 3x - \sqrt{5}\}$$
$$= \frac{1}{2\sqrt{5}}\{2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}\}$$
$$= \frac{1}{2\sqrt{5}}\{\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5})\}$$
$$= \frac{1}{2\sqrt{5}}\{(2x + \sqrt{5})(\sqrt{5}x - 1)\}$$

$\Rightarrow \quad x = \frac{1}{\sqrt{5}}, -\frac{\sqrt{5}}{2}$

Hence, the zeroes of $p(x)$ are $\frac{1}{\sqrt{5}}, -\frac{\sqrt{5}}{2}$.

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

Ans.

Let $p(x) = x^3 - 6x^2 + 3x + 10$

and (a), $(a + b)$ and $(a + 2b)$ are the zeroes of $p(x)$.

We know:

Sum of the zeroes = –(coefficient of x^2) ÷ coefficient of x^3

$\Rightarrow a + (a + b) + (a + 2b) = -(-6)$

$\Rightarrow 3a + 3b = 6$

$\Rightarrow a + b = 2$

$\Rightarrow a = 2 - b$ ...(i)

Product of all the zeroes = –(constant term) ÷ coefficient of x^3

$a(a + b)(a + 2b) = -10$

$(2 - b)(2)(2 + b) = -10$ [Using eqn. (i)]

$(2 - b)(2 + b) = -5$

$4 - b^2 = -5$

$\Rightarrow b^2 = 9$

$\Rightarrow b = \pm 3$

When $b = 3$, $a = 2 - 3 = -1$ [Using equation (i)]

$\Rightarrow a = -1$ when $b = 3$.

When $b = -3$, $a = 2 - (-3) = 5$ [Using equation (i)]

$\Rightarrow a = 5$ when $b = -3$.

Case 1: when $a = -1$ and $b = 3$

The zeroes of the polynomial are:

$a = -1$

$a + b = -1 + 3 = 2$

$a + 2b = -1 + 2(3) = 5$

$\Rightarrow$ –1, 2 and 5 are the zeroes.

Case 2: when $a = 5$ and $b = -3$

The zeroes of the polynomial are:

$a = 5$

$a + b = 5 - 3 = 2$

$a + 2b = 5 - 2(3) = -1$

$\Rightarrow$ –1, 2 and 5 are the zeroes.

In both the cases, the zeroes of the polynomial are –1, 2, 5.

3. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes. [CBSE 2010]

Ans.

Let $p(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

As $\sqrt{2}$ is one of the zeroes of $p(x)$.

$\Rightarrow g(x) = (x - \sqrt{2})$ is one of the factors of $p(x)$.

$$
\begin{array}{l}
(x-\sqrt{2}) \overline{)\ 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}\ (\ 6x^2 + 7\sqrt{2}x + 4} \\
\quad 6x^3 - 6\sqrt{2}x^2 \\
\quad - \quad + \\
\hline
\qquad 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
\qquad 7\sqrt{2}x^2 - 14x \\
\qquad - \quad + \\
\hline
\qquad\qquad 4x - 4\sqrt{2} \\
\qquad\qquad 4x - 4\sqrt{2} \\
\qquad\qquad - \quad + \\
\hline
\qquad\qquad 0
\end{array}
$$

By division algorithm

Dividend = (divisor)(quotient) + remainder

i.e., $p(x) = g(x)q(x) + r(x)$

Clearly $r(x) = 0$ and $q(x) = 6x^2 + 7\sqrt{2}x + 4$

$\Rightarrow 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

$= (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4)$

$= (x - \sqrt{2})\{6x^2 + (3\sqrt{2}x + 4\sqrt{2}x) + 4\}$

(by splitting the middle term)

$= (x - \sqrt{2})\{6x^2 + 3\sqrt{2}x + 4\sqrt{2}x + 4\}$

$= (x - \sqrt{2})\{3\sqrt{2}x(\sqrt{2}x + 1) + 4(\sqrt{2}x + 1)\}$

$= (x - \sqrt{2})(\sqrt{2}x + 1)(3\sqrt{2}x + 4)$

$\Rightarrow x = \sqrt{2}, -\frac{1}{\sqrt{2}}$ or $-\frac{4}{3\sqrt{2}}$

Thus, the other two zeroes are $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ and $-\frac{4}{3\sqrt{2}}$ or $-\frac{2\sqrt{2}}{3}$.

4. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials. [CBSE 2015, 14, 12]

Ans.

Let $g(x) = x^2 + 2x + k$

and $p(x) = 2x^4 + x^3 - 14x^2 + 5x + 6$

Then, if $g(x)$ is a factor of $p(x)$

By division algorithm:

Dividend = (divisor) (quotient) + remainder

$\Rightarrow p(x) = g(x)q(x) + r(x)$,

where $q(x)$ is the quotient and $r(x)$ is the remainder which will be equal to zero.

i.e., $r(x) = 0$

$$
\begin{array}{l}
\qquad\qquad 2x^2 - 3x - 8 - 2k \\
x^2 + 2x + k \overline{)\ 2x^4 + x^3 - 14x^2 + 5x + 6} \\
\quad 2x^4 + 4x^3 + 2kx^2 \\
\quad - \quad - \quad - \\
\hline
\qquad -3x^3 - 14x^2 - 2kx^2 + 5x + 6 \\
\qquad -3x^3 - 6x^2 - 3kx \\
\qquad + \quad + \quad + \\
\hline
\qquad\quad -8x^2 - 2kx^2 + 5x + 3kx + 6 \\
\qquad\quad -8x^2 - 2kx^2 - 16x - 4kx - 8k - 2k^2 \\
\qquad\quad + \quad + \quad + \quad + \quad + \quad + \\
\hline
\qquad\qquad\qquad (21 + 7k)x + 2k^2 + 8k + 6
\end{array}
$$

Clearly, $q(x) = 2x^2 - 3x - 8 - 2k$

and $r(x) = (21 + 7k)x + (2k^2 + 8k + 6)$

Now, $r(x) = 0$

By comparing the coefficients,

$21 + 7k = 0$ and $2k^2 + 8k + 6 = 0$

$2k^2 + 8k + 6 = 0$

$\Rightarrow k^2 + 4k + 3 = 0$

$\Rightarrow k^2 + (k + 3k) + 3 = 0$

$\Rightarrow k(k + 1) + 3(k + 1) = 0$

$\Rightarrow \quad (k+1)(k+3) = 0$

$\Rightarrow \quad k = -1, -3$

Also, $\quad 21 + 7k = 0$

Case1: $k = -1$

$21 + 7(-1) = 0$

$\Rightarrow \quad 21 - 7 = 14 \neq 0$

Hence, $k = -1$ is rejected.

Case2: $k = -3$

$21 + 7(-3) = 0$

$\Rightarrow \quad 21 - 21 = 0$

Therefore, the value of k is -3.

$g(x) = x^2 + 2x - 3$

$x^2 + 2x - 3 = 0$

$x^2 + (-x + 3x) - 3 = 0$

$x(x-1) + 3(x-1) = 0$

$(x-1)(x+3) = 0$

$\Rightarrow \quad x = 1, -3$

$q(x) = 2x^2 - 3x - 8 - 2(-3) = 2x^2 - 3x - 2$

$2x^2 - 3x - 2 = 0$

$2x^2 - (4x - x) - 2 = 0$

$2x(x-2) + (x-2) = 0$

$(x-2)(2x+1) = 0$

$\Rightarrow \quad x = 2, -\frac{1}{2}$

Now, we know $g(x)$ and $q(x)$ are factors of $p(x)$.

Therefore, the zeroes of $g(x)$ and $q(x)$ will be the zeroes of $p(x)$.

Hence, the zeroes of $p(x) = -3, -\frac{1}{2}$, 1 and 2.

5. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial. [CBSE 2010]

Ans.

Let $\quad p(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$

As $\sqrt{5}$ is one of the zeroes of $p(x)$.

$\Rightarrow (x - \sqrt{5})$ is one of the factors of $p(x)$.

$$
\begin{array}{r}
x^2 - 2\sqrt{5}x + 3 \\
x - \sqrt{5} \overline{)\, x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}} \\
\underline{x^3 - \sqrt{5}x^2} \quad (-,+) \\
-2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
\underline{-2\sqrt{5}x^2 + 10x} \quad (+,-) \\
3x - 3\sqrt{5} \\
\underline{3x - 3\sqrt{5}} \quad (-,+) \\
0
\end{array}
$$

Dividend = (divisor) (quotient) + remainder

$\Rightarrow \quad p(x) = g(x)q(x) + r(x)$

$= (x - \sqrt{5})(x^2 - 2\sqrt{5}x + 3) + 0$

$x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$

$= (x - \sqrt{5})(x^2 - 2\sqrt{5}x + 3)$

$= (x - \sqrt{5})[x^2 - \{(\sqrt{5} + \sqrt{2})x + (\sqrt{5} - \sqrt{2})x\} + 3)$

$= (x - \sqrt{5})[x\{x - (\sqrt{5} + \sqrt{2})\} - (\sqrt{5} - \sqrt{2})\{x - (\sqrt{5} + \sqrt{2})\} + 3]$

$= (x - \sqrt{5})\{x - (\sqrt{5} + \sqrt{2})\}\{x - (\sqrt{5} - \sqrt{2})\}$

So, all the zeroes of the given polynomial are $(\sqrt{5} + \sqrt{2})$, $(\sqrt{5} - \sqrt{2})$ and $\sqrt{5}$.

6. For which values of a and b are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

Ans.

Let $\quad p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$

and $\quad q(x) = x^3 + 2x^2 + a$.

Since, the zeroes of the polynomial $q(x)$ are also zeroes of $p(x)$, we can say that $q(x)$ is a factor of $p(x)$.

Then, by division algorithm:

$$
\begin{array}{r}
x^2 - 3x + 2 \\
x^3 + 2x^2 + a \overline{)\, x^5 - x^4 - 4x^3 + 3x^2 + 3x + b} \\
\underline{x^5 + 2x^4 \quad + ax^2} \quad (-,-,-) \\
-3x^4 - 4x^3 + (3-a)x^2 + 3x + b \\
\underline{-3x^4 - 6x^3 \quad - 3ax} \quad (+,+,+) \\
+2x^3 + (3-a)x^2 + (3+3a)x + b \\
\underline{+2x^3 + 4x^2 \quad + 2a} \quad (-,-,-) \\
-(a+1)x^2 + 3(1+a)x + b - 2a
\end{array}
$$

But remainder:

$r(x) = -(a+1)x^2 + 3(1+a)x + b - 2a = 0$

$\Rightarrow -(a+1)x^2 + 3(1+a)x + b - 2a$

$= 0.x^2 + 0.x + 0$

On comparing the coefficients of x^2 and constant term, we get

$-(a+1) = 0 \quad \Rightarrow \quad a = -1$

and $b - 2a = 0 \quad \Rightarrow \quad b = 2a = 2(-1) = -2$

$\Rightarrow \quad b = -2$

For $a = -1$ and $b = -2$, the zeroes of $q(x)$ are also the zeroes of the polynomial $p(x)$.

$\Rightarrow \quad q(x) = x^3 + 2x^2 - 1$

and $\quad p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$.

We know that:

Dividend = (divisor) (quotient) + remainder

$\Rightarrow \quad p(x) = g(x)q(x) + r(x)$

$p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$

$= q(x) \cdot g(x) + 0$

$= (x^3 + 2x^2 - 1)(x^2 - 3x + 2) + 0$

$= (x^3 + 2x^2 - 1)(x - 1)(x - 2)$

Therefore, 1 and 2 are the zeroes of $p(x)$ that are not the zeroes of $q(x)$.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. If sum and product of the zeroes of $ky^2 + 2y + 3$ are equal. Find k.

Ans.

$$\text{Sum of the zeroes} = -\frac{b}{a} = \frac{-2}{k}$$

$$\text{Product of zeroes} = \frac{c}{a} = \frac{3}{k}$$

Sum of the zeroes = Product of zeroes

$$\frac{-2}{k} = \frac{3}{k}$$

2. If one of the zeroes of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is equal in magnitude but opposite in sign of the other, then find the value of k.

Ans.

$$f(x) = 4x^2 - 8kx - 9$$

Let one of the zeroes of the polynomial be α and the other zeroes be $-\alpha$

$$\text{Sum of zeroes} = \left(-\frac{b}{a}\right) = \frac{8k}{4}$$

$$\alpha + (-\alpha) = 0$$

So, $\quad \frac{8k}{4} = 0 \Rightarrow k = 0$

3. Can $(x - 5)$ be the remainder on division of a polynomial $p(x)$ by $(x + 8)$?

Ans. No.

We know that we cannot divide the polynomials which have same degree.

As we can see that degree of $(x - 5)$ = degree of $(x + 8)$

So, they are not divisible.

4. If the zeroes of a polynomial $x^2 - 8x + k = 0$, are the HCF of (6, 12), then find the value of k.

Ans.

HCF of (6, 12) = 6

So, 6 is one of the roots of the polynomial.

$$f(x) = x^2 - 8x + k = 0$$

$$f(6) = (6)^2 - 8(6) + k = 0$$

$$36 - 48 + k = 0$$

$$-12 + k = 0 \Rightarrow k = 12.$$

5. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans.

Let α and β are zeroes of the $2x^2 - 5x - 3$

$$\alpha + \beta = -\frac{b}{a} = \frac{5}{2} \quad \text{...(i)}$$

$$\alpha\beta = \frac{c}{a} = -\frac{3}{2} \quad \text{...(ii)}$$

According to the question,

2α and 2β are zeroes of $x^2 + px + q$

$$2\alpha + 2\beta = -p \Rightarrow 2(\alpha + \beta) = -p$$

$$2 \times \left(\frac{5}{2}\right) = -p \quad \text{[from eqn. (i)]}$$

$$p = -5$$

$$2\alpha \times 2\beta = q \Rightarrow 4\alpha\beta = q$$

$$4 \times \left(-\frac{3}{2}\right) = q \quad \text{[from eqn. (ii)]}$$

$$q = -6$$

Hence, $p = -5$ and $q = -6$.

6. If 2 and -3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$; then find the values of a.

Ans.

$$p(x) = x^2 + (a + 1)x + b$$

So, 2 is a zero of $p(x) \Rightarrow p(2) = 0$

$$2^2 + (a + 1)2 + b = 0$$

$$2a + b = -6 \quad \text{...(i)}$$

Also, $\quad p(-3) = 0$

$$(-3)^2 + (a + 1)(-3) + b = 0$$

$$-3a + b = -6 \quad \text{...(ii)}$$

Solving (i) and (ii), we get

$a = 0, \quad b = -6.$

7. Find a quadratic polynomial whose zeroes are 1 and -3. Verify the relation between the coeffcients and zeroes of polynomial.

Ans.

Sum of zeroes, $\quad S = 1 + (-3) = -2 \quad \text{...(i)}$

Product of zeroes, $\quad P = 1 \times (-3) = -3 \quad \text{...(ii)}$

Quadratic polynomial

$$p(x) = x^2 - Sx + P$$

$$= x^2 - (-2)x - 3 = x^2 + 2x - 3$$

Here, $a = 1, b = 2, c = -3$

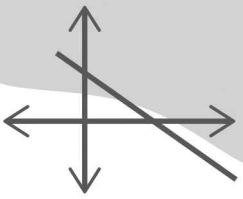
$$-\frac{b}{a} = -\frac{2}{1} = -2$$

Sum of zeroes $= -\frac{b}{a}$ [using eqn. (i)]

Also, $\frac{c}{a} = -\frac{3}{1} = -3$

Product of zeroes $= \frac{c}{a} = -3$ [using eqn. (ii)]

3 Pair of Linear Equations in Two Varibales



EXERCISE 3.1

Choose the correct answer from the given four options:

1. **Graphically, the pair of equations $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represents two lines which are:**

(A) **intersecting at exactly one point.**

(B) **intersecting at exactly two points.**

(C) **coincident**

(D) **parallel** **[CBSE 2013]**

Ans. (D)

Explanation: The given equations are:

$6x - 3y + 10 = 0$ ...(i)

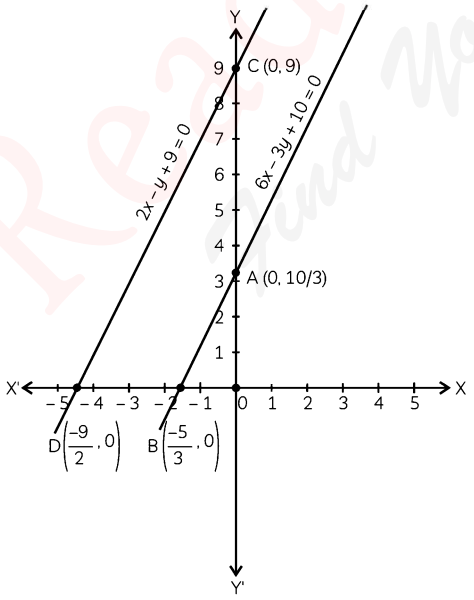
Also, $2x - y + 9 = 0$ ...(ii)

Table for $6x - 3y + 10 = 0$,

x	0	$-\frac{5}{3}$
y	$\frac{10}{3}$	0

Table for $2x - y + 9 = 0$,

x	0	$-\frac{9}{2}$
y	9	0



Hence, the pair of equations represents two parallel lines.

2. **The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has:**

(A) **a unique solution**

(B) **exactly two solutions**

(C) **infinitely many solutions**

(D) **no solution** **[CBSE 2020, 17, 15]**

Ans. (D)

Explanation: The given equations are:

$x + 2y + 5 = 0$

$-3x - 6y + 1 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1; b_1 = 2; c_1 = 5$

$a_2 = -3; b_2 = -6; c_2 = 1$

$\frac{a_1}{a_2} = -\frac{1}{3}; \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3}; \frac{c_1}{c_2} = \frac{5}{1} = 5$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent. Hence, the pair of equations has no solution.

3. **If a pair of linear equations is consistent, then the lines will be:**

(A) **parallel**

(B) **always coincident**

(C) **intersecting or coincident**

(D) **always intersecting**

Ans. (C)

Explanation: The conditions for a pair of linear equations to be consistent are:

- Intersecting lines having unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

OR

- Coincident or dependent lines having infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

4. **The pair of equations $y = 0$ and $y = -7$ has:**

(A) **one solution**

(B) **two solutions**

(C) **infinitely many solutions**

(D) **no solution**

Ans. (D)

Explanation: We know that equation of the form $y = a$ is a line parallel to the x-axis at a distance 'a' from it.

The given pair of equations are $y = 0$ and $y = -7$.

$y = 0$ is the equation of the x-axis and $y = -7$ is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines.

We know that parallel lines never intersect. So, there is no solution for these lines.

5. **The pair of equations $x = a$ and $y = b$ graphically represents lines which are:**

(A) **parallel**

(B) **intersecting at (b, a)**

(C) **coincident**

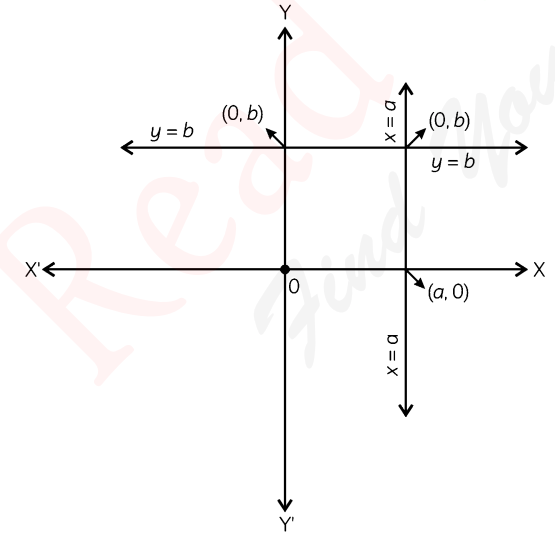
(D) **intersecting at (a, b)**

Ans. (D)

Explanation: We know that $x = a$ is the equation of a straight line parallel to the y-axis at a distance of 'a' from it.

Again, $y = b$ is the equation of a straight line parallel to the x-axis at a distance of 'b' from it.

So, the pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b) as shown below:



Hence, the two lines are intersecting at (a, b).

6. **For what value of k do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?**

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$

(C) 2 (D) –2

[CBSE 2019, 12, 10]

Ans. (C)

Explanation: The given equation of lines are

$$3x - y + 8 = 0$$

and $6x - ky + 16 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3; b_1 = -1; c_1 = 8$

$a_2 = 6; b_2 = -k; c_2 = 16$

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-1}{-k} = \frac{1}{k}; \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$

We know that the condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{k} = \frac{1}{2}$$

$$\Rightarrow \quad k = 2$$

7. **If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is:**

(A) $-\frac{5}{4}$ (B) $\frac{2}{5}$

(C) $\frac{15}{4}$ (D) $\frac{3}{2}$

[CBSE 2015, 11, 10]

Ans. (C)

Explanation: The given equation of lines are

$$3x + 2ky = 2$$

and $2x + 5y + 1 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3; b_1 = 2k; c_1 = -2$

$a_2 = 2; b_2 = 5; c_2 = 1$

$\frac{a_1}{a_2} = \frac{3}{2}; \frac{b_1}{b_2} = \frac{2k}{5}; \frac{c_1}{c_2} = \frac{-2}{1}$

We know that the condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{2k}{5}$$

$$\Rightarrow \quad 15 = 4k$$

$$\Rightarrow \quad k = \frac{15}{4}$$

For $k = \frac{15}{4}$,

$$\frac{2k}{5} = \frac{30}{20} = \frac{3}{2} \neq \frac{-2}{1}$$

Thus, $k = \frac{15}{4}$

8. **The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is:**

(A) 3 (B) –3

(C) –12 (D) no value

[CBSE 2019, 12, 10]

Ans. (D)

Explanation: The given equation of lines are

$$cx - y = 2$$

and $$6x - 2y = 3$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = c$; $b_1 = -1$; $c_1 = -2$

$a_2 = 6$; $b_2 = -2$; $c_2 = -3$

$\frac{a_1}{a_2} = \frac{c}{6}$; $\frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$

We know that the condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{c}{6} = \frac{1}{2} = \frac{2}{3}$$

But according to the above values, $\frac{1}{2} \neq \frac{2}{3}$ i.e., $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Also,

$$\Rightarrow \quad \frac{c}{6} = \frac{1}{2} \quad \text{and} \quad \frac{c}{6} = \frac{2}{3}$$

$$\Rightarrow \quad c = 3 \quad \text{and} \quad c = 4$$

Since c has different values and also $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, we can say that there exists no value of c for which the given equations have infinitely many solutions.

9. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be:

(A) $10x + 14y + 4 = 0$

(B) $-10x - 14y + 4 = 0$

(C) $-10x + 14y + 4 = 0$

(D) $10x - 14y = -4$

Ans. (D)

Explanation: In a pair of dependent linear equation, one equation is just a multiple of another equation. Thus, the second equation is

$$k(-5x + 7y - 2) = 0$$

Putting $k = 2$, we get

$$-10x + 14y - 4 = 0$$

Moving it to the other side

$$10x - 14y = -4$$

$\therefore$ (D) option is correct.

10. A pair of linear equations which has a unique solution $x = 2$, $y = -3$ is:

(A) $x + y = -1$ $\quad 2x - 3y = -5$

(B) $2x + 5y = -11$ $\quad 4x + 10y = -22$

(C) $2x - y = 1$ $\quad 3x + 2y = 0$

(D) $x - 4y - 14 = 0$ $\quad 5x - y - 13 = 0$

Ans. (D)

Explanation: If $x = 2$ and $y = -3$ is a unique solution of any pair of equation, then these values must satisfy that pair of equations.

Putting the values in the equations for every option and checking it -

For case (A):

The given equations are

$$x + y = -1$$

and $$2x - 3y = -5$$

Putting $x = 2$, $y = -3$ in the LHS of the equation

$$x + y = -1,$$

we get $2 - 3 = -1 = RHS$

Putting $x = 2$, $y = -3$ in the LHS of the equation

$$2x - 3y = -5,$$

we get $2 \times 2 - 3 \times (-3) = 4 + 9 = 13 \neq -5 \neq$ RHS

Since $x = 2$, $y = -3$ is satisfying only one of the two equations, option (A) is false.

Now, for case (B):

It is a pair of dependent linear equations and hence, has infinitely many solutions. (not a unique solution)

Now, for case (C):

The given equations are

$$2x - y = 1$$

and $$3x + 2y = 0$$

Putting $x = 2$, $y = -3$ in the LHS of the equation

$$2x - y = 1,$$

we get $2 \times 2 - (-3) = 4 + 3 = 7 \neq 1 \neq$ RHS

Putting $x = 2$, $y = -3$ in the LHS of the equation

$$3x + 2y = 0,$$

we get $3 \times 2 + 2 \times (-3) = 6 - 6 = 0 =$ RHS

Since $x = 2$, $y = -3$ is satisfying only one of the two equations, option (C) is false.

Now, for case (D):

The given equations are

$$x - 4y - 14 = 0$$

and $$5x - y - 13 = 0$$

Putting $x = 2$, $y = -3$ in the LHS of the equation

$$x - 4y - 14 = 0,$$

we get $2 - 4 \times (-3) - 14 = 2 + 12 - 14 = 0 =$ RHS

Putting $x = 2$, $y = -3$ in the LHS of the equation

$$5x - y - 13 = 0,$$

we get $5 \times 2 - (-3) - 13 = 10 + 3 - 13 = 0 =$ RHS

Since $x = 2$, $y = -3$ is satisfying both the equations, option (D) is true.

Hence, $x = 2$, $y = -3$ is the unique solution for these equations.

11. **If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b respectively are:**

(A) 3 and 5 (B) 5 and 3

(C) 3 and 1 (D) –1 and –3

[CBSE 2010]

Ans. (C)

Explanation: Since $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, these values must satisfy the given pair of equations.

Putting the values in the equations, we have

$$a - b = 2 \quad \text{...(i)}$$

and $$a + b = 4 \quad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$2a = 6 \quad \text{or} \quad a = 3$$

Putting the value of a in equation (ii), we get

$$3 + b = 4 \quad \text{or} \quad b = 1$$

Hence, option (C) is correct.

12. **Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins respectively are:**

(A) 35 and 15 (B) 35 and 20

(C) 15 and 35 (D) 25 and 25

Ans. (D)

Explanation: Let number of ₹ 1 coins = x

and number of ₹ 2 coins = y.

It is given that,

Total number of coins = $x + y = 50$ ...(i)

Also, amount of money with her

= (Number of ₹ 1 coins × 1) + (Number of ₹ 2 coins × 2)

Now, by the given condition:

$\Rightarrow \quad x(1) + y(2) = 75$

$\Rightarrow \quad x + 2y = 75$ ...(ii)

On subtracting eq. (i) from eq. (ii), we get

$(x + 2y) - (x + y) = (75 - 50)$

$\Rightarrow \quad y = 25$

Putting $y = 25$ in eq. (i), we get

$x + 25 = 50$

$\Rightarrow \quad x = 25$

Hence, Aruna has 25 ₹ 1 coins and 25 ₹ 2 coins.

13. **The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father respectively are:**

(A) 4 and 24 (B) 5 and 30

(C) 6 and 36 (D) 3 and 24

[CBSE 2019, 15]

Ans. (C)

Explanation: Let the present age of the father be 'x' years and the present age of the son be 'y' years.

According to the given condition,

$$x = 6y \quad \text{...(i)}$$

After four years their age will be

Father's age = $x + 4$

Son's age = $y + 4$

According to the given condition,

$$(x + 4) = 4(y + 4)$$

$\Rightarrow \quad x + 4 = 4y + 16$

$\Rightarrow \quad x - 4y - 12 = 0$ ...(ii)

Substituting the value of x from eq. (i) in eq. (ii), we get

$6y - 4y - 12 = 0$

$\Rightarrow \quad 2y = 12 \quad \Rightarrow \quad y = 6$

Putting $y = 6$ in eq. (i), we get

$x = 6(6) = 36$

Hence, the present age of the father is 36 years and the age of the son is 6 years.

EXERCISE 3.2

1. **Do the following pairs of linear equations have no solution? Justify your answer.**

(i) $2x + 4y = 3$ $\quad 12y + 6x = 6$

(ii) $x = 2y$ $\quad y = 2x$

(iii) $3x + y - 3 = 0$ $\quad 2x + \frac{2}{3}y = 2$

[CBSE 2017, 15]

Ans.

We know that the condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \text{(parallel lines)}$$

(i) Yes, $2x + 4y = 3$ and $12y + 6x = 6$ pair of linear equations has no solution.

The given pair of equations is

$$2x + 4y - 3 = 0$$

and $6x + 12y - 6 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 2, b_1 = 4, c_1 = -3;$

$a_2 = 6, b_2 = 12, c_2 = -6;$

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}; \ \frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}; \ \frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution.

(ii) No, $x = 2y$ and $y = 2x$ pair of linear equations have a unique solution.

The given pair of equations is

$x = 2y$ or $x - 2y = 0$

and $y = 2x$ or $2x - y = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = -2, c_1 = 0;$

$a_2 = 2, b_2 = -1, c_2 = 0;$

$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{-1} = 2$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of linear equations has a unique solution.

(iii) No, $3x + y - 3 = 0$ and $2x + \frac{2}{3}y = 2$ have infinitely many solutions.

The given pair of equations is

$3x + y - 3 = 0$

and $2x + \frac{2}{3}y = 2$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3, b_1 = 1, c_1 = -3;$

$a_2 = 2, b_2 = 2/3, c_2 = -2;$

$\frac{a_1}{a_2} = \frac{3}{2}; \frac{b_1}{b_2} = \frac{1}{\frac{2}{3}} = \frac{3}{2}; \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given pair of linear equations is coincident lines and has infinitely many solutions.

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + \frac{1}{7}y = 3$ $\quad 7x + 3y = 7$

(ii) $-2x - 3y = 1$ $\quad 6y + 4x = -2$

(iii) $\frac{x}{2} + y + \frac{2}{5} = 0$ $\quad 4x + 8y + \frac{5}{16} = 0$

[CBSE 2017, 15]

Ans.

We know that the condition for coincident solution is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(i) No, $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$ do not represent a pair of coincident lines but have a unique solution.

The given pair of linear equations is

$3x + \frac{1}{7}y = 3$

and $7x + 3y = 7$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3, b_1 = \frac{1}{7}, c_1 = -3;$

$a_2 = 7, b_2 = 3, c_2 = -7;$

$\frac{a_1}{a_2} = \frac{3}{7}; \frac{b_1}{b_2} = \frac{1}{21}; \frac{c_1}{c_2} = \frac{-3}{-7} = \frac{3}{7}$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of linear equations has a unique solution.

(ii) Yes, $-2x - 3y = 1$ and $6y + 4x = -2$ are coincident.

The given pair of linear equations is

$-2x - 3y - 1 = 0$

and $4x + 6y + 2 = 0;$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = -2, b_1 = -3, c_1 = -1;$

$a_2 = 4, b_2 = 6, c_2 = 2;$

$\frac{a_1}{a_2} = -\frac{2}{4} = -\frac{1}{2}; \frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}; \frac{c_1}{c_2} = -\frac{1}{2}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given pair of linear equations is coincident.

(iii) No, $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$ have no solution.

The given pair of linear equations is

$\frac{x}{2} + y + \frac{2}{5} = 0$

and $4x + 8y + \frac{5}{16} = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = \frac{1}{2}, b_1 = 1, c_1 = \frac{2}{5};$

$a_2 = 4, b_2 = 8, c_2 = \frac{5}{16};$

$\frac{a_1}{a_2} = \frac{1}{8}; \frac{b_1}{b_2} = \frac{1}{8}; \frac{c_1}{c_2} = \frac{32}{25}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution.

3. Are the following pairs of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$ $\quad 4y + 3x = 12$

(ii) $\frac{3}{5}x - y = \frac{1}{2}$ $\quad \frac{1}{5}x - 3y = \frac{1}{6}$

(iii) $2ax + by = a$ $\quad 4ax + 2by - 2a = 0;\ a, b \neq 0$

(iv) $x + 3y = 11$ $\quad 2(2x + 6y) = 22$

[CBSE 2017, 16]

Ans.

We know that the conditions for a pair of linear equations to be consistent are:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [unique solution]

or $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

[coincident or infinitely many solutions]

(i) No, $-3x - 4y = 12$ and $4y + 3x = 12$ are inconsistent.

The given pair of linear equations is

$-3x - 4y - 12 = 0$

and $3x + 4y - 12 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = -3, b_1 = -4, c_1 = -12;$

$a_2 = 3, b_2 = 4, c_2 = -12;$

$\frac{a_1}{a_2} = -\frac{3}{3} = -1;\ \frac{b_1}{b_2} = -\frac{4}{4} = -1;\ \frac{c_1}{c_2} = \frac{-12}{-12} = 1$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equations has no solution i.e., it is inconsistent.

(ii) Yes, $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$ are consistent.

The given pair of linear equations is

$\frac{3}{5}x - y = \frac{1}{2} = 0$

and $\frac{1}{5}x - 3y = \frac{1}{6} = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = \frac{3}{5}, b_1 = -1, c_1 = -\frac{1}{2};$

$a_2 = \frac{1}{5}, b_2 = -3, c_2 = -\frac{1}{6};$

$\frac{a_1}{a_2} = \frac{3}{5} \div \frac{1}{5} = \frac{3}{1};\ \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3};$

$\frac{c_1}{c_2} = \frac{-1}{2} \div \frac{-1}{6} = \frac{3}{1}$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of linear equations has a unique solution i.e., consistent.

(iii) Yes, $2ax + by = a$ and $4ax + 2by - 2a = 0$ are consistent.

The given pair of linear equations is

$2ax + by - a = 0$

and $4ax + 2by - 2a = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 2a, b_1 = b, c_1 = -a;$

$a_2 = 4a, b_2 = 2b, c_2 = -2a;$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2};\ \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2};\ \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Hence, the given pair of linear equations has infinitely many solutions i.e., it isconsistent

(iv) No, $x + 3y = 11$ and $2(2x + 6y) = 22$ are inconsistent

The given pair of linear equations is

$x + 3y - 11 = 0$

and $2x + 6y - 11 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = 3, c_1 = -11$

$a_2 = 2, b_2 = 6, c_2 = -11$

$\frac{a_1}{a_2} = \frac{1}{2};\ \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2};\ \frac{c_1}{c_2} = \frac{-11}{-11} = 1$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution i.e., inconsistent

4. For the pair of equations

$\lambda x + 3y = -7$

$2x + 6y = 14$

to have infinitely many solutions, the value of λ should be 1. Is this statement true? Give reasons.

Ans.

No, for no value of λ will the given pair of linear equations has infinitely many solutions.

The given pair of linear equations is

$\lambda x + 3y + 7 = 0$

and $2x + 6y - 14 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = \lambda, b_1 = 3, c_1 = 7;$

$a_2 = 2, b_2 = 6, c_2 = -14;$

$\frac{a_1}{a_2} = \frac{\lambda}{2};\ \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2};\ \frac{c_1}{c_2} = \frac{7}{-14} = -\frac{1}{2}$

For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{\lambda}{2} = \frac{1}{2}$ and $\frac{\lambda}{2} = -\frac{1}{2}$

$\Rightarrow \quad \lambda = 1$ and $\lambda = -1$

Since, λ does not have a unique value so for no value of λ will the given pair of linear equations have infinitely many solutions.

5. For all real values of c, the pair of equations

$x - 2y = 8$

$5x - 10y = c$

have a unique solution. Justify whether it is true or false.

Ans. False

The given pair of equations will not have a unique solution for any value of c.

The given pair of linear equations is

$$x - 2y - 8 = 0$$

and $\quad 5x - 10y - c = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = -2, c_1 = -8;$

$a_2 = 5, b_2 = -10, c_2 = -c;$

$$\frac{a_1}{a_2} = \frac{1}{5};\ \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5};\ \frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$$

But for $c = 40$ (any real value), the ratio will be

$$\frac{c_1}{c_2} = \frac{8}{40} = \frac{1}{5}$$

when $c = 40$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{5}$$

Thus the given pair of linear equations will have infinitely many solutions for $c = 40$.

Also, when $c \neq 40$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus the given pair of linear equations will have no solution for $c \neq 40$.

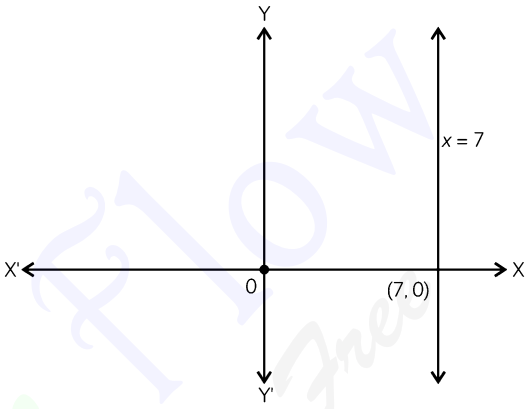
Hence, for any value of c, the system of linear equations does not have a unique solution.

6. The line represented by $x = 7$ is parallel to the x-axis. Justify whether the statement is true or not.

Ans. False

The line represented by $x = 7$ is not parallel to the x-axis.

Explanation: The line represented by $x = 7$ is of the form $x = a$. The graph of the equation is a line parallel to the y-axis and perpendicular to the x-axis.



Hence, the given statement is not true.

EXERCISE 3.3

1. For which value(s) of λ do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have:

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Ans.

The given pair of linear equations is

$$\lambda x + y - \lambda^2 = 0$$

and $\quad x + \lambda y - 1 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = \lambda, b_1 = 1, c_1 = -\lambda^2;$

$a_2 = 1, b_2 = \lambda, c_2 = -1;$

$$\frac{a_1}{a_2} = \frac{\lambda}{1};\ \frac{b_1}{b_2} = \frac{1}{\lambda};\ \frac{c_1}{c_2} = \frac{-\lambda^2}{-1} = \frac{\lambda^2}{1}$$

(i) For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \quad \text{and} \quad \frac{\lambda}{1} \neq \frac{\lambda^2}{1}$$

$$\lambda^2 - 1 = 0 \quad \text{and} \quad \lambda^2 \neq \lambda$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad (\lambda^2 - \lambda) \neq 0$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad \lambda(\lambda - 1) \neq 0$$

$$\lambda = 1, -1 \quad \text{and} \quad \lambda \neq 0, 1$$

Here, we take only $\lambda = -1$.

Hence for $\lambda = -1$, the pair of linear equations has no solution.

(ii) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \quad \text{and} \quad \frac{\lambda}{1} = \frac{\lambda^2}{1}$$

$$\lambda^2 - 1 = 0 \quad \text{and} \quad \lambda^2 = \lambda$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad (\lambda^2 - \lambda) = 0$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad \lambda(\lambda - 1) = 0$$

$$\lambda = 1, -1 \quad \text{and} \quad \lambda = 0, 1$$

$\lambda = 1$ satisfies both the equations.

Hence, for $\lambda = 1$, the pair of linear equations has infinitely many solutions.

(iii) For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\lambda^2 - 1 \neq 0$$

$$(\lambda - 1)(\lambda + 1) \neq 0$$

$$\lambda \neq 1, -1$$

Hence, for all real values of λ except ± 1, the given pair of equations has a unique solution.

2. For which value(s) of k will the pair of equations

$kx + 3y = k - 3$

$12x + ky = k$

have no solution? [CBSE 2015, 11]

Ans.

The given pair of linear equations is

$$kx + 3y = k - 3$$

and $$12x + ky = k$$

$$kx + 3y - (k - 3) = 0$$

and $$12x + ky - k = 0$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = k, b_1 = 3, c_1 = -(k - 3)$

$a_2 = 12, b_2 = k, c_2 = -k$

$$\frac{a_1}{a_2} = \frac{k}{12}; \frac{b_1}{b_2} = \frac{3}{k}; \frac{c_1}{c_2} = \frac{-(k-3)}{-k} = \frac{k-3}{k}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{k}{12} = \frac{3}{k} \neq \frac{k-3}{k}$$

Taking the first two parts, we get

$$\frac{k}{12} = \frac{3}{k}$$

$$\Rightarrow \quad k^2 = 36$$

$$\Rightarrow \quad k = \pm 6$$

Taking the last two parts, we get

$$\frac{3}{k} \neq \frac{k-3}{k}$$

$$\Rightarrow \quad 3k \neq k(k - 3)$$

$$\Rightarrow \quad 3k - k(k - 3) \neq 0$$

$$\Rightarrow \quad k(3 - k + 3) \neq 0$$

$$\Rightarrow \quad k(6 - k) \neq 0$$

$$\Rightarrow \quad k \neq 0 \text{ and } k \neq 6$$

Hence, the required value of k for which the given pair of linear equations has no solution is -6.

3. For which values of a and b will the following pair of linear equations have infinitely many solutions?

$x + 2y = 1$

$(a - b)x + (a + b)y = a + b - 2$ [CBSE 2013, 11]

Ans.

The given pair of linear equations is

$$x + 2y = 1$$

and $$(a - b)x + (a + b)y = a + b - 2$$

$$x + 2y - 1 = 0$$

and $$(a - b)x + (a + b)y - (a + b - 2) = 0$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = 2, c_1 = -1$

$a_2 = (a - b), b_2 = (a + b), c_2 = -(a + b - 2)$

$$\frac{a_1}{a_2} = \frac{1}{a-b}; \frac{b_1}{b_2} = \frac{2}{a+b};$$

$$\frac{c_1}{c_2} = \frac{-1}{-(a+b-2)} = \frac{1}{a+b-2}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{a-b} = \frac{2}{a+b} = \frac{1}{a+b-2}$$

Taking the first two parts

$$\frac{1}{a-b} = \frac{2}{a+b}$$

$$\Rightarrow \quad a + b = 2(a - b)$$

$$\Rightarrow \quad 2a - a = 2b + b$$

$$\Rightarrow \quad a = 3b \qquad \text{...(i)}$$

Taking the last two parts,

$$\frac{2}{a+b} = \frac{1}{a+b-2}$$

$$\Rightarrow \quad 2(a + b - 2) = (a + b)$$

$$\Rightarrow \quad 2a + 2b - 4 = a + b$$

$$\Rightarrow \quad a + b = 4 \qquad \text{...(ii)}$$

Putting the value of a from eq. (i) in eq. (ii), we get

$$\Rightarrow \quad 3b + b = 4$$

$$\Rightarrow \quad 4b = 4$$

$$\Rightarrow \quad b = 1$$

Putting the value of b in eq. (i), we get

$$a = 3(1) = 3$$

The values $(a, b) = (3, 1)$ satisfies all the parts.

Hence, the required values of a and b are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.

4. Find the value(s) of p in (i) to (iv) and p and q in (v) for the following pair of equations:

(i) $3x - y - 5 = 0$ and $6x - 2y - p = 0$, if the lines represented by these equations are parallel. [CBSE 2015, 11, 10]

(ii) $-x + py = 1$ and $px - y = 1$, if the pair of equations has no solution. [CBSE 2015, 11, 10]

(iii) $-3x + 5y = 7$ and $2px - 3y = 1$, if the lines represented by these equations are intersecting at a unique point. [CBSE 2016, 13]

(iv) $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$, if the pair of equations has a unique solution. [CBSE 2016, 13]

(v) $2x + 3y = 7$ and $2px + py = 28 - qy$, if the pair of equations has infinitely many solutions. [CBSE 2013, 11]

Ans.

(i) The given pair of linear equations is

$$3x - y - 5 = 0$$

and $$6x - 2y - p = 0$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3, b_1 = -1, c_1 = -5$;

$a_2 = 6, b_2 = -2, c_2 = -p$;

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-5}{-p} = \frac{5}{p}$

It is given that the lines represented by these equations are parallel, therefore

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{1}{2} \neq \frac{5}{p}$$

Taking the last two parts, we get

$$\frac{1}{2} \neq \frac{5}{p}$$

$$\Rightarrow \quad p \neq 10$$

Hence, the given pair of linear equations is parallel for all real values of p except 10.

(ii) The given pair of linear equations is

$$-x + py = 1$$

and $$px - y - 1 = 0$$

$$-x + py - 1 = 0$$

and $$px - y - 1 = 0$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = -1, b_1 = p, c_1 = -1$;

$a_2 = p, b_2 = -1, c_2 = -1$;

$\frac{a_1}{a_2} = \frac{-1}{p}$; $\frac{b_1}{b_2} = \frac{p}{-1}$; $\frac{c_1}{c_2} = \frac{-1}{-1} = 1$

It is given that the pair of linear equations has no solution i.e., both lines are parallel to each other. For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$-\frac{1}{p} = \frac{p}{-1} \neq \frac{1}{1}$$

Taking the first two parts

$$-\frac{1}{p} = \frac{p}{-1}$$

$$\Rightarrow \quad p^2 = 1$$

$$\Rightarrow \quad p = \pm 1$$

Taking the last two parts, we get

$$\frac{p}{-1} \neq \frac{1}{1}$$

$$\Rightarrow \quad p \neq -1$$

Hence, the given pair of linear equations has no solution for $p = 1$.

(iii) The given pair of linear equations is

$$-3x + 5y = 7$$

and $$2px - 3y = 1$$

$$-3x + 5y - 7 = 0$$

and $$2px - 3y - 1 = 0$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = -3, b_1 = 5, c_1 = -7$;

$a_2 = 2p, b_2 = -3, c_2 = -1$;

$\frac{a_1}{a_2} = \frac{-3}{2p}$; $\frac{b_1}{b_2} = \frac{5}{-3}$; $\frac{c_1}{c_2} = \frac{-7}{-1} = 7$

It is given that the lines are intersecting at a unique point i.e., they have a unique solution. For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{-3}{2p} \neq \frac{5}{-3}$$

$$\Rightarrow \quad 9 \neq 10p$$

$$\Rightarrow \quad p \neq \frac{9}{10}$$

Hence, the lines represented by these equations are intersecting at a unique point for all real values of p except $\frac{9}{10}$.

(iv) The given pair of linear equations is

$$2x + 3y - 5 = 0$$

and $$px - 6y - 8 = 0$$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 2, b_1 = 3, c_1 = -5$;

$a_2 = p, b_2 = -6, c_2 = -8$;

$\frac{a_1}{a_2} = \frac{2}{p}$; $\frac{b_1}{b_2} = \frac{3}{-6} = -\frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$

It is given that the lines are intersecting at a unique point i.e., they have a unique solution. For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2}{p} \neq -\frac{1}{2}$$

$$\Rightarrow \quad p \neq -4$$

Hence, the pair of linear equations has a unique solution for all values of p except -4.

(v) The given pair of linear equations is

$$2x + 3y = 7$$

and $$2px + py = 28 - qy$$

$$2x + 3y - 7 = 0$$

and $2px + (p + q)y - 28 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 2, b_1 = 3, c_1 = -7$;

$a_2 = 2p, b_2 = (p + q), c_2 = -28$;

$\frac{a_1}{a_2} = \frac{2}{2p} = \frac{1}{p}$; $\frac{b_1}{b_2} = \frac{3}{(p+q)}$; $\frac{c_1}{c_2} = \frac{-7}{-28} = \frac{1}{4}$

It is given that the pair of linear equations has infinitely many solutions i.e., both lines are coincident. For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{p} = \frac{3}{(p+q)} = \frac{1}{4}$$

Taking the first and third parts, we get

$$\frac{1}{p} = \frac{1}{4}$$

$\Rightarrow \quad p = 4$

Again, taking the last two parts, we get

$$\frac{3}{(p+q)} = \frac{1}{4}$$

$\Rightarrow \quad (p + q) = 12$

But $\quad p = 4 \quad \Rightarrow \quad q = 8$

Here, we see that the values of $p = 4$ and $q = 8$ satisfy all three parts.

Hence, the pair of equations has infinitely many solutions for all values of $p = 4$ and $q = 8$.

5. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.

Ans.

The given pair of linear equations is

$$x - 3y = 2$$

and $\quad -2x + 6y = 5$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = -3, c_1 = -2;$

$a_2 = -2, b_2 = 6, c_2 = -5;$

$\frac{a_1}{a_2} = \frac{1}{-2} = -\frac{1}{2}; \ \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}; \ \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

Here, $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus the two given lines are parallel.

Hence, the two straight paths represented by the given equations never cross each other because they are parallel to each other.

6. Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

Ans.

We know that the condition for the pair of system to have a unique solution is

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Let the equations be

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

It is given that $x = -1$ and $y = 3$ is the unique solution of these two equations, then it must satisfy the above equations.

$\Rightarrow \quad a_1(-1) + b_1(3) + c_1 = 0$

$\Rightarrow \quad -a_1 + 3b_1 + c_1 = 0 \quad \ldots(i)$

and $\quad a_2(-1) + b_2(3) + c_2 = 0$

$\Rightarrow \quad -a_2 + 3b_2 + c_2 = 0 \quad \ldots(ii)$

The restricted values of a_1, a_2 and b_1, b_2 are only

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \quad \ldots(iii)$$

So, all the real values of a_1, a_2, b_1, b_2 except those which satisfy eq. (iii) and satisfy eq. (i), and eq. (ii) will have the solution $x = -1$ and $y = 3$.

Hence, infinitely many pairs of linear equations are possible.

7. If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and $\frac{y}{x} - 2$.

Ans.

The given equations are

$$2x + y = 23 \quad \ldots(i)$$

$$4x - y = 19 \quad \ldots(ii)$$

On adding both equations, we get

$\Rightarrow \quad 6x = 42$

$\Rightarrow \quad x = 7$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 2(7) + y = 23$

$\Rightarrow \quad y = 23 - 14$

$\Rightarrow \quad y = 9$

We have $\quad 5y - 2x = 5(9) - 2(7)$

$$= 45 - 14$$

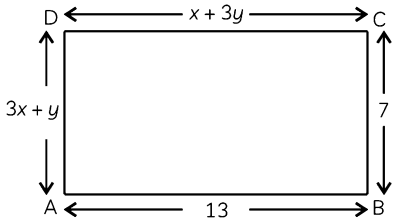
$$= 31$$

and $\quad \frac{y}{x} - 2 = \frac{9}{7} - 2$

$$= -\frac{5}{7}$$

Hence, the values of $(5y - 2x)$ and $\frac{y}{x} - 2$ are 31 and $-\frac{5}{7}$ respectively.

8. Find the values of x and y in the following rectangle.



[CBSE 2018]

Ans.

We know that the opposite sides of a rectangle are equal in length.

$\Rightarrow$ Length of side CD = Length of side AB

$\Rightarrow \quad x + 3y = 13 \quad \ldots(i)$

$\Rightarrow$ Length of side AD = Length of side BC

$\Rightarrow \quad 3x + y = 7 \quad \ldots(ii)$

On multiplying eq. (ii) by 3 and then subtracting eq. (i), we get,

$3(3x + y) - (x + 3y) = 7 \times 3 - 13$

$\Rightarrow \quad 9x + 3y - x - 3y = 21 - 13$

$\Rightarrow \quad 8x = 8$

$\Rightarrow \quad x = 1$

Putting the value of $x = 1$ in eq. (i), we get

$\Rightarrow \quad 1 + 3y = 13$

$\Rightarrow \quad 3y = 12$

$\Rightarrow \quad y = 4$

Therefore, the required values of x and y are 1 and 4 respectively.

9. Solve the following pairs of equations

(i) $x + y = 3.3$

$\frac{0.6}{3x - 2y} = -1, \quad 3x - 2y \neq 0$

(ii) $\frac{x}{3} + \frac{y}{4} = 4$

$\frac{5x}{6} - \frac{y}{8} = 4$

(iii) $4x + \frac{6}{y} = 15$

$6x - \frac{8}{y} = 14, \quad y \neq 0$ **[CBSE 2010]**

(iv) $\frac{1}{2x} - \frac{1}{y} = -1$

$\frac{1}{x} + \frac{1}{2y} = 8, \quad x, y \neq 0$ **[CBSE 2017]**

(v) $43x + 67y = -24$

$67x + 43y = 24$ **[CBSE 2011]**

(vi) $\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2, \quad a, b \neq 0$

(vii) $\frac{2xy}{x + y} = \frac{3}{2}$

$\frac{xy}{2x - y} = \frac{-3}{10}, \quad x + y \neq 0, 2x - y \neq 0$

Ans.

(i) The given pair of linear equations is

$x + y = 3.3$ ...(i)

and $\frac{0.6}{3x - 2y} = -1$

$\Rightarrow \quad 0.6 = -3x + 2y$

$\Rightarrow \quad 3x - 2y = -0.6$ ...(ii)

Multiplying eq. (i) by 2 and then adding with eq. (ii), we get

$(2x + 2y) + (3x - 2y) = 6.6 - 0.6$

$5x = 6$

$x = \frac{6}{5} = 1.2$

Putting the value of x in eq. (i), we get

$1.2 + y = 3.3$

$y = 3.3 - 1.2 = 2.1$

Hence, the required values of x and y are 1.2 and 2.1 respectively.

(ii) The given pair of linear equations is

$\frac{x}{3} + \frac{y}{4} = 4$

$\Rightarrow \quad 4x + 3y = 48$ ...(i)

$\frac{5x}{6} - \frac{y}{8} = 4$

$\Rightarrow \quad 20x - 3y = 96$ ...(ii)

Adding eq. (i) and (ii), we get

$24x = 144$

$\Rightarrow \quad x = 6$

Putting the value of x in eq. (i), we get

$4(6) + 3y = 48$

$\Rightarrow \quad 24 + 3y = 48$

$\Rightarrow \quad 3y = 24$

$\Rightarrow \quad y = 8$

Hence, the required values of x and y are 6 and 8 respectively.

(iii) The given pair of linear equations is

$4x + \frac{6}{y} = 15$

and $6x - \frac{8}{y} = 14$

Let $p = \frac{1}{y}$

The above equation will become

$4x + 6p = 15$ ...(i)

and $6x - 8p = 14$ ...(ii)

Multiplying eq. (i) by 8 and eq. (ii) by 6 and then adding both of them, we get

$(32x + 48p) + (36x - 48p) = 120 + 84$

$68x = 204$

$x = 3$

Putting the value of x in eq. (i), we get

$4(3) + 6p = 15$

$6p = 3$

$p = \frac{1}{2}$

$p = \frac{1}{y} = \frac{1}{2}$

$y = 2$

Hence, the required values of x and y are 3 and 2 respectively.

(iv) The given pair of linear equations is

$\frac{1}{2x} - \frac{1}{y} = -1$

and $\frac{1}{x} + \frac{1}{2y} = 8$

Let us take $u = \frac{1}{x}$ and $v = \frac{1}{y}$ in the above equations

$\frac{u}{2} - v = -1$

$\Rightarrow \quad u - 2v = -2$ ...(i)

and $u + \frac{v}{2} = 8$

$2u + v = 16$ ...(ii)

Multiplying eq. (ii) by 2 and then adding it with eq. (i), we get

$(4u + 2v) + (u - 2v) = (32 - 2)$

$\Rightarrow \quad 5u = 30$

$\Rightarrow \quad u = 6$

Putting the value of u in eq. (ii), we get

$$2(6) + v = 16$$
$$v = 16 - 12$$
$$v = 4$$
$$u = \frac{1}{x} = 6$$
$$x = \frac{1}{6}$$
$$v = \frac{1}{y} = 4$$
$$y = \frac{1}{4}$$

Hence, the required values of x and y are $\frac{1}{6}$ and $\frac{1}{4}$ respectively.

(v) The given pair of equations is

$$43x + 67y = -24 \quad ...(i)$$
$$67x + 43y = 24 \quad ...(ii)$$

Adding eq. (i) and eq. (ii), we get

$$110x + 110y = 0$$
$$x + y = 0 \quad ...(iii)$$

Subtracting (ii) from (i), we get

$$-24x + 24y = -48$$
$$-x + y = -2 \quad ...(iv)$$

Adding eq. (iii) and eq. (iv), we get

$$2y = -2$$
$$y = -1$$

Putting the value of y in eq. (iii), we get

$$x + y = 0$$
$$x + (-1) = 0$$
$$x = 1 \text{ and } y = -1$$

Hence, the required values of x and y are 1 and -1 respectively.

(vi) The given pair of linear equations is

$$\frac{x}{a} + \frac{y}{b} = a + b \quad ...(i)$$

and $$\frac{x}{a^2} + \frac{y}{b^2} = 2 \quad ...(ii)$$

On multiplying eq. (i) by $\frac{1}{a}$ and then subtracting from eq. (ii), we get

$$\left(\frac{x}{a^2} + \frac{y}{b^2}\right) - \left(\frac{x}{a^2} + \frac{y}{ab}\right) = 2 - \left(1 + \frac{b}{a}\right)$$
$$\Rightarrow \quad y\left(\frac{1}{b^2} - \frac{1}{ab}\right) = 1 - \frac{b}{a}$$
$$\Rightarrow \quad y = b^2$$

Putting the value of y in eq. (ii), we get

$$\frac{x}{a^2} + \frac{b^2}{b^2} = 2$$
$$\Rightarrow \quad \frac{x}{a^2} = 1$$
$$\Rightarrow \quad x = a^2$$

Hence, the required values of x and y are a^2 and b^2 respectively.

(vii) The given pair of equations is

$$\frac{2xy}{x+y} = \frac{3}{2}$$
$$\Rightarrow \quad \frac{x+y}{2xy} = \frac{2}{3}$$
$$\Rightarrow \quad \frac{x}{2xy} + \frac{y}{2xy} = \frac{2}{3}$$
$$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = \frac{4}{3} \quad ...(i)$$

and $$\frac{xy}{2x-y} = -\frac{3}{10}$$
$$\Rightarrow \quad \frac{2x-y}{xy} = -\frac{10}{3}$$
$$\Rightarrow \quad \frac{2x}{xy} - \frac{y}{xy} = -\frac{10}{3}$$
$$\Rightarrow \quad \frac{2}{y} - \frac{1}{x} = -\frac{10}{3} \quad ...(ii)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then the pair of equations becomes

$$u + v = \frac{4}{3} \quad ...(iii)$$

and $$2v - u = -\frac{10}{3} \quad ...(iv)$$

Adding both equations, we get

$$\Rightarrow \quad 3v = \frac{4}{3} - \frac{10}{3} = -\frac{6}{3}$$
$$\Rightarrow \quad 3v = -2$$
$$\Rightarrow \quad v = -\frac{2}{3}$$

Putting the value of v in eq. (iii), we get

$$\Rightarrow \quad -\frac{2}{3} + u = \frac{4}{3}$$
$$\Rightarrow \quad u = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$
$$\Rightarrow \quad x = \frac{1}{u} = \frac{1}{2}$$

and $$y = \frac{1}{v} = -\frac{3}{2}$$

Hence, the required values of x and y are $\frac{1}{2}$ and $-\frac{3}{2}$ respectively.

10. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$. Hence, find λ, if $y = \lambda x + 5$.

Ans.

The given pair of equations is

$$\frac{x}{10} + \frac{y}{5} - 1 = 0$$
$$\Rightarrow \quad x + 2y - 10 = 0$$
$$\Rightarrow \quad x + 2y = 10 \quad ...(i)$$

and $$\frac{x}{8} + \frac{y}{6} = 15$$
$$\Rightarrow \quad 3x + 4y - 360 = 0$$
$$\Rightarrow \quad 3x + 4y = 360 \quad ...(ii)$$

Multiplying eq. (i) by 2 and subtracting it from eq. (ii), we get

$(3x + 4y) - (2x + 4y) = 360 - 20$

$\Rightarrow \quad x = 340$

Putting the value of x in eq. (i), we get

$340 + 2y = 10$

$\Rightarrow \quad 2y = -330$

$\Rightarrow \quad y = -165$

It is given that $y = \lambda x + 5$

Putting the values of x and y in the above equation, we get

$y = \lambda x + 5$

$\Rightarrow \quad -165 = \lambda(340) + 5$

$\Rightarrow \quad -\lambda(340) = 5 + 165$

$\Rightarrow \quad -\lambda(340) = 170$

$\Rightarrow \quad \lambda = \frac{-170}{340} = \frac{-1}{2}$

Hence, the solution of the pair of equations is $x = 340$, $y = -165$ and the required value of λ is $-\frac{1}{2}$.

11. By the graphical method, find whether the following pairs of equations are consistent or not. If consistent, solve them.

(i) $3x + y + 4 = 0$ and $6x - 2y + 4 = 0$

(ii) $x - 2y = 6$ and $3x - 6y = 0$

(iii) $x + y = 3$ and $3x + 3y = 9$ [CBSE 2014]

Ans.

(i) $3x + y + 4 = 0$ and $6x - 2y + 4 = 0$

The given pair of equations is

$3x + y + 4 = 0$

and $6x - 2y + 4 = 0$

When $x = 0$, then $y = -4$

When $x = -1$, then $y = -1$

When $x = -2$, then $y = 2$

x	0	-1	-2
y	-4	-1	2

And $6x - 2y + 4 = 0$

$y = 3x + 2$

When $x = 0$, then $y = 2$

When $x = -1$, then $y = -1$

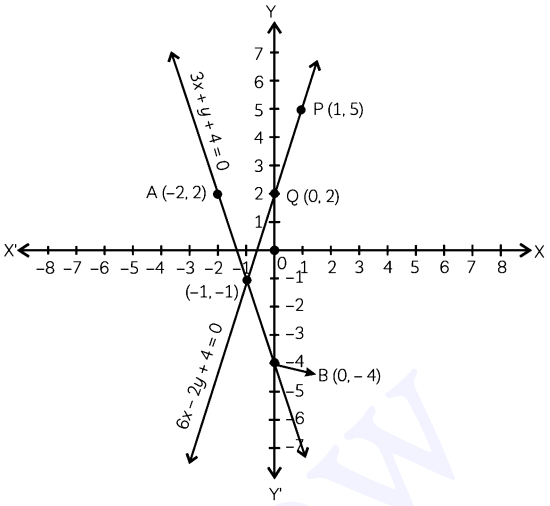
When $x = 1$, then $y = 5$

x	0	-1	1
y	2	-1	5

Plotting the points B(0, –4) and A(–2, 2), we get the straight tine AB.

Plotting the points Q(0, 2) and P(1, 5) we get the straight line PQ.

The lines AB and PQ intersect at C(–1, –1).



Thus, the pair of equations is consistent and has solution $x = -1$, $y = -1$.

(ii) $x - 2y = 6$ and $3x - 6y = 0$

The given pair of equations is

$x - 2y = 6$ and $3x - 6y = 0$

$x - 2y - 6 = 0$ and $3x - 6y = 0$

$x - 2y = 6$

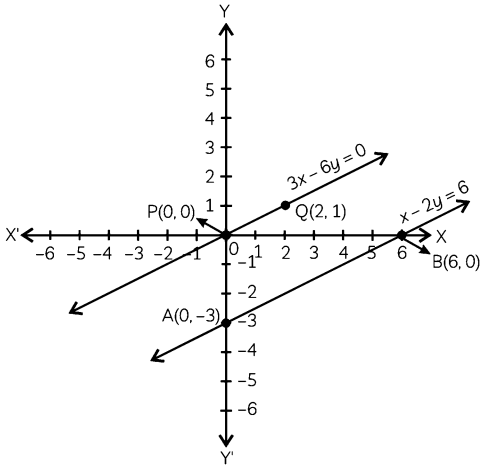
x	0	6
y	-3	0

$3x - 6y = 0$

x	0	2
y	0	1

Plotting the points A(0, –3) & B(6, 0), we get line AB.

Again plotting the points P(0, 0) & Q(2, 1) we get line PQ.



Since the lines are parallel, the pair of equations is inconsistent.

(iii) $x + y = 3$ and $3x + 3y = 9$

The given pair of equations is

$x + y = 3$ and $3x + 3y = 9$

Now, $x + y = 3$

$\Rightarrow \quad y = 3 - x$

If $x = 0$, then $y = 3$

If $x = 3$, then $y = 0$

If $x = 2$, then $y = 1$

x	0	3	2
y	3	0	1

and $3x + 3y = 9$

$\Rightarrow \quad y = \frac{9 - 3x}{3}$

If $x = 0$ then $y = 3$

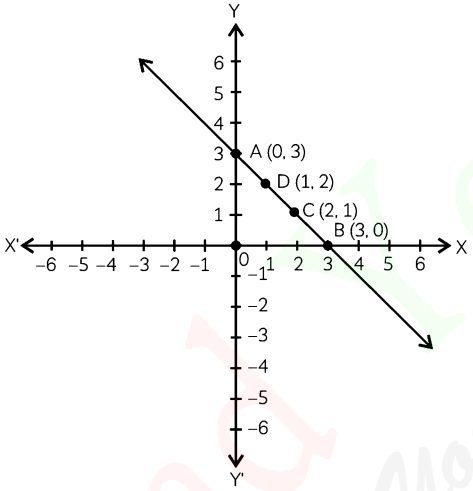
if $x = 1$, then $y = 2$

if $x = 3$, then $y = 0$.

x	0	1	3
y	3	2	0

Plotting the points A(0, 3) and B(3, 0), we get the line AB.

Again, plotting the points A(0, 3) and D(1, 2) and B(3, 0), we get the line ADB.



Since the two lines are coincident, the pair of equations is consistent with infinitely many solutions.

12. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the Y-axis and find the area of this triangle.

[CBSE 2017, 15, 13]

Ans.

The given pair of linear equations is

$2x + y = 4$

$\Rightarrow \quad y = 4 - 2x$

If $x = 0$, then $y = 4$

If $x = 2$, then $y = 0$

Table for line $2x + y = 4$

x	0	2
y	4	0

$2x - y = 4$

$\Rightarrow \quad y = 2x - 4$

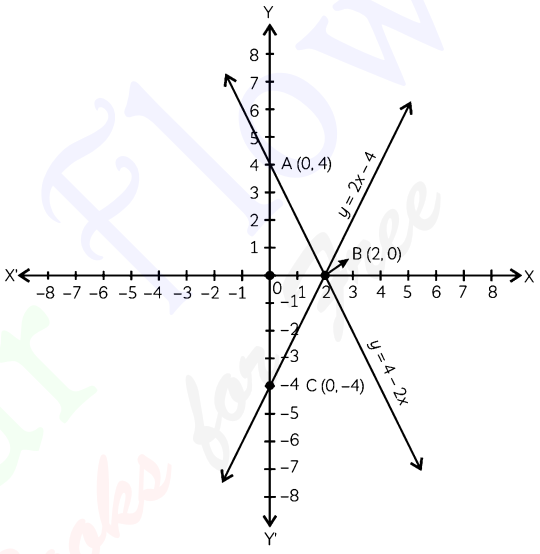
If $x = 0$, then $y = -4$

If $x = 2$, then $y = 0$

Table for line $2x - y = 4$

x	0	2
y	−4	0

Graphical representation of both the equations:



Here, two lines and the Y-axis form a ΔABC, with vertices A(0, 4), B(2, 0) and C(0, −4).

Required area of ΔABC

$= 2 \times \Delta AOB$

$= 2 \times \frac{1}{2} \times 4 \times 2$

$= 8$ sq. units

Hence, the required area of the triangle is 8 sq. units.

13. Write an equation for a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?

Ans.

The given equations are

$x + y = 2$ ...(i)

$2x - y = 1$ ...(ii)

Adding eq. (i) and (ii), we have

$3x = 3 \quad \Rightarrow \quad x = 1$

Substituting $x = 1$ in eq. (i), we have

$y = 1$

So, the solution is $x = 1$ and $y = 1$ and the point that represents the solution is (1, 1).

We also know that an infinite number of lines can pass through a given point, say (1, 1).

Hence, infinite lines can pass through the intersection point of the linear equations $x + y = 2$ and $2x - y = 1$ i.e., E(1, 1).

14. If $(x + 1)$ is a factor of $(2x^3 + ax^2 + 2bx + 1)$, then find the values of a and b, given that $2a - 3b = 4$.

Ans.

We know that if $(x + \alpha)$ is a factor of $f(x) = ax^2 + bx + c$, then $f(-\alpha) = 0$.

It is given that $(x + 1)$ is a factor of $f(x) = (2x^3 + ax^2 + 2bx + 1)$, then, $f(-1) = 0$

$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$\Rightarrow -2 + a - 2b + 1 = 0$

$\Rightarrow a - 2b - 1 = 0$ ...(i)

Also, it is given that $2a - 3b = 4$

$3b = 2a - 4$

$b = \frac{2a - 4}{3}$

Putting the value of b in eq. (i), we get

$\Rightarrow a - 2\left(\frac{2a - 4}{3}\right) - 1 = 0$

$\Rightarrow 3a - 2(2a - 4) - 3 = 0$

$\Rightarrow 3a - 4a + 5 = 0$

$\Rightarrow a = 5$

Putting the value of a in eq. (i), we get

$5 - 2b - 1 = 0$

$\Rightarrow 2b = 4$

$\Rightarrow b = 2$

Hence, the required values of a and b are 5 and 2 respectively.

15. The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y.

Ans.

It is given that x, y and 40° are the angles of a triangle.

We know that the sum of all the angles of a triangle is 180°

$\Rightarrow x + y + 40 = 180$

$\Rightarrow x + y = 140$ ...(i)

Also, it is given that the difference of angles x and y is

$x - y = 30$ ...(ii)

Adding eq. (i) and (ii), we get

$(x + y) + (x - y) = 140 + 30$

$\Rightarrow 2x = 170$

$\Rightarrow x = 85$

Putting the value of x in eq. (i), we get

$\Rightarrow 85 + y = 140$

$\Rightarrow y = 140 - 85$

$y = 55$

Hence, the required values of x and y are 85° and 55° respectively.

16. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now? [CBSE 2019]

Ans.

Let Salim's present age be x years and his daughter's present age be y years.

By the given condition, two years ago, Salim was thrice as old as his daughter.

$\Rightarrow (x - 2) = 3(y - 2)$

$\Rightarrow x - 2 = 3y - 6$

$\Rightarrow x - 3y = -4$ ...(i)

By the second condition, six years later, Salim will be four years older than twice her age.

$(x + 6) = 2(y + 6) + 4$

$\Rightarrow x + 6 = 2y + 16$

$\Rightarrow x - 2y = 10$ ...(ii)

Subtracting eq. (i) from eq. (ii), we get

$(x - 2y) - (x - 3y) = 10 - (-4)$

$\Rightarrow y = 14$

Putting the value of y in eq. (ii), we get

$x - 2(14) = 10$

$\Rightarrow x = 38$

Hence, Salim and his daughter's ages are 38 years and 14 years, respectively.

17. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father. [CBSE 2020, 19]

Ans.

Let father's present age (in years) be x and his two children's ages be y and z years respectively.

By the given condition, the age of the father is twice the sum of the ages of his two children.

$x = 2(y + z)$ ...(i)

After 20 years, his age will be equal to the sum of the ages of his children.

$\Rightarrow (x + 20) = (y + 20) + (z + 20)$

$\Rightarrow x = y + z + 20$

$\Rightarrow y + z = x - 20$ ...(ii)

Putting the value of $(y + z)$ in eq. (i) we get the present age of the father

$\Rightarrow x = 2(x - 20)$

$\Rightarrow x = 40$

Hence, the father's age is 40 years.

18. Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4:5. Find the numbers. **[CBSE 2019]**

Ans.

Let the two numbers be x and y.

It is given that these two numbers are in the ratio 5:6.

$$\Rightarrow \quad \frac{x}{y} = \frac{5}{6}$$

$$\Rightarrow \quad y = \frac{6}{5}x$$

Also, it is given that if 8 is subtracted from each of the numbers, then the ratio becomes 4:5.

$$\Rightarrow \quad \frac{(x-8)}{(y-8)} = \frac{4}{5}$$

$$\Rightarrow \quad 5(x-8) = 4(y-8)$$

$$\Rightarrow \quad 5x - 4y = 8$$

Putting the value of y in eq. (ii), we get

$$5x - 4\left(\frac{6x}{5}\right) = 8$$

$$\Rightarrow \quad 25x - 24x = 8 \times 5$$

$$\Rightarrow \quad x = 40$$

Putting the value of x in eq. (i), we get

$$\Rightarrow \quad y = \frac{6}{5} \times 40$$

$$= 6 \times 8 = 48$$

Hence, the required numbers are 40 and 48.

19. There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But, if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

Ans.

Let the number of students in hall A and B be x and y respectively.

By the given condition, to make the number of students equal in each hall, 10 students are sent from A to B

$$\Rightarrow \quad x - 10 = y + 10$$

$$\Rightarrow \quad x - y = 20 \quad \text{...(i)}$$

Also, it is given that if 20 students are sent from B to A, the number of students in A becomes double the number of students in B

$$\Rightarrow \quad (x + 20) = 2(y - 20)$$

$$x - 2y = -60 \quad \text{...(ii)}$$

Subtracting eq. (ii) from eq. (i), we get

$$(x - y) - (x - 2y) = 20 - (-60)$$

$$\Rightarrow \quad y = 80$$

Putting the value of y in eq. (i), we get

$$x - 80 = 20$$

$$\Rightarrow \quad x = 100$$

Hence, 100 students are in hall A and 80 students are in hall B.

20. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for book kept for four days. Find a fixed charges and the charge for each extra day. **[CBSE 2019]**

Ans.

Let Latika take a fixed charge of ₹ x for the first two days and an additional charge of ₹ y for each day thereafter.

It is given that Latika paid ₹ 22 for a book kept for six days

$\Rightarrow$ For the first two days = ₹ x

For the remaining days = ₹ $4y$

$$x + 4y = 22 \quad \text{...(i)}$$

Also, it is given that Anand paid ₹ 16 for a book kept for four days

$\Rightarrow$ For the first two days = ₹ x

For the remaining days = ₹ $2y$

$$x + 2y = 16 \quad \text{...(ii)}$$

Now, subtracting eq. (ii) from eq. (i), we get

$$(x + 4y) - (x + 2y) = 22 - 16$$

$$\Rightarrow \quad 2y = 6$$

$$\Rightarrow \quad y = 3$$

Putting the value of y in eq. (ii), we get

$$x + 2(3) = 16$$

$$\Rightarrow \quad x = 10$$

Hence, the fixed charge is ₹ 10 and the charge for each extra day is ₹ 3.

21. In a competitive examination, one mark is awarded for each correct answer, while $\frac{1}{2}$ mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Ans.

Let x be the number of correct answers, then:

Marks awarded for correct answer = $x \times 1 = x$

Total no. of questions attempted = 120

Number of wrong answers = $(120 - x)$

Marks deducted for wrong answers

$$= (120 - x) \times \frac{1}{2} = \frac{120 - x}{2}$$

Total marks awarded to Jayanti = 90

$$\Rightarrow \quad x - \left(\frac{120 - x}{2}\right) = 90$$

$$\Rightarrow \quad x + \frac{x}{2} - 60 = 90$$

$$\Rightarrow \quad \frac{3x}{2} = 150$$

$\Rightarrow \quad x = \frac{150 \times 2}{3}$

$\Rightarrow \quad x = 100$

Hence, Jayanti answered 100 questions correctly.

22. The angles of a cyclic quadrilateral ABCD are $\angle A = (6x + 10)°$, $\angle B = (5x)°$, $\angle C = (x + y)°$ and $\angle D = (3y - 10)°$.

Find x and y and hence the values of the four angles.

Ans.

It is given that

$\angle A = (6x + 10)°$

$\angle B = (5x)°$

$\angle C = (x + y)°$ and

$\angle D = (3y - 10)°$.

We know that by property of cyclic quadrilateral:

Sum of opposite angles = 180°

$\angle A + \angle C = 180°$

$\Rightarrow \quad (6x + 10) + (x + y) = 180°$

$\Rightarrow \quad 6x + 10 + x + y = 180°$

$\Rightarrow \quad 7x + y = 170° \quad ...(i)$

Also, $\angle B + \angle D = 180°$

$\Rightarrow \quad 5x + (3y - 10) = 180°$

$\Rightarrow \quad 5x + 3y = 180° + 10°$

$\Rightarrow \quad 5x + 3y = 190° \quad ...(ii)$

Multiplying eq. (i) by 3 and then subtracting eq. (ii) from it, we get

$3(7x + y) - (5x + 3y) = 3(170) - 190$

$\Rightarrow \quad 16x = 320$

$\Rightarrow \quad x = 20°$

Putting the value of $x = 20°$ in eq. (i), we get

$7(20) + y = 170$

$\Rightarrow \quad y = 30°$

$\angle A = (6x + 10)°$

$= (6 \times 20 + 10)°$

$= (120 + 10)° = 130°$

$\angle B = (5x)°$

$= (5 \times 20)°$

$= 100°$

$\angle C = (x + y)°$

$= (20 + 30)°$

$= 50°$

$\angle D = (3y - 10)°$

$= (3 \times 30 - 10)°$

$= (90 - 10)°$

$= 80°$

Hence, the required values of x and y are 20° and 30° respectively, mand the values of the four angles i.e., $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are 130°, 100°, 50°, and 80° respectively.

EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$2x + y = 6$ and

$2x - y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis. [CBSE 2016]

Ans.

The given equations are

$2x + y = 6$

and $2x - y + 2 = 0$

Table for equation

$2x + y - 6 = 0$

$y = -2x + 6$

If $x = 0$, $y = 6 - 2(0) = 6$

If $x = 1$, $y = 6 - 2(1) = 6 - 2 = 4$

If $x = 2$, $y = 6 - 2(2) = 6 - 4 = 2$

x	0	1	2
y	6	4	2

Table for equation

$2x - y + 2 = 0$,

$y = 2x + 2$

If $x = 0$, $y = 2(0) + 2 = 0 + 2 = 2$

If $x = 1$, $y = 2(1) + 2 = 2 + 2 = 4$

If $x = 2$, $y = 2(2) + 2 = 4 + 2 = 6$

x	0	1	2
y	2	4	6

Let area of triangle formed with x -axis = T_1

T_1 = Area of ΔACE

$= \frac{1}{2} \times AC \times PE$

$T_1 = \frac{1}{2} \times 4 \times 4 = 8$

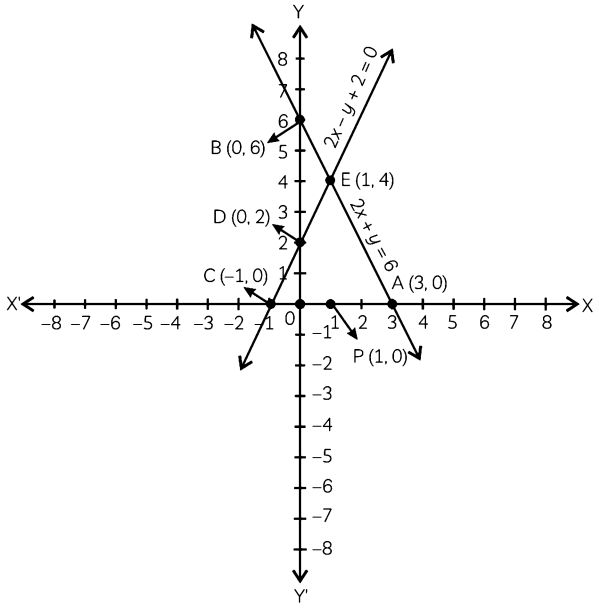
and area of triangle formed with y – axis = T_2

T_2 = Area of ΔBDE

$= \frac{1}{2} \times BD \times QE$

$T_2 = \frac{1}{2} \times 4 \times 1 = 2$

$T_1 : T_2 = 8:2 = 4:1$



2. Determine, graphically, the vertices of the triangle formed by the lines

***y* = *x*, 3*y* = *x* and *x* + *y* = 8. [CBSE 2017]**

Ans.

The given linear equations are

$y = x \quad 3y = x \quad x + y = 8$

Table for line $y = x$,

If $x = 1$, $y = 1$

If $x = 0$, $y = 0$

If $x = 2$, $y = 2$

x	0	1	2
y	0	1	2

Table for line $x = 3y$, $y = \frac{x}{3}$

If $x = 0$, $y = 0$

If $x = 3$, $y = 1$

If $x = 6$, $y = 2$

x	0	3	6
y	0	1	2

Table for line $x + y = 8$

$\Rightarrow \quad y = 8 - x$

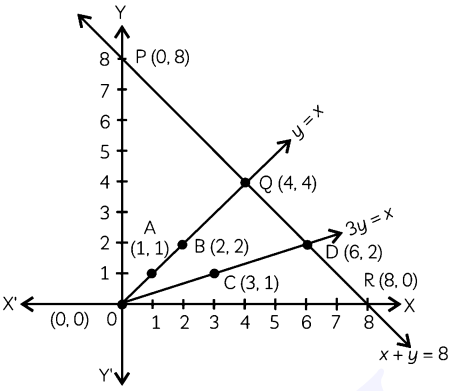
If $x = 0$, $y = 8$

If $x = 8$, $y = 0$

If $x = 4$, $y = 4$

x	0	4	8
y	8	4	0

Plotting $y = x$, $3y = x$ and $x + y = 8$, we get three lines OQ, OD and PR respectively, as shown below:



We see that lines OQ and OD intersect the line PR on Q and D respectively.

So, ΔOQD is formed by these lines. Hence, the vertices of the ΔOQD formed by the given lines are O(0, 0), Q(4, 4) and D(6, 2).

3. Draw the graphs of the equations *x* = 3, *x* = 5 and 2*x* – *y* – 4 = 0. Also find the area of the quadrilateral formed by the lines and the x–axis.

Ans.

The given equation of the lines are

$x = 3 \quad x = 5 \quad$ and $\quad 2x - y - 4 = 0$

Table for line

$2x - y - 4 = 0$

$\Rightarrow \quad y = 2x - 4$

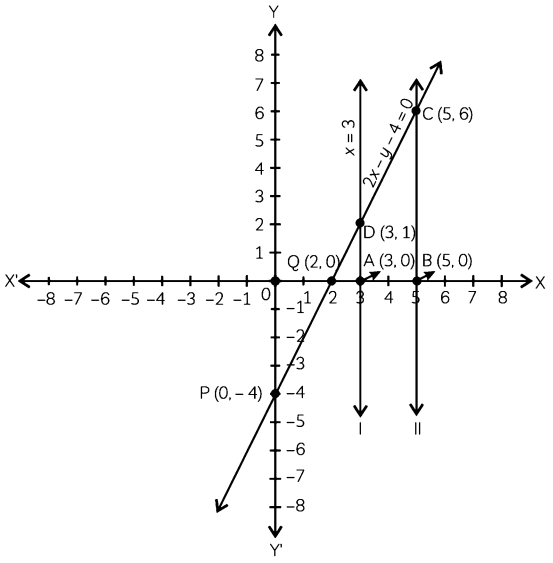
If $x = 0$, $y = -4$

If $x = 2$, $y = 0$

If $x = 4$, $y = 4$

x	0	2	4
y	–4	0	4

Plotting $x = 3$ and $x = 5$ and $2x - y - 4 = 0$, we obtain three lines I, II and III respectively, forming a quadrilateral ABCD with the X-axis as shown below:



From the graph, we get,

$$AB = OB - OA = 5 - 3 = 2$$

$$AD = 2$$

$$BC = 6$$

We know that the quadrilateral ABCD is a trapezium.

Area of Quadrilateral ABCD

$= \frac{1}{2}$ × (distance between parallel lines) × (sum of parallel sides)

$= \frac{1}{2} \times (AB) \times (AD + BC)$

$= \frac{1}{2} \times 2 \times (2 + 6)$

= 8 sq. units

Hence, the area of the required quadrilateral is 8 square units.

4. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation, find the cost of a pen and a pencil box.

[CBSE 2015]

Ans.

Let the cost of a pen be ₹ x

and the cost of a pencil box be ₹ y.

According to the question,

$\Rightarrow \quad 4x + 4y = 100$

$\Rightarrow \quad x + y = 25 \quad ...(i)$

$3x = y + 15$

$\Rightarrow \quad 3x - y = 15 \quad ...(ii)$

Adding equations (i) and (ii), we get

$4x = 40$

$\Rightarrow \quad x = 10$

Putting the value of $x = 10$ in eq. (i), we get

$10 + y = 25$

$y = 25 - 10 = 15$

Hence, the cost of a pen is ₹ 10 and the cost of a pencil box is ₹ 15

5. Determine, algebraically, the vertices of the triangle formed by the lines

$3x - y = 3$,

$2x - 3y = 2$ and

$x + 2y = 8$

Ans.

The given equation of lines are:

$3x - y = 3 \quad ...(i)$

$2x - 3y = 2 \quad ...(ii)$

$x + 2y = 8 \quad ...(iii)$

Let lines (i), (ii) and (iii) represent the side of a ΔABC i.e., AB, BC and CA respectively.

On solving lines (i) and (ii), we will get the intersection point B.

Multiplying eq. (i) by 3 and then subtracting eq. (ii), we get

$\Rightarrow (9x - 3y) - (2x - 3y) = 9 - 2$

$\Rightarrow \quad 7x = 7$

$\Rightarrow \quad x = 1$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 3 \times 1 - y = 3$

$\Rightarrow \quad y = 0$

Hence, the coordinate of point or vertex B is (1, 0).

On solving lines (ii) and (iii), we will get the intersection point C.

Multiplying eq. (iii) by 2 and then subtracting eq. (ii), we get

$(2x + 4y) - (2x - 3y) = 16 - 2$

$\Rightarrow \quad 7y = 14$

$\Rightarrow \quad y = 2$

Putting the value of y in eq. (iii), we get

$\Rightarrow \quad x + 2(2) = 8$

$\Rightarrow \quad x = 4$

Hence, the coordinate of point or vertex C is (4, 2).

On solving lines (iii) and (i), we will get the intersecting point A.

Multiplying eq. (i) by 2 and then adding eq. (iii), we get

$(6x - 2y) + (x + 2y) = 6 + 8$

$\Rightarrow \quad 7x = 14$

$\Rightarrow \quad x = 2$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 3 \times 2 - y = 3$

$\Rightarrow \quad y = 3$

Hence, the coordinate of point or vertex A is (2, 3).

Hence, the vertices of the ΔABC formed by the given lines are A (2, 3), B(1, 0) and C (4, 2).

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus. **[CBSE 2013, 11]**

Ans.

Let the speed of the rickshaw and the bus be x km/hr and y km/hr, respectively.

We know that

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

and

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Case I:

Time taken by Ankita to travel 2 km by rickshaw,

$$t_1 = \frac{2}{x} \text{ hr}$$

Remaining distance = 14 – 2 = 12 km

Time taken by Ankita to travel remaining distance, i.e., 12 km by bus,

$$t_2 = \frac{12}{y} \text{ hr}$$

It is given that:

Total time taken by rickshaw and bus = $\frac{1}{2}$ hr

$$\Rightarrow \quad t_1 + t_2 = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2}{x} + \frac{12}{y} = \frac{1}{2} \qquad \text{...(i)}$$

Case II:

Time taken by Ankita to travel 4 km by rickshaw,

$$t_3 = \frac{4}{x} \text{ hr}$$

Remaining distance = 14 – 4 = 10 km

Time taken by Ankita to travel remaining distance i.e., 10 km by bus,

$$t_4 = \frac{10}{y} \text{ hr}$$

It is given that:

Total time taken by rickshaw and bus

$$= \left(\frac{1}{2} + \frac{9}{60}\right) \text{ hr}$$

$$= \left(\frac{1}{2} + \frac{3}{20}\right) \text{ hr}$$

$$t_3 + t_4 = \left(\frac{10}{20} + \frac{3}{20}\right) \text{ hr}$$

$$\frac{4}{x} + \frac{10}{y} = 13/20 \qquad \text{...(ii)}$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then eq. (i) and (ii) become

$$2u + 12v = \frac{1}{2} \qquad \text{...(iii)}$$

$$4u + 10v = \frac{13}{20} \qquad \text{...(iv)}$$

Multiplying eq. (iii) by 2 and then subtracting eq. (iv), we get

$$(4u + 24v) - (4u + 10v) = 1 - \frac{13}{20}$$

$$\Rightarrow \quad 14v = \frac{7}{20}$$

$$\Rightarrow \quad v = \frac{1}{40}$$

Putting the value of v in eq. (iii),

$$\Rightarrow \quad 2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$$

$$\Rightarrow \quad 2u = \frac{2}{10}$$

$$\Rightarrow \quad u = \frac{1}{10}$$

$$\Rightarrow \quad x = \frac{1}{u} = 10 \text{ km/hr}$$

$$\Rightarrow \quad y = \frac{1}{v} = 40 \text{ km/hr}$$

Hence, the speed of the rickshaw is 10 km/hr and the speed of bus is 40 km/hr.

7. A person, rowing at the speed of 5 km/hr in still water takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream. **[CBSE 2020, 14]**

Ans.

Let the speed of the stream be x km/hr.

Given speed of the person rowing in still water = 5 km/hr

The speed of the person rowing downstream = $(5 + x)$ km/hr

and the speed of the person rowing upstream = $(5 - x)$ km/hr.

We know that

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Time taken in rowing 40 km downstream,

$$t_1 = \frac{40}{5+x} \text{ hrs}$$

Time taken in rowing 40 km upstream,

$$t_2 = \frac{40}{5-x} \text{ hrs}$$

According to the given condition,

Time taken in rowing 40 km upstream = 3 × Time taken in rowing 40 km downstream

$$t_2 = t_1 \times 3$$

$$\Rightarrow \quad \frac{40}{5-x} = \frac{40}{5+x} \times 3$$

$$\Rightarrow \quad \frac{1}{5-x} = \frac{3}{5+x}$$

$$\Rightarrow \quad -3x + 15 = x + 5$$

$$\Rightarrow \quad -3x - x = 5 - 15$$

$$\Rightarrow \quad -4x = -10$$

$$\Rightarrow \quad x = \frac{10}{4}$$

$$\Rightarrow \quad x = 2.5 \text{ km/h}$$

Hence, the speed of the stream is 2.5 km/hr.

8. A motorboat can travel 30 km upstream and 28 km downstream in 7 hrs. It can travel 21 km upstream and return in 5 hrs. Find the speed of the boat in still water and the speed of the stream. **[CBSE 2019, 17, 12]**

Ans.

Let the speed of the boat in still water = x km/hr

the speed of the stream = y km/hr

the speed of the motorboat upstream = $(x - y)$ km/hr

the speed of the motorboat downstream = $(x + y)$ km/hr

Case I:

We know that $\text{time} = \frac{\text{distance}}{\text{speed}}$

Time taken by motorboat to travel 30 km upstream,

$$t_1 = \frac{30}{x-y} \text{ hrs}$$

Time taken by motorboat to travel 28 km downstream,

$$t_2 = \frac{28}{x+y} \text{ hrs}$$

According to the given condition,

$$t_1 + t_2 = 7 \text{ hrs}$$

$$\Rightarrow \quad \frac{30}{x-y} + \frac{28}{x+y} = 7 \qquad \text{...(i)}$$

Case II:

Time taken by motorboat to travel 21 km upstream,

$$t_3 = \frac{21}{x-y} \text{ hrs}$$

Time taken by motorboat to travel 21 km downstream,

$$t_4 = \frac{21}{x+y} \text{ hrs}$$

According to the given condition,

$$t_3 + t_4 = 5 \text{ hrs}$$

$$\Rightarrow \quad \frac{21}{x-y} + \frac{21}{x+y} = 5 \qquad \text{...(ii)}$$

Let $p = \frac{1}{x-y}$ and $q = \frac{1}{x+y}$

Putting these values in eq. (i) and eq. (ii) we get

$$30p + 28q = 7 \qquad \text{...(iii)}$$

and $\quad 21p + 21q = 5$

$$\Rightarrow \quad p + q = \frac{5}{21} \qquad \text{...(iv)}$$

Multiplying eq. (iv) by 28 and subtracting from eq. (iii), we get

$$(30p + 28q) - (28p + 28q) = 7 - \frac{140}{21}$$

$$\Rightarrow \quad 2p = 7 - \frac{20}{3}$$

$$\Rightarrow \quad 2p = \frac{1}{3}$$

$$\Rightarrow \quad p = \frac{1}{6}$$

Putting the value of p in eq. (iv), we get

$$\Rightarrow \quad \frac{1}{6} + q = \frac{5}{21}$$

$$\Rightarrow \quad q = \frac{5}{21} - \frac{1}{6}$$

$$= \frac{10-7}{42} = \frac{3}{42}$$

$$\Rightarrow \quad q = \frac{1}{14}$$

We know that

$$p = \frac{1}{x-y} \text{ and } q = \frac{1}{x+y}$$

$$\frac{1}{x-y} = \frac{1}{6}$$

$$(x - y) = 6 \qquad \text{...(v)}$$

$$\frac{1}{x+y} = \frac{1}{14}$$

$$(x + y) = 14 \qquad \text{...(vi)}$$

Adding eq. (v) and (vi), we get

$$2x = 20$$

$$x = 10$$

Putting the value of x in eq. (v), we get

$$10 - y = 6$$

$$y = 4$$

Hence, the speed of the motorboat in still water is 10 km/hr and the speed of the stream is 4 km/hr.

9. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number. [CBSE 2017, 12, 11, 10]

Ans.

Let the tens and ones digits be x and y respectively. Then, two-digit number,

$$N = 10x + y$$

Case I:

Multiplying the sum of the digits by 8 and then subtracting 5

$$N = 8(x + y) - 5$$

$$\Rightarrow \quad 10x + y = 8x + 8y - 5$$

$$\Rightarrow \quad 10x - 8x + y - 8y = -5$$

$$\Rightarrow \quad 2x - 7y = -5 \qquad \text{...(i)}$$

Case II:

Multiplying the difference of the digits by 16 and then adding 3

$$N = 16(x - y) + 3$$

$$\Rightarrow \quad 10x + y = 16x - 16y + 3$$

$$\Rightarrow \quad -6x + 17y = 3$$

$$\Rightarrow \quad 6x - 17y = -3 \qquad \text{...(ii)}$$

Multiplying eq. (i) by 3 and then subtracting from eq. (ii), we get

$$\Rightarrow (6x - 17y) - (6x - 21y) = -3 + 15$$

$$\Rightarrow \quad 4y = 12$$

$$\Rightarrow \quad y = 3$$

Putting the value of y in eq. (i), we get

$$\Rightarrow \quad 2x - 7(3) = -5$$

$$\Rightarrow \quad 2x = -5 + 21 = 16$$

$$\Rightarrow \quad x = 8$$

Hence, the required two-digit number

$$= 10x + y$$

$$= 10(8) + 3 = 80 + 3$$

$$= 83$$

Hence, the required number = 83.

10. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from station A to B costs ₹ 2530. Also, one reserved first class ticket and one reserved first class half ticket from station A to B costs ₹ 3810. Find the full first class fare from station A to B and also the reservation charges for a ticket. [CBSE 2019]

Ans.

Let the cost of full fare from station A to B = ₹ x

and the reservation charges per ticket = ₹ y.

Cost of one reserved first class ticket from station A to B = ₹ 2530

$\Rightarrow \quad x + y = 2530 \quad ...(i)$

Cost of one reserved first class ticket and one reserved first class half ticket from station A to B = ₹ 3810

$\Rightarrow \quad (x + y) + \left(\frac{x}{2} + y\right) = 3810$

$\Rightarrow \quad \left(\frac{3x}{2} + 2y\right) = 3810$

$\Rightarrow \quad (3x + 4y) = 7620 \quad ...(ii)$

Multiplying eq. (i) by 4 and then subtracting from eq. (ii), we get

$(3x + 4y) - (4x + 4y) = 7620 - 10120$

$-x = -2500$

$x = 2500$

Putting the value of x in eq. (i), we get

$2500 + y = 2530$

$y = 30$

Hence, full first class fare from station A to B is ₹ 2500 and the reservation for a ticket is ₹ 30.

11. A shopkeeper sells a saree at a profit of 8% and a sweater at a discount of 10%, thereby getting a sum ₹ 1008. If she had sold the saree at a profit of 10% and the sweater at a discount of 8%, she would have got ₹ 1028. Find the cost of the saree and the list price (price before discount) of the sweater.

Ans.

Let the cost price of a saree = ₹ x

and the list price of sweater = ₹ y

Case I:

(S. P. of saree at 8% profit) + (S.P. of a sweater at 10% discount) = ₹1008

$\Rightarrow$ (100 + 8) % of x + (100 – 10) % of y = 1008

$\Rightarrow \quad \frac{108x + 90y}{100} = 1008$

$\Rightarrow \quad 108x + 90y = 100800$

$\Rightarrow \quad 6x + 5y = 5600$

Multiplying above eq. by 46, we get

$\Rightarrow \quad 276x + 230y = 257600 \quad ...(i)$

Case II:

(S.P. of saree at 10% profit) + (S.P. of a sweater at 8% discount) = ₹1028

$\Rightarrow$ (100 + 10)% of x + (100–8%) of y = 1028

$\Rightarrow$110% of x + 92% of y = 1028

$\Rightarrow \quad 110x + 92y = 102800$

$\Rightarrow \quad 55x + 46y = 51400$

Multiplying above eq. by 5, we get

$\Rightarrow \quad 275x + 230y = 257000 \quad ...(ii)$

Subtracting eq. (ii) from eq. (i), we get

$\Rightarrow (276x + 230y) - (275x + 230y)$

$= 257600 - 257000$

$x = 600$

Putting the value of x in the above equation, we get

$6x + 5y = 5600$

$\Rightarrow \quad 6(600) + 5y = 5600$

$\Rightarrow \quad 5y = 5600 - 3600$

$\Rightarrow \quad y = \frac{2000}{5}$

$\Rightarrow \quad y = 400$

Hence, the cost price of the saree and the list price (price before discount) of the sweater are ₹ 600 and ₹ 400, respectively.

12. Susan invested a certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

Ans.

Let the money invested in scheme A = ₹ x

and the money invested in scheme B = ₹ y

Case I:

Susan invested ₹ x at 8% p.a. + Susan invested ₹ y at 9% p.a. and received ₹1860 as annual interest.

We know that simple interest,

$$SI = \frac{\text{Principal} \times \text{rate} \times \text{time}}{100}$$

Interest earned when ₹ x invested at 8% per annum on scheme A,

$$SI_1 = \frac{x \times 8 \times 1}{100} = \frac{8x}{100}$$

Interest earned when ₹ y invested at 9% per annum on scheme B,

$$SI_2 = \frac{y \times 9 \times 1}{100} = \frac{9y}{100}$$

Interest at 8% per annum on scheme A + Interest at 9% per annum on scheme B = 1860

$\Rightarrow \quad \frac{8x}{100} + \frac{9y}{100} = 1860$

$\Rightarrow \quad 8x + 9y = 186000 \quad ...(i)$

Case II:

Susan invested ₹ y at 8% p.a. + Susan invested ₹ x at 9% p.a. and received ₹ (1860 + 20) as annual interest.

Interest earned when ₹ y invested at 8% per annum on scheme A,

$$SI_1 = \frac{y \times 8 \times 1}{100} = \frac{8y}{100}$$

Interest earned when ₹ x invested at 9% per annum on scheme B,

$$SI_2 = \frac{x \times 9 \times 1}{100} = \frac{9x}{100}$$

Interest at 8% per annum on scheme A + Interest at 9% per annum on scheme B = 1880

$$\Rightarrow \quad \frac{8y}{100} + \frac{9x}{100} = 1880$$

$$\Rightarrow \quad 9x + 8y = 188000 \quad \text{...(ii)}$$

Multiplying eq. (i) by 9 and eq. (ii) by 8 and then subtracting them, we get

$$(72x + 81y) - (72x + 64y) = 1674000 - 1504000$$

$$\Rightarrow \quad 81y - 64y = 170000$$

$$\Rightarrow \quad 17y = 170000$$

$$\Rightarrow \quad y = 10000$$

Putting the value of y in eq. (i), we get

$$\Rightarrow \quad 8x + 9(10000) = 186000$$

$$\Rightarrow \quad 8x = 186000 - 90000$$

$$\Rightarrow \quad 8x = 96000$$

$$\Rightarrow \quad x = 12000$$

Hence, Susan invested ₹ 12000 and ₹ 10000 in schemes A and B respectively.

13. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

Ans.

Let the number of bananas in lot A = x

and the number of bananas in lot B = y

Case I:

Sold the first lot at the rate of ₹ 2 for 3 bananas + Sold the second lot at the rate of ₹ 1 per banana = Amount received

S.P. of 3 bananas of lot A = ₹ 2

$$\Rightarrow \text{S.P. of 1 banana of lot A} = ₹ \frac{2}{3}$$

$$\Rightarrow \text{S.P. of } x \text{ bananas of lot A} = ₹ \frac{2x}{3}$$

S.P. of 1 banana of lot B = ₹ 1

$$\Rightarrow \text{S.P. of } y \text{ bananas of lot B} = ₹ y$$

As per given condition

$$\frac{2x}{3} + y = 400$$

$$2x + 3y = 1200 \quad \text{...(i)}$$

Case II:

Sold the first lot at the rate of ₹ 1 per banana + Sold the second lot at the rate of ₹ 4 for 5 bananas = Amount received

S.P. of 1 banana of lot A = ₹ 1

$$\Rightarrow \text{S.P. of } x \text{ bananas of lot A} = ₹ x$$

S.P. of 5 bananas of lot B = ₹ 4

$$\Rightarrow \text{S.P. of 1 banana of lot B} = ₹ \frac{4}{5}$$

$$\Rightarrow \text{S.P. of } y \text{ bananas of lot B} = ₹ \frac{4y}{5}$$

As per the given condition

$$x + \frac{4y}{5} = 460$$

$$5x + 4y = 2300 \quad \text{...(ii)}$$

Multiplying eq. (i) by 4 and eq. (ii) by 3 and then subtracting them, we get

$$(8x + 12y) - (15x + 12y) = 4800 - 6900$$

$$\Rightarrow \quad -7x = -2100$$

$$\Rightarrow \quad x = 300$$

Putting the value of x in eq. (i), we get

$$2x + 3y = 1200$$

$$\Rightarrow \quad 2(300) + 3y = 1200$$

$$\Rightarrow \quad 3y = 1200 - 600$$

$$\Rightarrow \quad 3y = 600$$

$$\Rightarrow \quad y = 200$$

Total number of bananas

= Number of bananas in lot A + Number of bananas in lot B

$= (x + y)$

$= (300 + 200)$

$= 500$

Hence, the total number of bananas he had is 500.

1. When will the system $kx - y = 2$ and $6x - 2y = 3$ has a unique solution only?

Ans.

A pair of linear pair has unique solution only when,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Then, $\frac{k}{6} \neq \frac{-1}{-2}$

So, $k \neq 3$.

2. Ratio between the girls and boys in a class of 40 students is 2 : 3. Five new students joined the class. How many of them must be boys so that the ratio between girls and boys becomes 4 : 5?

Ans.

Let number of girls = $2x$

and number of boys = $3x$

Total students, $2x + 3x = 40$

$x = 8$

So, number of girls = $2 \times 8 = 16$

and number of boys = $3 \times 8 = 24$

Let out of 5 students, y denotes number of boys.

Then, number of girls = $5 - y$

According to question,

$$\frac{16+5-y}{24+y} = \frac{4}{5}$$

$5(21 - y) = 4(24 + y)$

$105 - 5y = 96 + 4y$

$9y = 105 - 96$

$9y = 9$

$y = 1$

So, there must be one boy among five new students.

3. In a ΔABC, ∠A = x°, ∠B = 3x° and ∠C = y°. If 3y° – 5x° = 30° prove that the triangle is right angled.

Ans.

We know that,

$\angle A + \angle B + \angle C = 180°$

[Sum of interior angles of triangle ABC is 180°]

$x + 3x + y = 180°$

$4x + y = 180$ ...(i)

$3y - 5x = 30$ [Given] ...(ii)

Multiply equation (i) by 3

$12x + 3y = 540$ ...(iii)

Subtraction (ii) and (iii), we get

$-17x = -510$

$x = 30°$

Putting value of x in equation (i), we get

$4 \times 30° + y = 180°$

$y = 60°$

$\angle A = 30°$,

$\angle B = 3 \times 30° = 90°$

And $\angle C = 60°$

Hence, ΔABC is right angled triangle at B.

4. A and B each has a certain number of mangoes. A says to B, "If you give 30 of your mangoes, I will have twice as many as left with you." B replies "If you give me 10, I will have thrice as many as left with you". How many mangoes does coach have?

Ans.

Let number of mangoes with A be x.

Number of mangoes with B be y.

According to the question,

$x + 30 = 2(y - 30)$

$x + 30 = 2y - 60$

$x - 2y = -90$ ...(i)

$y + 10 = 3(x - 10)$

$y + 10 = 3x - 30$

$3x - y = 40$ ...(ii)

Multiplying equation (ii) by 2 and subtracting from equation (i), we get

$-5x = -170$

$x = 34$

Putting $x = 34$ in equation (i), we get

$34 - 2y = -90$

$-2y = -90 - 34$

$-2y = -124$

$y = 62$

So, number of mangoes with A are 34 and number of mangoes with B are 62.

5. A man wished to give ₹ 12 to each person and found that he fell short of ₹ 6 when he wanted to give to all the persons present. He, therefore, distributed ₹ 9 to each person and found that ₹ 9 were left over. How much money did he have and how many persons were there?

Ans.

Let number of persons = x

Money share per person = y

Therefore, Total money = ₹ xy

According to the question,

$12 \times x = xy + 6$

$12x - 6 = xy$ ...(i)

$9x = xy - 9$

$9x + 9 = xy$ ...(ii)

Equating (i) and (ii), we get

$12x - 6 = 9x + 9$

$3x = 15$

Put the value of x in equation (i). Then

$12 \times 5 - 6 = x \times y$

$\Rightarrow$ $xy = 54$

So, he have ₹ 54 and there were 5 persons.

4 Quadratic Equations

EXERCISE 4.1

Choose the correct answer from the given four options in the following questions:

1. Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(B) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

(C) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$

(D) $x^3 - x^2 = (x - 1)^3$

Ans. (D)

Explanation: Given equation is

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$

$\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$

$\Rightarrow 10x = 18$

$\Rightarrow 10x - 18 = 0$

This equation is not of the form $ax^2 + bx + c$, $a \neq 0$.

Thus, it is not a quadratic equation.

(B) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

$\Rightarrow -2x^2 = 10x - 2x^2 - 2 + \frac{2x}{5}$

$\Rightarrow 50x + 2x - 10 = 0$

$\Rightarrow 52x - 10 = 0$

This is also not a quadratic equation as it is also not of the form $ax^2 + bx = c = 0$, $a \neq 0$.

(C) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k - 1$

$\Rightarrow (-1 + 1)x^2 + \frac{3x}{2} = 7$

$\Rightarrow \frac{3x}{2} = 7$

$\Rightarrow 3x - 14 = 0$

This is also not a quadratic equation as it is also not of the form $ax^2 + bx = c = 0$, $a \neq 0$.

(D) $x^3 - x^2 = (x - 1)^3$

$x^3 - x^2 = x^3 - 3x^2(1) + 3x(1)^2 - (1)^3$

$[\because (a - b)^3 = a^3 - 3a^2b + 3b^2a - b^3]$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$\Rightarrow -x^2 + 3x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

This represents a quadratic equation because it is of the form

$ax^2 + bx + c = 0$, $a \neq 0$.

2. Which of the following is not a quadratic equation?

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$

(B) $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

(D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Ans. (C)

Explanation: It is given that

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$

$\Rightarrow 2(x^2 + 1 - 2x) = 4x^2 - 2x + 1$

$\Rightarrow 2x^2 + x^2 - 4x = 4x^2 - 2x + 1$

$\Rightarrow 2x^2 + 2x - 1 = 0$

This represents a quadratic equation as it is of the form $ax^2 + bx + c = 0$, $a \neq 0$.

(B) $2x - x^2 = x^2 + 5$

$\Rightarrow x^2 + 5 + x^2 - 2x = 0$

$\Rightarrow 2x^2 - 2x + 5 = 0$

This also represents a quadratic equation.

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

$\Rightarrow 2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$

$\Rightarrow 3x^2 + 3 + 2\sqrt{6}x - 3x^2 + 5x = 0$

$\Rightarrow (5 + 2\sqrt{6})x + 3 = 0$

This does not represent a quadratic equation as it is not of the form

$ax^2 + bx + c = 0$, $a \neq 0$.

(D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

$x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^3$

$4x^2 - 3 = 0$

This represents a quadratic equation.

Definition

An equation which is of the form $ax^2 + bx + c = 0$, $a \neq o$ is called a quadratic eguation.

3. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

[CBSE 2012]

Ans. (C)

Explanation:

(A) Putting the value of $x = 2$ in $x^2 - 4x + 5 = 0$, we get

$(2)^2 - 4(2) + 5 = 0$

$\Rightarrow \quad 4 - 8 + 5 = 0$

$\Rightarrow \quad 1 \neq 0$

So, $x = 2$ is not a root of $x^2 - 4x + 5 = 0$.

(B) Putting the value of $x = 2$ in $x^2 + 3x - 12 = 0$, we get

$(2)^2 + 3(2) - 12 = 0$

$\Rightarrow \quad 4 + 6 - 12 = 0$

$\Rightarrow \quad -2 \neq 0$

So, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$.

(C) Putting the value of $x = 2$ in $2x^2 - 7x + 6 = 0$, we get

$2(2)^2 - 7(2) + 6 = 0$

$\Rightarrow \quad 8 - 14 + 6 = 0$

$\Rightarrow \quad 0 = 0$

So, $x = 2$ is the root of the equation $2x^2 - 7x + 6 = 0$.

(D) Putting the value of $x = 2$ in $3x^2 - 6x - 2 = 0$, we get

$3(2)^2 - 6(2) - 2 = 0$

$\Rightarrow \quad 12 - 12 - 2 = 0$

$\Rightarrow \quad -2 \neq 0$

So, $x = 2$ is not the root of the equation $3x^2 - 6x - 2 = 0$.

Trick Applied

A real number α is said to be the root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.

4. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is:

(A) 2 (B) –2

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

[CBSE 2018, 15, 10]

Ans. (A)

Explanation: It is given that $\frac{1}{2}$ is the root of the equation

$$x^2 + kx - \frac{5}{4} = 0$$

$$\therefore \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\Rightarrow \quad \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\Rightarrow \quad \frac{1 + 2k - 5}{4} = 0$$

$$\Rightarrow \quad 2k - 4 = 0$$

$$\Rightarrow \quad k = 2$$

5. Which of the following equations has the sum of its roots as 3?

(A) $2x^2 - 3x + 6 = 0$

(B) $-x^2 + 3x - 3 = 0$

(C) $\sqrt{2}\,x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

(D) $3x^2 - 3x + 3 = 0$

Ans. (B)

Explanation:

We know that sum of the roots $= -\frac{b}{a}$

On comparing the given equations with $ax^2 + bx + c = 0$:

(A) $2x^2 - 3x + 6 = 0$

$a = 2, b = -3, c = 6$

$\Rightarrow$ Sum of its roots $= -\frac{b}{a} = -\frac{(-3)}{2} = \frac{3}{2}$

(B) $-x^2 + 3x - 3 = 0$

$a = -1, b = 3, c = -3$

$\Rightarrow$ Sum of its roots $= -\frac{b}{a} = -\left(\frac{3}{-1}\right) = 3$

(C) $\sqrt{2}\,x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

$\Rightarrow a = \sqrt{2}, b = \frac{3}{\sqrt{2}}, c = 1$

$\Rightarrow$ Sum of its roots $= -\frac{b}{a} = -\frac{\left(\frac{-3}{\sqrt{2}}\right)}{\sqrt{2}} = \frac{3}{2}$

(D) $3x^2 - 3x + 3 = 0$

$\Rightarrow a = 3, b = -3$ and $c = 3$

$\Rightarrow$ Sum of its roots $= -\frac{b}{a} = -\frac{(-3)}{3} = 1$

6. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is:

(A) 0 only (B) 4

(C) 8 only (D) 0, 8

[CBSE 2020, 19, 17, 15, 14, 12, 10]

Ans. (D)

Explanation: The given equation is:

$$2x^2 - kx + k = 0$$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 2, b = -k, c = k$.

For equal roots, the discriminant must be zero

i.e. $D = b^2 - 4ac = 0$

$\Rightarrow \quad (-k)^2 - 4(2)(k) = 0$

$\Rightarrow \quad k^2 - 8k = 0$

$\Rightarrow \quad k(k - 8) = 0 \quad \Rightarrow \quad k = (0, 8)$

Hence, values of k are 0, 8.

7. **Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?**

(A) $\frac{1}{8}$ (B) $\frac{1}{64}$

(C) $\frac{1}{4}$ (D) $\frac{9}{64}$ **[CBSE 2015]**

Ans. (B)

Explanation: The given equation is:

$$9x^2 + \frac{3x}{4} - \sqrt{2} = 0$$

$$\Rightarrow \quad (3x)^2 + \frac{1}{4}.3x - \sqrt{2} = 0$$

Let $3x = y$, then

$$y^2 + \frac{y}{4} - \sqrt{2} = 0$$

$$\Rightarrow y^2 + \frac{y}{4} + \left(\frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 - \sqrt{2} = 0$$

$$\Rightarrow \quad \left(y + \frac{1}{8}\right)^2 = \frac{1}{64} + \sqrt{2}$$

$$\Rightarrow \quad \left(y + \frac{1}{8}\right)^2 = \frac{1 + 64\sqrt{2}}{64}$$

Thus, $\frac{1}{64}$ must be added and subtracted to solve the given equation.

8. **The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has:**

(A) two distinct real roots

(B) two equal real roots

(C) no real roots

(D) more than 2 real roots

Ans. (C)

Explanation: We know that if $D = b^2 - 4ac < 0$ for a quadratic equation $ax^2 + bx + c = 0$, then roots are not real.

The given equation is:

$$2x^2 - \sqrt{5}\,x + 1 = 0$$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = -\sqrt{5}, c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (-\sqrt{5})^2 - 4(2)(1)$$

$$D = 5 - 8 = -3$$

$$\Rightarrow \quad D = -3 < 0$$

Since the discriminant is negative, the given equation has no real roots.

9. **Which of the following equations has two distinct real roots?**

(A) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(B) $x^2 + x - 5 = 0$

(C) $x^2 + 3x + 2\sqrt{2} = 0$

(D) $5x^2 - 3x + 1 = 0$

Ans. (B)

Explanation: We know that if $D = b^2 - 4ac > 0$ for the quadratic equation $ax^2 + bx + c = 0$, then its roots are real and distinct.

(A) The given equation is:

$$2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$$

$$\Rightarrow \quad a = 2, b = -3\sqrt{2}, c = \frac{9}{4}$$

$$\Rightarrow \quad D = b^2 - 4ac$$

$$= (-3\sqrt{2})^2 - 4(2)\left(\frac{9}{4}\right)$$

$$= 18 - 18 = 0$$

So, the equation has real and equal roots.

(B) The given equation is:

$$x^2 + x - 5 = 0$$

$$\Rightarrow \quad a = 1, b = 1, c = -5$$

$$\Rightarrow \quad D = b^2 - 4ac$$

$$= (1)^2 - 4(1)(-5)$$

$$= 1 + 20 = 21 > 0$$

So, the equation has two distinct real roots.

(C) The given equation is:

$$x^2 + 3x + 2\sqrt{2} = 0$$

$$\Rightarrow \quad a = 1, b = 3, c = 2\sqrt{2}$$

$$\Rightarrow \quad D = b^2 - 4ac$$

$$= (3)^2 - 4(1)(2\sqrt{2})$$

$$= 9 - 8\sqrt{2} < 0$$

So, the equation has no real roots.

(D) The given equation is:

$$5x^2 - 3x + 1 = 0$$

$$\Rightarrow \quad a = 5, b = -3, c = 1$$

$$\Rightarrow \quad D = b^2 - 4ac$$

$$= (-3)^2 - 4(5)(1)$$

$$= 9 - 20$$

$$= -11 < 0$$

So, the equation has no real roots.

Trick Applied

A quadratic equation $ax^2 + bx + c = 0$ has:

(i) two distinct real roots if $b^2 - 4ac > 0$.

(ii) two equal roots if $b^2 - 4ac = 0$.

(iii) no real roots if $b^2 - 4ac < 0$.

10. **Which of the following equations has no real roots?**

(A) $x^2 - 4x + 3\sqrt{2} = 0$ **(B)** $x^2 + 4x - 3\sqrt{2} = 0$

(C) $x^2 - 4x - 3\sqrt{2} = 0$ **(D)** $3x^2 + 4\sqrt{3}x + 4 = 0$

Ans. (A)

Explanation: We know for the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

- If $D = b^2 - 4ac > 0$, then its roots are distinct and real.

- If $D = b^2 - 4ac = 0$, then its roots are real and equal.
- If $D = b^2 - 4ac < 0$, then roots are not real and imaginary.

(A) The given equation is:

$x^2 - 4x + 3\sqrt{2} = 0$

$\Rightarrow \quad a = 1, b = -4, c = 3\sqrt{2}$

$\Rightarrow \quad D = b^2 - 4ac$

$= (-4)^2 - 4(1)(3\sqrt{2})$

$= 16 - 12\sqrt{2}$

$= 16 - 12\ (1.414)$

$= 16 - 16.92$

$= -0.92 < 0$

$\because$ D < 0, Therefore, the equation has no real roots.

(B) The given equation is:

$x^2 + 4x - 3\sqrt{2} = 0$

$\Rightarrow \quad a = 1, b = 4, c = -3\sqrt{2}$

$\Rightarrow \quad D = b^2 - 4ac$

$= (4)^2 - 4(1)(-3\sqrt{2})$

$= (4)^2 + 12\sqrt{2}$

$D = 16 + 12\sqrt{2} > 0$

So, the equation has two distinct real roots.

(C) The given equation is:

$x^2 - 4x - 3\sqrt{2} = 0$

$\Rightarrow \quad a = 1, b = -4, c = -3\sqrt{2}$

$\Rightarrow \quad D = b^2 - 4ac$

$= (-4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

So, the equation has two distinct real roots,

(D) The given equation is:

$3x^2 + 4\sqrt{3}x + 4 = 0$

$\Rightarrow \quad a = 3, \ b = 4\sqrt{3}$ and $c = 4$

$\Rightarrow \quad D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0$

So, the equation has two equal real roots.

Thus $x^2 - 4x + 3\sqrt{2} = 0$ has no real roots.

11. $(x^2 + 1)^2 - x^2 = 0$ has:

(A) four real roots **(B) two real roots**

(C) no real root **(D) one real root**

Ans. (C)

Explanation: The given equation is:

$(x^2 + 1)^2 - x^2 = 0$

$\Rightarrow x^4 + 1 + 2x^2 - x^2 = 0$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \quad x^4 + x^2 + 1 = 0$

Let $x^2 = y$

$(x^2)^2 + x^2 + 1 = 0$

$\Rightarrow \quad y^2 + y + 1 = 0$

On comparing with $ay^2 + by + c = 0$, we get

$a = 1, b = 1, c = 1$

$\Rightarrow \quad D = b^2 - 4ac$

$= (1)^2 - 4(1)(1)$

$= -3 < 0$

As D < 0, thus, we can say equation has no real roots

for $\quad y^2 + y + 1 = 0$

i.e. $\quad x^4 + x^2 + 1 = 0$

or $\quad (x^2 + 1)^2 - x^2 = 0$ has no real roots.

EXERCISE 4.2

1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i) $x^2 - 3x + 4 = 0$

(ii) $2x^2 + x - 1 = 0$

(iii) $2x^2 - 6x + \frac{9}{2} = 0$

(iv) $3x^2 - 4x + 1 = 0$

(v) $(x + 4)^2 - 8x = 0$

(vi) $(x - \sqrt{2})^2 - 2(x + 1) = 0$

(vii) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

(viii) $x(1 - x) - 2 = 0$

(ix) $(x - 1)(x + 2) + 2 = 0$

(x) $(x + 1)(x - 2) + x = 0$

Ans.

We know that the quadratic equation $ax^2 + bx + c = 0$ has two distinct real root if

$b^2 - 4ac > 0$

Comparing given equations with $ax^2 + bx + c = 0$

(i) The given equation is:

$x^2 - 3x + 4 = 0$

$a = 1, b = -3, c = 4.$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (-3)^2 - 4(1)(4)$

$D = 9 - 16 = -7 < 0$

i.e. $\quad D < 0$

Hence, the equation $x^2 - 3x + 4 = 0$ has no real roots.

(ii) The given equation is:

$2x^2 + x - 1 = 0$

$a = 2, b = 1, c = -1,$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (1)^2 - 4(2)(-1)$

$\Rightarrow \quad D = 1 + 8 = 9 > 0$

i.e. $\quad D > 0$

Hence, the equation $2x^2 + x - 1 = 0$ has two distinct real roots.

(iii) The given equation is:

$2x^2 - 6x + \frac{9}{2} = 0$

$a = 2, b = -6, c = \frac{9}{2}.$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (-6)^2 - 4(2)\left(\frac{9}{2}\right)$

$D = 36 - 36 = 0$

i.e. $\quad D = 0$

Hence, the equation $2x^2 - 6x + \frac{9}{2} = 0$ has real & equal roots.

(iv) The given equation is:

$3x^2 - 4x + 1 = 0$

$a = 3, b = -4, c = 1$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (-4)^2 - 4(3)(1)$

$D = 16 - 12 = 4 > 0$

i.e. $\quad D > 0$

Hence, the equation $3x^2 - 4x + 1 = 0$ has two distinct real roots.

(v) The given equation is:

$(x + 4)^2 - 8x = 0$

$x^2 + 16 + 8x - 8x = 0$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$x^2 + 16 = 0$

or $\quad x + 0x + 16 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = 0, c = 16$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (0)^2 - 4(1)(16)$

$= -64 < 0$

i.e. $\quad D < 0$

Hence, the equation $(x + 4)^2 - 8x = 0$ has no real roots.

(vi) The given equation is:

$(x - \sqrt{2})^2 - 2(x + 1) = 0$

$\Rightarrow \quad x^2 + 2 - 2\sqrt{2}x - 2x - 2 = 0$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$x^2 - (2\sqrt{2} + 2)x + 0 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -(2\sqrt{2} + 2), c = 0$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= [-(2\sqrt{2} + 2)]^2 - 4(1)(0)$

$= (2\sqrt{2} + 2)^2 > 0$

i.e. $\quad D > 0$

$[\because$ the square of any no. is $> 0]$

Hence, the equation $(x - \sqrt{2})^2 - 2(x + 1) = 0$ has two distinct real roots.

(vii) The given equation is:

$\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

$a = \sqrt{2}, b = -\frac{3}{\sqrt{2}}, c = \frac{1}{\sqrt{2}}$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= \left(-\frac{3}{\sqrt{2}}\right)^2 - 4(\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{9}{2} - 4$

$= \frac{9-8}{2} = \frac{1}{2} > 0$

i.e. $\quad D > 0$

Hence, the equation $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$ has two distinct real roots.

(viii) The given equation is:

$x(1 - x) - 2 = 0$

$\Rightarrow \quad x - x^2 - 2 = 0$

$\Rightarrow \quad -x^2 + x - 2 = 0$

$\Rightarrow \quad x^2 - x + 2 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -1, c = 2$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (-1)^2 - 4(1)(2)$

$= 1 - 8 = -7 < 0$

i.e. $\quad D < 0$

Hence, given equation $x(1 - x) - 2 = 0$ has no real roots.

(ix) The given equation is:

$(x - 1)(x + 2) + 2 = 0$

$x^2 + 2x - x - 2 + 2 = 0$

$x^2 + x + 0 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = 1, c = 0$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (1)^2 - 4(1)(0)$

$= 1 - 0 = 1 > 0$

i.e. $D > 0$

Hence, the given equation $(x - 1)(x + 2) + 2 = 0$ has two distinct real roots.

(x) The given equation is:

$(x + 1)(x - 2) + x = 0$

$x^2 - 2x + x - 2 + x = 0$

$\Rightarrow x^2 - 2 = 0$

$\Rightarrow x^2 + 0x - 2 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = 0, c = -2$

$\Rightarrow$ Discriminant,

$D = b^2 - 4ac$

$= (0)^2 - 4(1)(-2)$

$= 8 > 0$

i.e. $D > 0$

Hence, the given equation $(x + 1)(x - 2) + x = 0$ has two distinct real roots.

2. Write whether the following statements are true or false. Justify your answers.

(i) Every quadratic equation has exactly one root.

(ii) Every quadratic equation has at least one real root.

(iii) Every quadratic equation has at least two roots.

(iv) Every quadratic equations has at most two roots.

(v) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

(vi) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

Ans.

(i) False, a quadratic equation has at most two roots.

For example: $x^2 = 1$ is a quadratic equation with two roots.

(ii) False, a quadratic equation may not have any real roots.

For example: $x^2 + 1 = 0$ has no real root.

(iii) False, a quadratic equation may not have any real root.

For example: $x^2 + 2 = 0$ has no real roots.

(iv) True, because every quadratic polynomial has atmost two zeroes.

(v) True, because if in $ax^2 + bx + c = 0$, a and c have opposite signs, then $ac < 0$ and so $b^2 - 4ac > 0$. That means $D > 0$ i.e. equation has real roots.

(vi) True, because if in $ax^2 + bx + c = 0$, a and c have the same sign and $b = 0$, then

$b^2 - 4ac = -4ac < 0.$

So, $D < 0$ i.e. equation has no real roots.

3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Ans.

No, the given statement is not always true.

Consider the quadratic equation

$8x^2 - 2x - 1 = 0$

By splitting the middle term,

$8x^2 - 4x + 2x - 1 = 0$

$4x(2x - 1) + 1(2x - 1) = 0$

$(4x + 1)(2x - 1) = 0$

If $4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

So, the given equation has integral coefficients but no integral roots.

Hence, the given statement is false.

4. Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

Ans.

Yes, there exists a quadratic equation whose coefficients are rational but both of its roots are irrational .

Consider the quadratic equation

$x^2 - 6x + 7 = 0$

Here, $D = b^2 - 4ac$

$= (-6)^2 - 4(1)(7)$

$\Rightarrow D = 36 - 28 = 8$

Since Discriminant is not a perfect square, therefore it will have irrational roots.

The roots will be

$\frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2}$

The roots will be $3 \pm \sqrt{2}$

i.e. $3 + \sqrt{2}$ and $3 - \sqrt{2}$, are both irrational.

5. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

Ans.

Yes, there exists a quadratic equation whose coefficients are all distinct irrational coefficients. but both the roots are rational.

Consider the quadratic equation

$\sqrt{3}x^2 - 7\sqrt{3}x + 12\sqrt{3} = 0$

$\sqrt{3}x^2 - 4\sqrt{3}x - 3\sqrt{3}x + 12\sqrt{3} = 0$

[on splitting the middle terms]

$\sqrt{3}x(x-4) - 3\sqrt{3}(x-4) = 0$

$(x-4)(\sqrt{3}x - 3\sqrt{3}) = 0$

If $x - 4 = 0$, then $x = 4$

If $\sqrt{3}x - 3\sqrt{3} = 0$, then $x = 3$

i.e. equation with irrational coefficients

$\sqrt{3}x^2 - 7\sqrt{3}x + 12\sqrt{3} = 0$ has roots, 3 and 4.

6. Is 0.2 a root of the equation $x^2 - 0.4 = 0$? Justify.

Ans.

No, because 0.2 does not satisfy the quadratic equation i.e.

$$x^2 - 0.4 = (0.2)^2 - 0.4$$
$$= 0.04 - 0.4$$
$$= -0.36 \neq 0$$

7. If $b = 0$ and $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify.

Ans.

It is given that $b = 0$ and $c < 0$.

The given quadratic equation is:

$$x^2 + bx + c = 0$$

On putting $b = 0$ in this equation, we get

$$x^2 + 0x + c = 0$$
$$x^2 + c = 0$$
$$\Rightarrow \quad x^2 = -c$$

Here, $c < 0 \Rightarrow -c > 0$

$$\Rightarrow \quad x = \pm\sqrt{-c}$$

Hence, the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign.

EXERCISE 4.3

1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

(i) $2x^2 - 3x - 5 = 0$

(ii) $5x^2 + 13x + 8 = 0$

(iii) $-3x^2 + 5x + 12 = 0$

(iv) $-x^2 + 7x - 10 = 0$

(v) $x^2 + 2\sqrt{2}x - 6 = 0$

[CBSE 2017, 16, 15, 14, 11]

(vi) $x^2 - 3\sqrt{5}x + 10 = 0$

[CBSE 2017, 16, 15, 14, 11]

(vii) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

[CBSE 2017, 16, 15, 14, 11]

Ans.

(i) The given equation is: $2x^2 - 3x - 5 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -3$ and $c = -5$

By quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{9 + 40}}{4}$$
$$= \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4}$$
$$\Rightarrow \quad x = \frac{3+7}{4} \quad \text{or} \quad x = \frac{3-7}{4}$$
$$\Rightarrow \quad x = \frac{10}{4} = \frac{5}{2} \quad \text{or} \quad x = -\frac{4}{4} = -1$$

Hence, roots of the given equations are $\frac{5}{2}$ and -1.

(ii) The given equation is: $5x^2 + 13x + 8 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 5$, $b = 13$, $c = 8$

By quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(+13) \pm \sqrt{(13)^2 - 4(5)(8)}}{2(5)}$$
$$= \frac{-13 \pm \sqrt{169 - 160}}{10}$$
$$= \frac{-13 \pm 3}{10}$$
$$\Rightarrow \quad x = \frac{-13+3}{10} \quad \text{or} \quad x = \frac{-13-3}{10}$$
$$\Rightarrow \quad x = -\frac{10}{10} = -1 \quad \text{or} \quad x = -\frac{16}{10} = -\frac{8}{5}$$

Hence, roots of the given equation are -1 and $-\frac{8}{5}$.

(iii) The given equation is: $-3x^2 + 5x + 12 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = -3$, $b = 5$ and $c = 12$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5 \pm \sqrt{(5)^2 - 4(-3)(12)}}{2(-3)}$$
$$= \frac{-5 \pm \sqrt{25 + 144}}{-6}$$
$$= \frac{-5 \pm \sqrt{169}}{-6} = \frac{-5 \pm 13}{-6}$$
$$\Rightarrow \quad x = \frac{-5+13}{-6} \quad \text{or} \quad x = \frac{-5-13}{-6}$$
$$\Rightarrow \quad x = \frac{8}{-6} = \frac{4}{-3} \quad \text{or} \quad x = -\frac{-18}{-6} = 3$$

Hence, the roots of the given equation are $-\frac{4}{3}$ and 3.

(iv) The given equation is: $-x^2 + 7x - 10 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = -1, b = 7, c = -10$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{(7)^2 - 4(-1)(-10)}}{2(-1)}$$

$$= \frac{-7 \pm \sqrt{49 - 40}}{-2}$$

$$= \frac{-7 \pm \sqrt{9}}{-2} = \frac{-7 \pm 3}{-2}$$

$\Rightarrow \quad x = \frac{-7+3}{-2}$ or $x = \frac{-7-3}{-2}$

$\Rightarrow \quad x = \frac{-4}{-2} = +2$ or $x = \frac{-10}{-2} = +5$

Hence, roots of the given equation are 2 and 5.

(v) The given equation is: $x^2 + 2\sqrt{2}x - 6 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = 2\sqrt{2}, c = -6$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8+24}}{2}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{32}}{2} = -\sqrt{2} \pm 2\sqrt{2}$$

$\Rightarrow \quad x = -\sqrt{2} + 2\sqrt{2}$ or $x = -\sqrt{2} - 2\sqrt{2}$

$\Rightarrow \quad x = \sqrt{2}$ or $x = -3\sqrt{2}$

Hence, the roots of the given equation are $\sqrt{2}$ and $-3\sqrt{2}$.

(vi) The given equation is: $x^2 - 3\sqrt{5}x + 10 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -3\sqrt{5}, c = 10$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3\sqrt{5}) \pm \sqrt{(-3\sqrt{5})^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{3\sqrt{5} \pm \sqrt{45-40}}{2}$$

$$= \frac{3\sqrt{5} \pm \sqrt{5}}{2}$$

$\Rightarrow \quad x = \frac{3\sqrt{5}+\sqrt{5}}{2}$ or $x = \frac{3\sqrt{5}-\sqrt{5}}{2}$

$\Rightarrow \quad x = \frac{4\sqrt{5}}{2} = 2\sqrt{5}$ or $x = \frac{2\sqrt{5}}{2} = \sqrt{5}$

Hence, the roots of the given equation are $2\sqrt{5}$ and $\sqrt{5}$.

(vii) The given equation is: $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = \frac{1}{2}, b = -\sqrt{11}, c = 1$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-\sqrt{11}) \pm \sqrt{(-\sqrt{11})^2 - 4\left(\frac{1}{2}\right)(1)}}{2\left(\frac{1}{2}\right)}$$

$$= \frac{\sqrt{11} \pm \sqrt{11-2}}{2\left(\frac{1}{2}\right)}$$

$$= \frac{\sqrt{11} \pm \sqrt{9}}{1} = \frac{\sqrt{11} \pm 3}{1}$$

$\Rightarrow \quad x = \sqrt{11} + 3$ or $x = \sqrt{11} - 3$

Hence, roots of the given equation are $\sqrt{11} + 3$ and $\sqrt{11} - 3$.

Trick Applied

- *Compare the given equation with $ax^2 + bx + c = 0$, to get a, b and c.*
- *Use the following quadratic formula for finding roots of the equation*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Find the roots of the following quadratic equations by the factorisation method:

(i) $2x^2 + \frac{5}{3}x - 2 = 0$

(ii) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ **[CBSE 2016, 12]**

(iv) $3x^2 + 5\sqrt{5}x - 10 = 0$ **[CBSE 2016, 12]**

(v) $21x^2 - 2x + \frac{1}{21} = 0$

Ans.

(i) The given equation is: $2x^2 + \frac{5x}{3} - 2 = 0$

Multiplying both the sides by 3, we get

$$6x^2 + 5x - 6 = 0$$

Splitting the middle term, we have

$$6x^2 + 9x - 4x - 6 = 0$$

$$3x(2x + 3) - 2(2x + 3) = 0$$

$\Rightarrow \quad (2x + 3)(3x - 2) = 0$

$\Rightarrow \quad 2x + 3 = 0$ or $3x - 2 = 0$

$x = -\frac{3}{2}$ or $x = \frac{2}{3}$

Hence, the roots of the given equation are $-\frac{3}{2}$ and $\frac{2}{3}$

(ii) The given equation is: $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

Multiplying both the sides by 5, we get

$$2x^2 - 5x - 3 = 0$$

Splitting the middle term, we have

$$2x^2 - 6x + x - 3 = 0$$
$$2x(x - 3) + 1(x - 3) = 0$$
$$(2x + 1)(x - 3) = 0$$
$$2x + 1 = 0 \text{ or } x - 3 = 0$$
$$x = -\frac{1}{2} \text{ or } x = 3$$

Hence, the roots of the equation are $-\frac{1}{2}$ and 3.

(iii) The given equation is: $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

Splitting the middle term, we have

$$3\sqrt{2}x^2 - 6x + x - \sqrt{2} = 0$$
$$3\sqrt{2}x^2 - 3\sqrt{2}.\sqrt{2}x + x - \sqrt{2} = 0$$
$$3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$$
$$(x - \sqrt{2})(3\sqrt{2}x + 1) = 0$$
$$x - \sqrt{2} = 0 \text{ or } 3\sqrt{2}x + 1 = 0$$
$$x = \sqrt{2} \text{ or } x = \frac{-1}{3\sqrt{2}} = \frac{-\sqrt{2}}{6}$$

Hence, the roots of the equation are $\sqrt{2}$ and $\frac{-\sqrt{2}}{6}$.

(iv) The given equation is: $3x^2 + 5\sqrt{5}x - 10 = 0$

Splitting the middle term, we h

$$3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$$
$$3x^2 + 6\sqrt{5}x - \sqrt{5}x - (2\sqrt{5})(\sqrt{5}) = 0$$
$$3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$$
$$(x + 2\sqrt{5})(3x - \sqrt{5}) = 0$$
$$\Rightarrow (x + 2\sqrt{5}) = 0 \text{ or } (3x - \sqrt{5}) = 0$$
$$\Rightarrow x = -2\sqrt{5} \text{ or } x = \frac{\sqrt{5}}{3}$$

Hence, roots of the given equation: $3x^2 + 5\sqrt{5}x - 10$ are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{3}$.

(v) The given equation is: $21x^2 - 2x + \frac{1}{21} = 0$

Multiplying both the sides by 21, we get

$$(21)(21)x^2 - (21)(2x) + \frac{1}{21}(21) = 0$$
$$441x^2 - 42x + 1 = 0$$

Splitting the middle term, we have

$$441x^2 - 21x - 21x + 1 = 0$$
$$21x(21x - 1) - 1(21x - 1) = 0$$
$$(21x - 1)(21x - 1) = 0$$
$$\Rightarrow x = \frac{1}{21} \text{ or } x = \frac{1}{21}$$

Hence, the roots of the given equation: $21x^2 - 2x + \frac{1}{21} = 0$ are $\frac{1}{21}$ and $\frac{1}{21}$.

Trick Applied

- *If any coefficient of quadratic equation $ax^2 + bx + c = 0$ is in fractional form, make all coefficients in integral form.*
- *Use the factorisation method, to get the required roots of the given quadratic equation.*

EXERCISE 4.4

1. Find whether the following equations have real roots. If real roots exist, find them.

(i) $8x^2 + 2x - 3 = 0$

(ii) $-2x^2 + 3x + 2 = 0$

(iii) $5x^2 - 2x - 10 = 0$

(iv) $\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

(v) $x^2 + 5\sqrt{5}x - 70 = 0$

Ans.

(i) The given equation is: $8x^2 + 2x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 8, b = 2, c = -3$

Discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4(8)(-3)$$
$$= 4 + 96 = 100 > 0$$

Hence, the equation $8x^2 + 2x - 3 = 0$ has two distinct real roots, as $D > 0$.

Roots, $x = \frac{-b \pm \sqrt{D}}{2a}$

$$= \frac{-2 \pm \sqrt{100}}{16} = \frac{-2 \pm 10}{16}$$
$$\Rightarrow x = \frac{-2 + 10}{16} = \frac{8}{16} = \frac{1}{2}$$

or $x = \frac{-2 - 10}{16} = \frac{-12}{16} = \frac{-3}{4}$

Roots are $\frac{1}{2}$ and $\frac{-3}{4}$.

(ii) The given equation is: $-2x^2 + 3x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = -2, b = 3, c = 2$

Discriminant, $D = b^2 - 4ac$

$$= (3)^2 - 4(-2)(2)$$
$$= 9 + 16 = 25 > 0$$

Hence, the equation $-2x^2 + 3x + 2 = 0$ has two distinct real roots as $D > 0$.

Roots, $x = \frac{-b \pm \sqrt{D}}{2a}$

$$= \frac{-3 \pm \sqrt{25}}{2(-2)} = \frac{-3 \pm 5}{-4}$$
$$\Rightarrow x = \frac{-3 + 5}{-4} \text{ or } x = \frac{-3 - 5}{-4}$$
$$\Rightarrow x = -\frac{2}{4} = \frac{-1}{2} \text{ or } x = \frac{-8}{-4} = 2$$

Roots are $\frac{-1}{2}$ and 2.

(iii) The given equation is: $5x^2 - 2x - 10 = 0$

On comparing with $ax^2 + bx + c$, we get

$a = 5, b = -2, c = -10$

Discriminant, $D = b^2 - 4ac$

$= (-2)^2 - 4(5)(-10)$

$= 4 + 200 = 204 > 0$

Hence, the equation $5x^2 - 2x - 10 = 0$ has two distinct real roots as $D > 0$.

Roots, $x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-(-2) \pm \sqrt{204}}{2(5)}$

$= \frac{2 \pm \sqrt{204}}{10}$

$= \frac{2 \pm 2\sqrt{51}}{10} = \frac{1 \pm \sqrt{51}}{5}$

$\Rightarrow \quad x = \frac{1 + \sqrt{51}}{5} = \frac{1}{5} + \frac{\sqrt{51}}{5}$

or $\quad x = \frac{1 - \sqrt{51}}{5} = \frac{1}{5} - \frac{\sqrt{51}}{5}$

Roots are $\frac{1}{5} + \frac{\sqrt{51}}{5}$ or $\frac{1}{5} - \frac{\sqrt{51}}{5}$

(iv) The given equation is:

$\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

$\Rightarrow \quad \frac{x - 5 + 2x - 3}{(2x-3)(x-5)} = 1$

$\Rightarrow \quad \frac{3x - 8}{(2x-3)(x-5)} = 1$

$\Rightarrow \quad 3x - 8 = (2x - 3)(x - 5)$

$\Rightarrow \quad 3x - 8 = 2x^2 - 10x - 3x + 15$

$\Rightarrow \quad 3x - 8 = 2x^2 - 13x + 15$

$2x^2 - 16x + 23 = 0$

Comparing with $ax^2 + bx + c$, we get

$a = 2, b = -16, c = 23$

Discriminant, $D = b^2 - 4ac$

$= (-16)^2 - 4(2)(23)$

$= 256 - 184 = 72 > 0$

Hence, the equation $\frac{1}{2x-3} + \frac{1}{x-5}$ has two distinct real roots as $D > 0$.

Roots, $x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-(-16) \pm \sqrt{72}}{2(2)}$

$= \frac{16 \pm 6\sqrt{2}}{4}$

$= \frac{8 \pm 3\sqrt{2}}{2} = 4 \pm \frac{3\sqrt{2}}{2}$

$\Rightarrow \quad x = 4 + \frac{3\sqrt{2}}{2}$ or $x = 4 - \frac{3\sqrt{2}}{2}$

Roots are $4 + \frac{3\sqrt{2}}{2}$ and $4 - \frac{3\sqrt{2}}{2}$.

(v) The given equation is: $x^2 + 5\sqrt{5}x - 70 = 0$

On comparing with $ax^2 + bx + c$, we get

$a = 1, b = 5\sqrt{5}, c = -70$

Discriminant, $D = b^2 - 4ac$

$= (5\sqrt{5})^2 - 4(1)(-70)$

$= 125 + 280 = 405 > 0$

Hence, the equation $x^2 + 5\sqrt{5}x - 70 = 0$ has two distinct real roots as $D > 0$.

Roots, $x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-5\sqrt{5} \pm \sqrt{405}}{2(1)}$

$= \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2}$

$\Rightarrow \quad x = \frac{-5\sqrt{5} + 9\sqrt{5}}{2}$ or $x = \frac{-5\sqrt{5} - 9\sqrt{5}}{2}$

$\Rightarrow \quad x = \frac{4\sqrt{5}}{2} = 2\sqrt{5}$ or $x = \frac{-14\sqrt{5}}{2} = -7\sqrt{5}$

Roots are $2\sqrt{5}$ or $-7\sqrt{5}$.

Trick Applied

- *Check for real roots. If $D \geq 0$, roots are real.*
- *If roots are real, use quadratic formula to obtain real roots:*

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

2. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Ans.

Let n be the required natural number.

According to the question:

Square of natural number diminished by 84 gives $n^2 - 84$.

Thrice of 8 more than given number $= 3(8 + n)$.

According to the question,

$n^2 - 84 = 3(8 + n)$

$\Rightarrow \quad n^2 - 84 = 24 + 3n$

$\Rightarrow \quad n^2 - 3n - 108 = 0$

Splitting the middle term, we have

$\Rightarrow \quad n^2 - 12n + 9n - 108 = 0$

$\Rightarrow \quad n(n - 12) + 9(n - 12) = 0$

$\Rightarrow \quad (n - 12)(n + 9) = 0$

$n = 12$ or $n = -9$

But $n \neq -9$ as n is a natural number.

Hence, the required natural number is 12.

3. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number. **[CBSE 2014]**

Ans.

Let n be the required natural number.

According to the question, number when increased by 12 = $n + 12$.

160 times number's reciprocal = $160\left(\frac{1}{n}\right) = \frac{160}{n}$

Now, by the given condition

$$n + 12 = \frac{160}{n}$$

$$\Rightarrow \quad n(n + 12) = 160$$

$$\Rightarrow \quad n^2 + 12n - 160 = 0$$

Splitting the middle term, we have

$$n^2 + 20n - 8n - 160 = 0$$

$$n(n + 20) - 8(n + 20) = 0$$

$$(n + 20)(n - 8) = 0$$

$$n = -20 \text{ or } +8$$

But $n \neq -20$ as n is a natural number.

Hence, the required number is 8.

4. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train. [CBSE 2018, 15, 11]

Ans.

Let the original speed of the train be x km/hr.

Then, new speed *i.e.*, the increased speed of the train = $(x + 5)$ km/hr.

It is given that distance = 360 km

We know that Time = $\frac{\text{Distance}}{\text{Speed}}$

Time taken by train when speed is x km/hr = $\frac{360}{x}$ hr.

Time taken by train when speed is $(x + 5)$ km/hr = $\frac{360}{(x+5)}$ hr.

According to the question,

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} = \frac{4}{5}$$

$$\left[\because 48\,\text{min} = \frac{48}{60}\,\text{hr}\right]$$

$$\Rightarrow \quad \frac{360(x+5) - 360(x)}{x(x+5)} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{360x + 1800 - 360x}{x^2 + 5x} = \frac{4}{5}$$

$$\Rightarrow \quad (1800)5 = 4(x^2 + 5x)$$

$$\Rightarrow \quad x^2 + 5x = 2250$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0$$

Splitting the middle term, we have

$$\Rightarrow x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow \quad x(x + 50) - 45(x + 50) = 0$$

$$\Rightarrow \quad (x + 50)(x - 45) = 0$$

$$\Rightarrow \quad x + 50 = 0 \text{ or } x - 45 = 0$$

But $x \neq -50$ because speed cannot be negative

$x - 45 = 0 \Rightarrow x = 45$.

Hence, the original speed of the train is 45 km/hr.

5. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Ans.

Let actual age of Zeba be x years.

Her age when she was 5 years younger = $(x - 5)$ years.

According to the condition given in question:

Square of her age = 11 more than 5 times her actual age

$$(x - 5)^2 = 11 + 5(x)$$

$$\Rightarrow \quad x^2 + 25 - 10x = 11 + 5x$$

$$[\because \ (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow x^2 - 10x - 5x + 25 - 11 = 0$$

$$\Rightarrow \quad x - 15x + 14 = 0$$

Splitting the middle term, we have

$$\Rightarrow \quad x^2 - 14x - x + 14 = 0$$

$$\Rightarrow \quad x(x - 14) - 1(x - 14) = 0$$

$$\Rightarrow \quad (x - 14)(x - 1) = 0$$

$$\Rightarrow \quad x = 14 \text{ or } x = 1$$

But $x \neq +1$ as in that case $(x - 5)$ will not be possible

$$\Rightarrow \quad x = 14$$

Hence, Zeba's age now is 14 years.

6. At present, Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha. [CBSE 2010]

Ans.

Let Nisha's present age be x years.

Then, Asha's present age = $(2 + x^2)$ [By the given conditon]

Now, when Nisha grows to her mother's present age i.e. after $\{(x^2 + 2) - x\}$ years.

Then, Asha's age will become $\{(x^2 + 2) - x\}$ years.

Now by the given condition,

Asha's age = 1 year less than 10 times present age of Nisha.

$$(2 + x^2) + \{(x^2 + 2) - x\} = 10x - 1$$

$$\Rightarrow \quad 2 + x^2 + x^2 + 2 - x = 10x - 1$$

$$\Rightarrow \quad 2x^2 - 11x + 5 = 0$$

Splitting the middle term, we have

$$\Rightarrow \quad 2x^2 - 10x - x + 5 = 0$$

$\Rightarrow \quad 2x(x-5) - 1(x-5) = 0$

$\Rightarrow \quad (x-5)(2x-1) = 0$

$\Rightarrow \quad (x-5)(2x-1) = 0$

$\Rightarrow \quad x = 5 \text{ or } x = \frac{1}{2}$

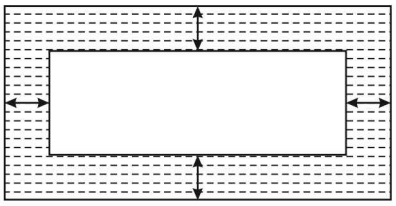
But $x \neq \frac{1}{2}$ as then Nisha's age $= \frac{1}{2}$. This means that her mother Asha's age $= (x^2 + 2) = \left(\frac{1}{4} + 2\right) = 2\frac{1}{4}$ years which is not possible.

Hence, the present age of Nisha = 5 years and

the present age of Asha $= x^2 + 2$

$= 5^2 + 2 = 25 + 2 = 27$ years

7. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2 [see Figure]. Find the length and breadth of the pond.

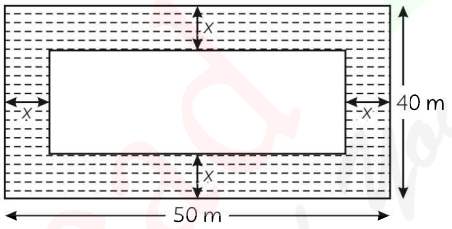


Ans.

Given: Rectangular lawn of dimensions = 50 m × 40 m.

A rectangular pond has to be constructed in the centre of the lawn.

Let the distance between the pond and the lawn be x m as shown in the figure:



Now, length of rectangular lawn, $L_1 = 50$ m

and breadth of rectangular lawn, $b_1 = 40$ m

Therefore, length of rectangular pond,

$L_2 = 50 - (x + x) = (50 - 2x)$ m

Breadth of rectangular pond,

$b_2 = 40 - (x + x) = (40 - 2x)$ m

Area of grass surrounding pond = 1184 m^2 [Given]

We know that:

Area of lawn − Area of pond = Area of grass

Area of lawn $= L_1 \times b_1 = 50 \times 40$

Area of pond $= L_2 \times b_2$

$= (50 - 2x)(40 - 2x)$

Area of grass = 1184

$\Rightarrow L_1 \times b_1 - L_2 \times b_2 = 1184$

$\Rightarrow 50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow 2000 - (2000 - 100x - 80x + 4x^2) = 1184$

$\Rightarrow 2000 - 2000 + 100x + 80x - 4x^2 - 1184 = 0$

$\Rightarrow -4x^2 + 180x - 1184 = 0$

$\Rightarrow 4x^2 - 180x + 1184 = 0$

$\Rightarrow x^2 - 45x + 296 = 0$

Splitting the middle term, we have

$\Rightarrow x^2 - 37x - 8x + 296 = 0$

$\Rightarrow x(x - 37) - 8(x - 37) = 0$

$\Rightarrow (x - 37)(x - 8) = 0$

$\Rightarrow x \neq 37$, as the length of the pool will be

$(50 - 2x) = 50 - 74 = -24$

and the breadth of pool will be $(40 - 2x) = 40 - 74 = -34$,

which is not possible as length and breadth cannot be negative.

$\Rightarrow \quad x = 8$

$\Rightarrow$ length of pond $= (50 - 2x) = 50 - 2(8)$

$= 50 - 16 = 34$ m

$\Rightarrow$ Breadth of pond $= (40 - 2x) = 40 - 2(8)$

$= 40 - 16 = 24$ m

Hence, the required length and breadth of the pond are 34 m and 24 m respectively.

8. At t minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

Ans.

It is given that at t minutes past 2 pm, the time needed by the minute hand to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ min.

$\Rightarrow \quad t + \left(\frac{t^2}{4} - 3\right) = 60$

[$\because$ time between 2 pm and 3 pm = 1 hour = 60 min.]

$\Rightarrow \quad 4t + t^2 - 12 = 240$

$\Rightarrow \quad t^2 + 4t - 12 - 240 = 0$

$\Rightarrow \quad t^2 + 4t - 252 = 0$

Splitting the middle term, we have

$t^2 + 18t - 14t - 252 = 0$

$\Rightarrow t(t + 18) - 14(t + 18) = 0$

$\Rightarrow \quad (t + 18)(t - 14) = 0$

$t = -18$ or $t = 14$.

But $t \neq -18$ as time cannot be negative

$\Rightarrow \quad t = 14$

Hence, the required value of t is 14 minutes.

1. If a and b are the roots of the equation $x^2 + ax - b = 0$, then find a and b.

Ans.

$x^2 + ax - b = 0$

Sum of the roots $= a + b$

$$= -\frac{b}{a} = -a$$

Product of roots $= ab$

$$= \frac{c}{a} = -b$$

So, $a + b = -a$

$\Rightarrow$ $b = -2a$

and, $ab = -b$

$\Rightarrow$ $a = -1$

Putting the value of a, we get

$b = -2 \times (-1) = 2$

Hence, $a = -1$ and $b = 2$.

2. Determine the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be thrice the other.

Ans.

Let the roots of the equation $ax^2 + bx + c$ be α and 3α.

Then, sum of the roots $= \alpha + 3\alpha$

$$= 4\alpha = -\frac{b}{a}$$

$$\alpha = -\frac{b}{4a}$$

Product of the roots $= \alpha \times 3\alpha$

$$= 3\alpha^2 = \frac{c}{a}$$

$$3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$$

$$\frac{3b^2}{16a^2} = \frac{c}{a}$$

$3b^2 = 16ac$, which is the required condition.

3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Ans. No.

Let's take an example,

$5x^2 - 7x + 2 = 0$

It has integral coefficient but its roots are 1 and $\frac{2}{5}$

Where, $\frac{2}{5}$ is not an integer.

4. If $x = 2$ and $m = 3$, the equation is $3x^2 - 2kx + 2m = 0$, find k.

Ans.

$3x^2 - 2kx + 2m = 0$

$x = 2$ and $m = 3$ [Given]

So, $3(2)^2 - 2k(2) + 2(3) = 0$

$12 - 4k + 6 = 0$

$-4k + 18 = 0$

$k = \frac{9}{2}$.

5. The product of two successive integral multiples of 5 is 1050. Determine the multiples.

Ans.

Let two successive integral multiples of 5 be x and $(x + 5)$

According to question,

$x(x + 5) = 1050$

$x^2 + 5x - 1050 = 0$

$(x - 30)(x + 35) = 0$

$x = 30$ or -35

When $x = 30$,

Multiples are 30 and $30 + 5 = 35$

When $x = -35$,

Multiples are -35 and $-35 + 5 = -30$

6. If a boy's age and his father's age amount together to 32 years. Fifth part of the product of their ages exceeds the boy's age by 10 years. Find how old they are.

Ans.

Let the age of the boy be x years.

So, age of his father be $(32 - x)$ years.

According to question,

$\frac{1}{5}x(32 - x) = x + 10$

$32x - x^2 = 5x + 50$

$x^2 - 27x + 50 = 0$

$(x - 2)(x - 25) = 0$

$x = 2, x = 25$

If $x = 25$,

Age of the boy = 25 years

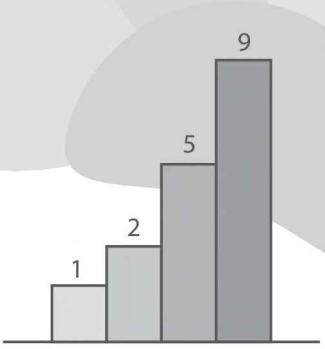
Age of his father = $32 - 25$

$= 7$ years [Not possible]

Hence, Age of the boy = 2 years

Age of his father = $32 - 2 = 30$ years.

5 Arithmetic Progressions



EXERCISE 5.1

Choose the correct option from the given four options:

1. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is:

(A) 6 (B) 7

(C) 20 (D) 28

[CBSE 2018, 11]

Ans. (D)

Explanation: It is given that

$d = -4$, $n = 7$, $a_n = 4$

We know that, in an AP,

$$a_n = a + (n - 1)d$$

$$\Rightarrow \quad 4 = a + (7 - 1)(-4)$$

$$\Rightarrow \quad 4 = a + 6(-4)$$

$$\Rightarrow \quad 4 = a - 24$$

$$\Rightarrow \quad a = 4 + 24 = 28$$

$$\Rightarrow \quad a = 28$$

2. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be:

(A) 0 (B) 3.5

(C) 103.5 (D) 104.5

Ans. (B)

Explanation: It is given that

$a = 3.5$, $d = 0$, $n = 101$

We know that in an AP,

$$a_n = a + (n - 1)d$$

$$= 3.5 + (101 - 1) \times 0$$

$$a_n = 3.5 + 0$$

$$\therefore \quad a_n = 3.5$$

3. The list of numbers –10, –6, –2, 2, ... is:

(A) an AP with $d = -16$

(B) an AP with $d = 4$

(C) an AP with $d = -4$

(D) not an AP

Ans. (B)

Explanation: The given list of number is

–10, –6, –2, 2, ...

Here, $a_1 = 10$, $a_2 = -6$, $a_3 = -2$ and $a_4 = 2$ ...

Since $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

From the above, we can see that each successive term has the same difference i.e. 4

Hence, the given list forms an AP with common difference: $d = 4$.

4. The 11^{th} term of the AP: $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is

(A) –20 (B) 20

(C) –30 (D) 30

[CBSE 2015, 14, 12]

Ans. (B)

Explanation: The given list of numbers is $-5, \frac{-5}{2}, 0, \frac{5}{2}$ ...

Here, $a = a_1 = -5$, $a_2 = \frac{-5}{2}$, $a_3 = 0$, $a_4 = \frac{5}{2}$

$$d = \frac{-5}{2} - (-5) = \frac{-5}{2} + 5 = \frac{5}{2} \qquad [\because d = a_2 - a_1]$$

$$\therefore \quad a_{11} = ?$$

We know that $a_n = a + (n - 1)d$

$$\Rightarrow \quad a_{11} = a + (11 - 1)d$$

$$= (-5) + (10)\left(\frac{5}{2}\right)$$

$$= -5 + (5)(5) = -5 + 25 = 20$$

$$\Rightarrow \quad a_{11} = 20$$

5. The first four terms of an AP, whose first term is –2 and the common difference is –2, are:

(A) –2, 0, 2, 4 (B) –2, 4, –8, 16

(C) –2, –4, –6, –8 (D) –2, –4, –8, –16

[CBSE 2012]

Ans. (C)

Explanation: It is given that the first term, $a = -2$ and common difference, $d = -2$

We know that

$$a_n = a + (n - 1)d$$

$$a_1 = -2 + (1 - 1)(-2) = -2$$

$$a_2 = -2 + (2 - 1)(-2) = -2 - 2 = -4$$

$$a_3 = -2 + (3 - 1)(-2)$$

$$= -2 + (2)(-2) = -2 - 4 = -6$$

$a_4 = -2 + (4 - 1)(-2)$

$= -2 + (3)(-2) = -2 - 6 = -8$

Hence, the first four terms of the AP are

$-2, -4, -6, -8$

6. The 21st term of the AP whose first two terms are –3 and 4 is:

(A) 17 (B) 137

(C) 143 (D) –143

Ans. (B)

Explanation: It is given that

$a_1 = -3 \quad a_2 = 4$

We know that

$a_n = a + (n - 1)d$

$\Rightarrow \quad a_1 = a + (1 - 1)d = a$

$\Rightarrow \quad a_2 = a + (2 - 1)d = a + d$

$\Rightarrow \quad a_1 = -3$ and $a_2 = a + d = 4$

$\Rightarrow \quad -3 + d = 4$

$\Rightarrow \quad d = 4 + 3 = 7$

$\therefore \quad a_{21} = a + (21 - 1)d = -3 + (20)7$

$= -3 + 140 = 137$

$\Rightarrow \quad a_{21} = 137$

7. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?

(A) 30 (B) 33

(C) 37 (D) 38 **[CBSE 2016]**

Ans. (B)

Explanation: It is given that

$a_2 = 13 \quad a_5 = 25 \quad a_7 = ?$

We know that $a_n = a + (n - 1)d$

$\Rightarrow \quad a_2 = a + (2 - 1)\, d = a + d = 13$

$\Rightarrow \quad a_5 = a + (5 - 1)\, d = a + 4d = 25$

$\Rightarrow \quad a + d = 13$ ...(i)

$\Rightarrow a + 4d = 25$ ...(ii)

Subtracting eq^n (i) from eq^n (ii), we get

$(a + 4d) - (a + d) = 25 - 13$

$3d = 12 \quad \Rightarrow \quad d = 4$

Putting this value of d in eq^n (1)

$a + 4 = 13 \quad \Rightarrow \quad a = 13 - 4 = 9$

$\therefore \quad a_7 = a + (7 - 1)d$

$= 9 + 6 \times 4$

$= 9 + 24 = 33$

$\Rightarrow \quad a_7 = 33$

8. Which term of the AP: 21, 42, 63, 84, ... is 210?

(A) 9th (B) 10th

(C) 11th (D) 12th **[CBSE 2012]**

Ans. (B)

Explanation: The given series is

21, 42, 63, 84, ...

Here, the first term, $a = 21$

and common difference,

$d = 42 - 21 = 21$

Let the n^{th} term of the given AP be 210

We know that $a_n = a + (n - 1)21$

$\Rightarrow \quad 210 = 21 + (n - 1)21$

$\Rightarrow \quad 210 = 21 + 21n - 21$

$\Rightarrow \quad 210 = 21n$

$\Rightarrow \quad n = 10$

Hence, 210 is the 10th term of the AP.

9. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(A) 5 (B) 20

(C) 25 (D) 30 **[CBSE 2011]**

Ans. (C)

Explanation: It is given that common difference, $d = 5$

$a_{18} - a_{13} = ?$

We know $a_n = a + (n - 1)d$

$a_{18} = a + (18 - 1)d = a + 17d$

$a_{13} = a + (13 - 1)d = a + 12d$

Now, $a_{18} - a_{13} = (a + 17d) - (a + 12d)$

$= a + 17d - a - 12d$

$= 5d = 5 \times 5 = 25$ [As $d = 5$]

$\Rightarrow \quad a_{18} - a_{13} = 25$

10. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(A) 8 (B) –8

(C) –4 (D) 4 **[CBSE 2017, 11]**

Ans. (A)

Explanation: It is given that

$a_{18} - a_{14} = 32$

We know that $a_n = a + (n - 1)d$

$a_{18} = a + (18 - 1)d = a + 17d$

$a_{14} = a + (14 - 1)d = a + 13d$

$\Rightarrow \quad a_{18} - a_{14} = (a + 17d) - (a + 13d)$

$\Rightarrow \quad 32 = a + 17d - a - 13d$

$\Rightarrow \quad 32 = 4d$

$\Rightarrow \quad d = 8$

Hence, we can say that common difference, $d = 8$.

11. Two APs have the same common difference. The first term of one of these is –1 and that of the other is –8. Then the difference between their 4th terms is:

(A) –1 (B) –8

(C) 7 (D) –9

Ans. (C)

Explanation: Let d be the common difference of the two AP's and a be the first term of the first

AP and A_1 be the first term of the second AP.
We have to find out

$|a_4 - A_4| = ?$

Also $a_1 = -1 \quad A_1 = -8$

We know that $a_n = a + (n-1)d$

$a_4 = a_1 + (4-1)d = a + 3d = -1 + 3d$

$A_4 = A_1 + (4-1)d = -8 + 3d$

Now, the difference between their 4th terms will be

$$|a_4 - A_4| = (-1 + 3d) - (-8 + 3d)$$
$$= -1 + 3d + 8 - 3d$$
$$= 7$$

Hence, the required difference is 7.

12. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be:

(A) 7 (B) 11

(C) 18 (D) 0 **[CBSE 2015, 12]**

Ans. (D)

Explanation: It is given that

$$7a_7 = 11a_{11}$$

We know that $a_n = a + (n-1)d$

$a_7 = a + (7-1)d = a + 6d$

$a_{11} = a + (11-1)d = a + 10d$

$$7a_7 = 11a_{11}$$
$$\Rightarrow \quad 7[a + 6d] = 11[a + 10d]$$
$$\Rightarrow \quad 7a + 42d = 11a + 110d$$
$$\Rightarrow \quad 4a + 68d = 0$$
$$\Rightarrow \quad 2(2a + 34d) = 0$$
$$\Rightarrow \quad 2a + 34d = 0$$
$$\Rightarrow \quad a + 17d = 0 \qquad \text{...(i)}$$

Now, 18th term of the AP

$= a_{18} = a + 17d = ?$

From eqn (i)

$$a + 17d = 0$$
$$\Rightarrow \quad a_{18} = 0$$

13. The 4th term from the end of the AP: –11, –8, –5, ... 49 is:

(A) 37 (B) 40

(C) 43 (D) 58 **[CBSE 2016]**

Ans. (B)

Explanation: The given list of AP is

–11, –8, –5, ... 49

Here, first term, $a = -11$

Last term, $l = 49$

common difference, $d = a_2 - a_1$

$= -8 - (-11)$

$= -8 + 11 = 3$

We know that the n^{th} term of an AP from the end is $a_n = l - (n-1)d$

$$\Rightarrow \quad a'_4 = 49 - (4-1)3$$
$$= 49 - (3)(3)$$
$$= 49 - 9 = 40$$
$$\Rightarrow \quad a'_4 = 40$$

i.e., 4th term from the end is 40

14. The famous mathematician associated with finding the sum of the first 100 natural numbers is:

(A) Pythagoras (B) Newton

(C) Gauss (D) Euclid

Ans. (C)

Explanation: Newton is famous for his laws of physics.

Pythagoras is famous for the pythagorean theorem of a right angled triangle.

Gauss is the famous mathematician associated with finding the sum of the first 100 natural numbers.

Euclid is most famous for his work in geometry.

15. If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is:

(A) 0 (B) 5

(C) 6 (D) 15

Ans. (A)

Explanation: It is given that the first term, $a = -5$ and common difference, $d = 2$

$S_6 = ?$

We know that the sum of n terms of an AP is

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$\Rightarrow \quad S_6 = \frac{6}{2}\{2(-5) + (6-1)(2)\}$$
$$\Rightarrow \quad S_6 = 3\{-10 + 5(2)\} = 3\{-10 + 10\} = 3(0) = 0$$
$$\Rightarrow \quad S_6 = 0$$

16. The sum of the first 16 terms of the AP: 10, 6, 2, ... is:

(A) –320 (B) 320

(C) –352 (D) –400 **[CBSE 2012]**

Ans. (A)

Explanation: The given series of AP is 10, 6, 2 ...

Here, the first term, $a = 10$

and common difference,

$$d = a_2 - a_1 = 6 - 10 = -4$$

Sum of 16 terms, $S_{16} = ?$

We know that $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$\Rightarrow \quad S_{16} = \frac{16}{2}\{2(10) + (16-1)(-4)\}$$
$$= 8[20 + 15(-4)\} = 8\{20 - 60\}$$

$= 8(-40) = -320$

$\Rightarrow S_{16} = -320$

17. **In an AP if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is:**

(A) 19 **(B) 21**

(C) 38 **(D) 42**

Ans. (C)

Explanation: It is given that the first term, $a = 1$

and n^{th} term, $a_n = 20$

Sum of n terms, $S_n = 399$

$n = ?$

We Know that

$$a_n = a + (n-1)d$$

$$\Rightarrow 20 = 1 + (n-1)d$$

$$\Rightarrow (n-1)d = 19 \quad \ldots (i)$$

Also we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow 399 = \frac{n}{2}\{2(1) + (n-1)d\}$$

$$\Rightarrow 798 = n[2 + (n-1)d]$$

$$\Rightarrow 798 = 2n + n(n-1)d \quad \ldots (ii)$$

Usnig equation (i) & (ii) we get

$$798 = 2n + 19n$$

$$\Rightarrow 798 = 21n$$

$$\Rightarrow n = \frac{798}{21} = 38$$

i.e., $n = 38$

18. **The sum of the first five multiples of 3 is:**

(A) 45 **(B) 55**

(C) 65 **(D) 75** **[CBSE 2018]**

Ans. (A)

Explanation: The first 5 multiples of 3 are 3, 6, 9, 12 and 15.

Here, first term, $a = 3$, common difference, $d = 3$ and number of terms $n = 5$

We know that $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$\Rightarrow S_5 = \frac{5}{2}\{2(3) + (5-1)3\} = \frac{5}{2}[6 + 4(3)]$$

$$= \frac{5}{2}[6 + 12] = \frac{5}{2}(18) = 5 \times 9 = 45$$

Hence, the sum of first 5 multiples is 45.

EXERCISE 5.2

1. **Which of the following form an AP? Justify your answer.** **[CBSE 2020]**

(i) –1, –1, –1, –1, ...

(ii) 0, 2, 0, 2, ...

(iii) 1, 1, 2, 2, 3, 3, ...

(iv) 11, 22, 33, ...

(v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

(vi) $2, 2^2, 2^3, 2^4, \ldots$

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Ans.

(i) Yes, it forms an A.P.

The given series of numbers is

$-1, -1, -1, -1, \ldots$

Here, $a_1 = -1$, $a_2 = -1$,

$a_3 = -1$, $a_4 = -1$

and common difference, d

$\Rightarrow a_2 - a_1 = -1 - (-1) = -1 + 1 = 0$

$a_3 - a_2 = -1 - (-1) = -1 + 1 = 0$

$a_4 - a_3 = -1 - (-1) = -1 + 1 = 0$

Clearly, the difference of successive terms is the same, therefore we can say that the given list of numbers forms an AP.

(ii) No, it does not form an AP

The given series of numbers is 0, 2, 0, 2 ...

Here, $a_1 = 0$ $a_2 = 2$

$a_3 = 0$ $a_4 = 2$

Difference between two succesive terms:

$a_2 - a_1 = 2 - 0 = 2$

$a_3 - a_2 = 0 - 2 = -2$

$a_4 - a_3 = 2 - 0 = 2$

Clearly, the difference of two successive terms is not the same, hence we can say that the given list of numbers does not form an A.P.

(iii) No. it does not form an AP.

The given series of numbers are 1, 1, 2, 2, 3, 3, ...

Here, $a_1 = 1$ $a_2 = 1$ $a_3 = 2$

$a_4 = 2$ $a_5 = 3$ $a_6 = 3$

Difference between two successive terms

$a_2 - a_1 = 1 - 1 = 0$

$a_3 - a_2 = 2 - 1 = 1$

$a_4 - a_3 = 2 - 2 = 0$

$a_5 - a_4 = 3 - 2 = 1$

Clearly, the difference of two successive terms is not the same, hence we can say that the given list of numbers does not form an AP.

(iv) Yes, it forms an AP.

The given series of numbers is 11, 22, 33 ...

Here, $a_1 = 11$ $a_2 = 22$ $a_3 = 33$

Difference between two successive terms,

$$a_2 - a_1 = 22 - 11 = 11$$

$$a_3 - a_2 = 33 - 22 = 11$$

Clearly, the difference of two successive terms is the same, hence we can say that the given list of numbers forms an AP.

(v) No, it does not form an AP

The given series of number is

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

Here, $a_1 = \frac{1}{2}, a_2 = \frac{1}{3}, a_3 = \frac{1}{4}, \ldots$

Difference between two successive terms

$$a_2 - a_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$$

$$a_3 - a_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \frac{-1}{12}$$

Clearly, the difference of two successive terms is not the same, hence we can say that the given list of numbers does not form an AP.

(vi) No, it does not form an AP.

The given series of number is $2, 2^2, 2^3, 2^4, \ldots$

Here, $a_1 = 2, a_2 = 2^2 = 4, a_3 = 2^3 = 8, a_4 = 2^4 = 16$

Difference between two successive term

$$a_2 - a_1 = 4 - 2 = 2$$

$$a_3 - a_2 = 8 - 4 = 4$$

$$a_4 - a_3 = 16 - 8 = 8$$

Clearly, difference of two successive terms is not the same, hence we can say that given list of numbers does not form an A.P.

(vii) Yes, it forms an A.P.

The given series of numbers are

$\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48} \ldots$

Here, $a_1 = \sqrt{3}, a_2 = \sqrt{12} = 2\sqrt{3}, a_3 = \sqrt{27} = 3\sqrt{3}, a_4 = \sqrt{48} = 4\sqrt{3}$.

Difference between two successive terms

$$a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$

$$a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

$$a_4 - a_3 = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$$

Clearly, the difference of two successive terms is the same, hence we can say that the given list of numbers forms an AP.

2. Justify whether it is true to say that $-1, -\frac{3}{2}, -2, \frac{5}{2}, \ldots$ form an AP as $a_2 - a_1 = a_3 - a_2$.

Ans. False

Explanation: The given series of numbers is $-1, \frac{-3}{2}, -2, \frac{5}{2}, \ldots$

Here, $a_1 = -1, a_2 = \frac{-3}{2}, a_3 = -2, a_4 = \frac{5}{2}, \ldots$

Difference between two successive terms

$$a_2 - a_1 = \frac{-3}{2} - (-1) = \frac{-3}{2} + 1 = \frac{-3+2}{2} = \frac{-1}{2}$$

$$a_3 = a_2 = -2 - \left(\frac{-3}{2}\right) = -2 + \frac{3}{2} = \frac{-4+3}{2} = \frac{-1}{2}$$

$$a_4 - a_3 = \frac{5}{2} - (-2) = \frac{5}{2} + 2 = \frac{5+4}{2} = \frac{9}{2}$$

We can see that $a_2 - a_1 = -\frac{1}{2}, a_3 - a_2 = -\frac{1}{2}$, $a_4 - a_3 = \frac{9}{2}$.

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2$

but $a_3 - a_2 \neq a_4 - a_3$

Clearly, the difference of two successive terms is not the same, hence we can say that the given list of numbers does not form an AP.

3. For the AP: –3, –7, –11, ..., can we directly find $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Ans. Yes, we can find.

The given list of numbers of an AP –3, –7, –11 ...

We know that $a_n = a + (n-1)d$

$\Rightarrow$ $a_{30} = a + (30-1)d = a + 29d$

$\Rightarrow$ $a_{20} = a + (20-1)d = a + 19d$

Now, $a_{30} - a_{20} = (a + 29d) - (a + 19d)$

$= a + 29d - a - 19d = 10d$ ...(i)

For the given series, common difference,

$d = -7 - (-3) = -7 + 3 = -4$

Hence, $a_3 - a_{20} = 10d = 10(-4) = -40$ [Using eqn. (i)]

$a_3 - a_{20} = -40$

4. Two AP's have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference betwee their 10^{th} terms is the same as the difference between their 21^{st} terms, which is the same as the difference between any two corresponding terms. Why?

Ans.

Let d be the common difference of two AP's

First term of first AP is 2.

First AP will be $2, 2 + d, 2 + 2d \ldots$

First term of second AP is 7 second AP will be $7, 7 + d, 7 + 2d \ldots$

We know that $a_n = a + (n-1)d$

10^{th} term of first AP $= 2 + 9d$

10^{th} term of second AP = $7 + 9d$

Difference of their 10^{th} term = $(7 + 9d) - (2 + 9d)$

$= 7 + 9d - 2 - 9d$

$= 5$

Again, 21^{st} term of first AP = $2 + 20d$

21^{st} term of second AP = $7 + 20d$

Difference of their 21^{st} term

$= (7 + 20d) - (2 + 20d)$

$= 7 + 20d - 2 - 20d = 5$

Thus, we can say that if a_n and b_n are n^{th} terms of first and second AP respectively, then

$b_n - a_n = [7 + (n - 1)d] - [2 + (n - 1)d]$

$= 7 + (n - 1)d - 2 - (n - 1)d = 5$

$\Rightarrow \quad b_n - a_n = 5$

Hence, the difference between any two corresponding terms of such AP's is the same as the difference between their first terms.

5. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer. [CBSE 2016, 11]

Ans. No. 0 is not a term of the given AP

The given AP is 31, 28, 25 ...

Here, $a = 31$ and common difference, $d = 28 - 31 = -3$

Let 0 be the n^{th} term of the given AP.

We know that

$a_n = a + (n - 1)d$

$\Rightarrow \quad 0 = 31 + (n - 1)(-3)$

$0 = 31 - 3n + 3$

$\Rightarrow \quad 3n = 34$

$\Rightarrow \quad n = \frac{34}{3} = 11\frac{1}{3}$

But n should be a positive integer. So, we can say that 0 is not a term of the given AP.

6. In which of the following situations, do the lists of numbers involved form an AP? Give reasons for your answers.

(i) The fee charged from a student every month by a school for the whole session, when the monthly fee is ₹ 400.

(ii) The fee charged every month by a school from classes I to XII, when the monthly fee for class I is ₹ 250, and it increases by ₹ 50 for the next higher class.

(iii) The amount of money in the account of Varun at the end of every year when ₹ 1000 is deposited at a simple interest of 10% per annum.

(iv) The number of bacteria in a certain food item after each second, when they double in every second.

Ans.

(i) Yes, the fee charged from a student every month forms an AP

Fee charged for

1^{st} month = 400

2^{nd} month = 400

3^{rd} month = 400

Difference between two successive terms = 400 – 400 = 0

Clearly, the common difference is the same for all the numbers; Hence, we can say that it forms an AP with common difference, d = 0

(ii) Yes, the fee charged every month for classes I to XII forms an AP.

Fee charged for

Class I = 250

Class II = (250 + 50) = 300

Class III = (250 + 2 × 50) = 350

Class IV = (250 + 3 × 50) = 400

i.e., 250, 300, 350, 400 ...

Clearly it forms an AP with common difference, d = 300 – 250 = 50

(iii) Yes, the amount of money in the account of Varun at the end of every year forms an AP.

We know that

Simple interest = $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

$= \frac{1000 \times 10 \times 1}{100} = 100$

Amount of money in Varun's account at the end of

year 1 = 1000 = 1000

year 2 = (1000 + 100 × 1) = 1100

year 3 = (1000 + 100 × 2) = 1200

i.e., 1000, 1100, 1200, 1300 ...

Clearly, it forms an AP with common difference, d = 100.

(iv) No, the number of bacteria ofter each second does not form an AP

Let no. of bacteria in certain food be x.

It is given that they double every second.

$\therefore$ Bacteria after every second will be

$x, 2x, 4x, 8x$ ...

Here, $a_1 = x$, $a_2 = 2x$, $a_3 = 4x$, $a_4 = 8x$ ...

$\Rightarrow \quad a_2 - a_1 = 2x - x = x$

and $\quad a_3 - a_2 = 4x - 2x = 2x$

Since, the difference between each successive term is not the same, the list does not form an AP.

7. Justify whether it is true to say that the follow-ing are the n^{th} terms of an AP:

(i) $2n - 3$ **(ii) $3n^2 + 5$**

(iii) $1 + n + n^2$

Ans.

(i) Yes, $(2n - 3)$ is the n^{th} term of an A.P.

It is given that

$$a_n = 2n - 3$$

Put $n = 1, a_1 = 2(1) - 3 = 2 - 3 = -1$

$n = 2, a_2 = 2(2) - 3 = 4 - 3 = 1$

$n = 3, a_3 = 2(3) - 3 = 6 - 3 = 3$

$n = 4, a_4 = 2(4) - 3 = 8 - 3 = 5$

List of numbers becomes $-1, 1, 3, 5$...

Here, $a_2 - a_1 = 1 - (-1) = 2$

$a_3 - a_2 = 3 - 1 = 2$

$a_4 - a_3 = 5 - 3 = 2$

Clearly, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Hence, $2n - 3$ is the n^{th} term of an AP.

(ii) No, $(3n^2 + 5)$ is not the n^{th} term of an AP.

It is given that $a_n = 3n^2 + 5$

Put $n = 1, a_1 = 3(1)^2 + 5 = 3 + 5 = 8$

$n = 2, a_2 = 3(2)^2 + 5 = 12 + 5 = 17$

$n = 3, a_3 = 3(3)^2 + 5 = 27 + 5 = 32$

List of number becomes 8, 17, 32 ...

Here, $a_2 - a_1 = 17 - 8 = 9$,

$a_3 - a_2 = 32 - 17 = 15$

Clearly, $a_2 - a_1 \neq a_3 - a_2$

Hence, $(3n^2 + 5)$ is not the n^{th} term of an AP.

(iii) No, $(1 + n + n^2)$ is not the n^{th} term of an AP.

It is given that $a_n = 1 + n + n^2$

Put $n = 1, \quad a_1 = 1 + (1) + (1)^2$

$= 1 + 1 + 1 = 3$

$n = 2, \quad a_2 = 1 + (2) + (2)^2$

$= 1 + 2 + 4 = 7$

$n = 3, \quad a_3 = 1 + (3) + (3)^2$

$= 1 + 3 + 9 = 13$

List of number becomes 3, 7, 13 ...

Here, $a_2 - a_1 = 7 - 3 = 4$

$a_3 - a_2 = 13 - 7 = 6$

Clearly, $a_2 - a_1 \neq a_3 - a_2$

Hence, $(1 + n + n^2)$ is not the n^{th} term of an AP.

EXERCISE 5.3

1. Match the APs given in column A with suitable common differences given in column B.

	Column A		Column B
(A_1)	2, –2, –6, –10, ...	(B_1)	$\frac{2}{3}$
(A_2)	$a = -18, n = 10, a_n = 0$	(B_2)	–5
(A_3)	$a = 0\ a_{10} = 6$	(B_3)	4
(A_4)	$a_2 = 13, a_4 = 3$	(B_4)	–4
		(B_5)	2
		(B_6)	$\frac{1}{2}$
		(B_7)	5

[CBSE 2019]

Ans.

$(A_1) \rightarrow (B_4)$

$(A_2) \rightarrow (B_5)$

$(A_3) \rightarrow (B_1)$

$(A_4) \rightarrow (B_2)$

Explanation:

$(A_1) \rightarrow 2, -2, -6, -10$ ...

Here, $a_1 = 2, a_2 = -2, a_3 = -6, a_4 = -10$

Common difference, $d = a_2 - a_1 = -2 - (2) = -4$ i.e., B_4

$(A_2) \rightarrow a = -18, n = 10, a_n = 0$

We know that $a_n = a + (n - 1)d$

$\Rightarrow \quad 0 = -18 + (10 - 1)d$

$\Rightarrow \quad 18 = 9d \Rightarrow d = 2$ i.e., B_5

$(A_3) \rightarrow a = 0, a_{10} = 6$

We know that $a_n = a + (n - 1)d$

when, $n = 10$ and $a_{10} = a + 9d$

$\Rightarrow \quad 6 = 0 + 9d \Rightarrow d = \frac{6}{9} = \frac{2}{3}$ i.e., B_1

$(A_4) \rightarrow a_2 = 13, a_4 = 3$

We know that $a_n = a + (n - 1)d$

$\Rightarrow \quad 13 = a + d$ ...(i)

and $a_4 = a + (4 - 1)d = a + 3d$

$\Rightarrow \quad 3 = a + 3d$ ...(ii)

On subtracting eq^n (i) from eq^n (ii), we get

$3 - 13 = (a + 3d) - (a + d)$

$-10 = a + 3d - a - d$

$\Rightarrow \quad -10 = 2d$

$\Rightarrow \quad d = -\frac{10}{2} = -5$ i.e., B_2

2. Verify that each of the following is an AP, and then write its next three terms.

(i) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

(ii) $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

(iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ **[CBSE 2014, 12]**

(iv) $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$

(v) $a, 2a + 1, 3a + 2, 4a + 3, \ldots$

Ans.

(i) The given series is $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots\ldots$

Here, $a_1 = 0$, $a_2 = \frac{1}{4}$, $a_3 = \frac{1}{2}$, $a_4 = \frac{3}{4}$

$a_2 - a_1 = \frac{1}{4} - 0 = \frac{1}{4}$ $\quad a_3 - a_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$a_4 - a_3 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

Clearly, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Hence, the given series forms an AP as they have a common difference, $d = \frac{1}{4}$

The next three terms are:

$a_5 = a_1 + 4d = 0 + 4\left(\frac{1}{4}\right) = 1$

$a_6 = a_1 + 5d = 0 + 5\left(\frac{1}{4}\right) = \frac{5}{4}$

$a_7 = a_1 + 6d = 0 + 6\left(\frac{1}{4}\right) = \frac{6}{4} = \frac{3}{2}$

(ii) The given series is $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

Here, $a_1 = 5$, $a_2 = \frac{14}{3}$, $a_3 = \frac{13}{3}$, $a_4 = 4$

$a_2 - a_1 = \frac{14}{3} - 5 = \frac{14-15}{3} = \frac{-1}{3}$

$a_3 - a_2 = \frac{13}{3} - \frac{14}{3} = \frac{13-14}{3} = \frac{-1}{3}$

$a_4 - a_3 = 4 - \frac{13}{3} = \frac{12-13}{3} = \frac{-1}{3}$

Clearly, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$.

Hence, the given series forms an AP as they have a common difference, $d = -\frac{1}{3}$

The next three terms are:

$a_5 = a_1 + 4d$

$= 5 + 4\left(\frac{-1}{3}\right) = 5 - \frac{4}{3} = \frac{15-4}{3} = \frac{11}{3}$

$a_6 = a_1 + 5d$

$= 5 + 5\left(\frac{-1}{3}\right) = 5 - \frac{5}{3} = \frac{15-5}{3} = \frac{10}{3}$

$a_7 = a_1 + 6d$

$= 5 + 6\left(\frac{-1}{3}\right) = 5 - 2 = 3$

(iii) The given series is $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

Here, $a_1 = \sqrt{3}$, $a_2 = 2\sqrt{3}$, $a_3 = 3\sqrt{3}$

$a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

Clearly, $a_2 - a_1 = a_3 - a_2 = \sqrt{3}$

Hence, the given series forms an AP as it has a common difference $d = \sqrt{3}$

The next three terms are

$a_4 = a_1 + 3d = \sqrt{3} + 3(\sqrt{3}) = 4\sqrt{3}$

$a_5 = a_1 + 4d = \sqrt{3} + 4(\sqrt{3}) = 5\sqrt{3}$

$a_6 = a_1 + 5d = \sqrt{3} + 5(\sqrt{3}) = 6\sqrt{3}$

(iv) The given series is:

$(a + b), (a + 1) + b, (a + 1) + (b + 1), \ldots$

Here, $a_1 = a + b$ $\quad a_2 = (a + 1) + b$

$a_3 = (a + 1) + (b + 1) \ldots$

$a_2 - a_1 = (a + 1) + b - (a + b)$

$= a + 1 + b - a - b = 1$

$a_3 - a_2 = (a + 1) + (b + 1) - [(a + 1) + b]$

$= a + 1 + b + 1 - a - 1 - b = 1$

Clearly, $a_2 - a_1 = a_3 - a_2 = 1$

Hence, the given series forms an AP with common difference, $d = 1$.

$a_4 = a_1 + 3d = (a + b) + 3(1)$

$= (a + 2) + (b + 1)$

$a_5 = a_1 + 4d = (a + b) + 4(1)$

$= (a + 2) + (b + 2)$

$a_6 = a_1 + 5d = (a + b) + 5(1)$

$= (a + 3) + (b + 2)$

(v) The given series is $a, 2a + 1, 3a + 2, 4a + 3 \ldots$

Here, $a_1 = a$, $a_2 = 2a + 1$, $a_3 = 3a + 2$, $a_4 = 4a + 3$

$a_2 - a_1 = 2a + 1 - a = a + 1$

$a_3 - a_2 = (3a + 2) - (2a + 1) = a + 1$

$a_4 - a_3 = (4a + 3) - (3a + 2) = a + 1$

Clearly, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Hence, the given series forms an AP with common difference, $d = a + 1$

The next three terms are:

$a_5 = a_1 + 4d = a + 4(a + 1) = 5a + 4$

$a_6 = a_1 + 5d = a + 5(a + 1) = 6a + 5$

$a_7 = a_1 + 6d = a + 6(a + 1) = 7a + 6$

3. Write the first three terms of the APs when a and d are as given below:

(i) $a = \frac{1}{2}, d = -\frac{1}{6}$

(ii) $a = -5, d = -3$

(iii) $a = \sqrt{2}, d = \frac{1}{\sqrt{2}}$

Ans.

(i) It is given that first term, $a = \frac{1}{2}$

and common difference, $d = \frac{-1}{6}$

We know that $a_n = a + (n-1)d$

First term $a_1 = a = \frac{1}{2}$

Second term $a_2 = a + d$

$$= \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$$

Third term $a_3 = a + 2d = \frac{1}{2} + 2\left(\frac{-1}{6}\right) = \frac{1}{2} - \frac{1}{3}$

$$= \frac{3-2}{6} = \frac{1}{6}$$

Hence, the required three terms are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$.

(ii) It is given that first term, $a = -5$

and common difference, $d = -3$

We know that $a_n = a + (n-1)d$

First term $a_1 = a = -5$

Second term $a_2 = a + d = -5 + (-3) = -8$

Third term $a_3 = a + 2d = -5 + 2(-3)$

$= -5 - 6 = -11$

Hence, the required three terms are -5, -8 and -11.

(iii) It is given that first term, $a = \sqrt{2}$

and common difference, $d = \frac{1}{\sqrt{2}}$

We know that $a_n = a_1 + (n-1)d$

First term $a_1 = a = \sqrt{2}$

Second term $a_2 = a + d = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

Third term $a_3 = a + 2d$

$$= \sqrt{2} + 2\left(\frac{1}{\sqrt{2}}\right) = \frac{2+2}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Hence, the required three terms are

$$\sqrt{2}, \frac{3}{\sqrt{2}} \text{ and } \frac{4}{\sqrt{2}}.$$

4. Find *a*, *b* and *c* such that the following numbers in AP: *a*, 7, *b*, 23, *c*. [CBSE 2012]

Ans.

It is given that *a*, 7, *b*, 23, *c* are in AP

They have a common difference

i.e., $7 - a = b - 7 = 23 - b = c - 23$

Taking second and third terms, we get

$b - 7 = 23 - b \Rightarrow 2b = 30 \Rightarrow b = 15$

Taking first and second terms, we get

$7 - a = b - 7$

$\Rightarrow$ $7 - a = 15 - 7$ [As $b = 15$]

$\Rightarrow$ $7 - a = 8 \Rightarrow a = -1$

Taking third and fourth terms, we get

$23 - b = c - 23$

$\Rightarrow$ $23 - 15 = c - 23$ [As $b = 15$]

$\Rightarrow$ $8 = c - 23 \Rightarrow c = 31$

Hence, $a = -1$, $b = 15$ and $c = 31$.

5. Determine the AP whose 5th term is 19 and the difference of the 8th term from the 13th term is 20. [CBSE 2011]

Ans.

Let the first term of *an* AP be *a* and commom difference be *d*.

It is given that $a_5 = 19$

and $a_{13} - a_8 = 20$

We know that $a_n = a + (n-1)d$

$a_5 = a + (5-1)d = a + 4d = 19$ ...(i)

Also, $a_{13} - a_8 = 20$

$\Rightarrow (a + 12d) - (a + 7d) = 20$

$\Rightarrow a + 12d - a - 7d = 20$

$\Rightarrow 5d = 20 \Rightarrow d = 4$

Putting the value of $d = 4$ in equation (i), we get

$a + 4d = 19$

$\Rightarrow$ $a + 4(4) = 19$

$\Rightarrow$ $a + 16 = 19$

$\Rightarrow$ $a = 3$

So, the required AP will be:

$a, a + d, a + 2d, a + 3d$ ...

i.e., $3, 3 + 4, 3 + 2(4), 3 + 3(4)$ ...

i.e., 3, 7, 11, 15 ...

6. The 26th, 11th and the last term of an AP are 0, 3 and $-\frac{1}{5}$ respectively. Find the common difference and the number of terms.

Ans.

It is given that:

26th term of AP, $a_{26} = 0$

11th term of AP, $a_{11} = 3$

Last term, $L = \frac{-1}{5}$

Let the AP contain *n* term, last term (L) is the n^{th} term.

Let first term be *a* and common difference be *d* of *an* AP.

We know that $a_n = a + (n-1)d$

Also, $a_{26} = 0$ [Given]

$\Rightarrow$ $a_{26} = a + (26-1)d$

$\Rightarrow$ $0 = a + 25d$

$\Rightarrow$ $a + 25d = 0$ ...(i)

Also, $a_{11} = 3$

$\Rightarrow \quad a + (11 - 1)\, d = 3$

$\Rightarrow \quad a + 10d = 3 \quad ...(ii)$

$\text{Last term } L = \frac{-1}{5}$

$a + (n - 1)\, d = \frac{-1}{5} \quad ...(iii)$

Subtracting equation (ii) from equation (i), we get

$\Rightarrow \quad (a + 25d) - (a + 10d) = 0 - 3$

$\Rightarrow \quad a + 25d - a - 10d = 0 - 3$

$\Rightarrow \quad 15d = -3$

$\Rightarrow \quad d = \frac{-3}{15} = \frac{-1}{5}$

Putting the value of d in equation (i)

$\Rightarrow \quad a + 25\left(\frac{-1}{5}\right) = 0$

$\Rightarrow \quad a - 5 = 0$

$\Rightarrow \quad a = 5$

Putting the value of a and d in equation (iii)

$\Rightarrow \quad a + (n - 1)\, d = -\frac{1}{5}$

$\Rightarrow 5 + (n - 1)\left(-\frac{1}{5}\right) = -\frac{1}{5}$

$\Rightarrow \quad 25 - (n - 1) = -1$

$\Rightarrow \quad 25 + 1 = (n - 1)$

$\Rightarrow \quad n = 27$

Hence, the common difference $= -\frac{1}{5}$

and number of terms = 27.

7. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

[CBSE 2011]

Ans.

Let a be the first term and d be the common difference of AP.

According to the question, it is given that

$a_5 + a_7 = 52$

and $a_{10} = 46$

We know that $a_n = a + (n - 1)\, d$

$a_5 = a + (5 - 1)\, d = a + 4d$

$a_7 = a + (7 - 1)\, d = a + 6d$

$a_{10} = a + (10 - 1)\, d = a + 9d$

Now, $a_5 + a_7 = 52$

$\Rightarrow \quad (a + 4d) + (a + 6d) = 52$

$\Rightarrow \quad 2a + 10d = 52$

$\Rightarrow \quad a + 5d = 26 \quad ...(i)$

Also, $a_{10} = 46$

$\Rightarrow \quad a + 9d = 46 \quad ...(ii)$

Subtracting equation (i) from equation (ii), we get

$(a + 9d) - (a + 5d) = 46 - 26$

$\Rightarrow \quad 4d = 20$

$\Rightarrow \quad d = 5$

Putting value of d in equation (i), we get

$a + 5(5) = 26$

$\Rightarrow \quad a + 25 = 26$

$\Rightarrow \quad a = 1$

So, the required AP is

$a, a + d, a + 2d, a + 3d, ...$

$1, 1 + 5, 1 + 2(5), 1 + 3(5) ...$

$1, 6, 11, 16 ...$

8. Find the 20th term AP whose 7th term is 24 less than the 11th term, first term being 12.

[CBSE 2012]

Ans.

Let a be the first term and d be the common difference of the AP.

It is given that first term, $a = 12$

and $a_7 = a_{11} - 24$

We know that $a_n = a + (n - 1)d$

$a_7 = a + (7 - 1)\, d = a + 6d$

$a_{11} = a + (11 - 1)\, d = a + 10d$

Now, $a_7 = a_{11} - 24$

$\Rightarrow \quad a + 6d = a + 10d - 24$

$\Rightarrow \quad 4d = 24$

$\Rightarrow \quad d = 6$

$\therefore$ 20th term of AP,

$a_{20} = a + (20 - 1)\, d$

$= 12 + 19(6) = 12 + 114$

$a_{20} = 126$

Hence, the required 20th term of AP is 126.

9. If the 9th term of an AP is 0, prove that its 29th term is twice its 19th term.

[CBSE 2016, 13, 12]

Ans.

Let a be the first term, d be the common difference and n be the number of terms of AP.

It is given that $a_9 = 0$

We know that $a_n = a + (n - 1)d$

$\Rightarrow \quad a_9 = a + (9 - 1)\, d = a + 8d = 0$

$\Rightarrow \quad a = -8d \quad ...(i)$

Now, $a_{19} = a + (19 - 1)\, d = a + 18d$

$a_{29} = a + (29 - 1)\, d = a + 28d$

Using equation (i), we get

$a_{19} = -8d + 18d = 10d$

$a_{29} = -8d + 28d = 20d$

Clearly, $a_{29} = 2 \times a_{19}$

Hence, its 29th term is twice its 19th term.

10. Find whether 55 is a term of the AP: 7, 10, 13, ... or not. If yes, find which term it is. **[CBSE 2016, 11]**

Ans.

The given AP is 7, 10, 13, ...

Let a the first term and d be the common difference and n be the number of term whose value is 55.

i.e., $a_n = 55$

We know that $a_n = a + (n - 1)d$

The given series is 7, 10, 13, ...

i.e., here, first term $a = 7$

Common difference $d = a_2 - a_1 = 10 - 7 = 3$

Now, $a_n = a + (n - 1)d$

$\Rightarrow$ $55 = 7 + (n - 1)3$

$\Rightarrow$ $55 = 7 + (3n - 3)$

$\Rightarrow$ $3n = 51 \Rightarrow n = 17$

Hence, the 17^{th} term of AP is 55.

11. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are three consecutive terms of an AP. **[CBSE 2016, 15, 14, 10]**

Ans.

It is given that

$(k^2 + 4k + 8)$, $(2k^2 + 3k + 6)$ and $(3k^2 + 4k + 4)$ are three consecutive terms of an AP.

$\Rightarrow$They will have common difference

i.e., $(2k^2 + 3k + 6) - (k^2 + 4k + 8)$

$= (3k^2 + 4k + 4) - (2k^2 + 3k + 6)$

= common difference

$\Rightarrow$ $2k^2 + 3k + 6 - k^2 - 4k - 8$

$= 3k^2 + 4k + 4 - 2k^2 - 3k - 6$

$\Rightarrow$ $k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow$ $-k = k$

$\Rightarrow$ $2k = 0 \Rightarrow k = 0$

12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

Ans.

Let $(a - d)$, a and $(a + d)$ be three parts of 207 such that these are in AP.

It is given that sum of these numbers = 207

$\Rightarrow$ $(a - d) + a + (a + d) = 207$

$\Rightarrow$ $3a = 207 \Rightarrow a = 69$

It is also given that product of two smaller parts = 4623

$\Rightarrow$ $a(a - d) = 4623$

$\Rightarrow$ $69(69 - d) = 4623$

$\Rightarrow$ $69 - d = \frac{4623}{69}$

$\Rightarrow$ $69 - d = 67$

$\Rightarrow$ $d = 69 - 67 = 2$

So, first part $= a - d = 69 - 2 = 67$,

second part $= a = 69$

and third part $= a + d = 69 + 2 = 71$

Hence, the three parts are 67, 69 and 71.

13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

Ans.

It is given that angles of a triangle are in AP.

Let A, B and C be the angles of a ΔABC if $\angle A$, $\angle B$ and $\angle C$ are in AP

$\Rightarrow$ $B = \frac{A + C}{2} \Rightarrow A + C = 2B$ ...(i)

We know that sum of the angles of ΔABC is 180

$A + B + C = 180$

$\Rightarrow$ $(A + C) + B = 180$

$2B + B = 180$ [Using equation (i)]

$\Rightarrow$ $3B = 180 \Rightarrow B = 60$

Let A be the greatest angle and C be the least angle. It is given that the greatest angle is twice the least.

$\Rightarrow$ $A = 2C$ ...(ii)

But according to equation (i)

$A + C = 2B$

$\Rightarrow$ $2C + C = 2B$ [Using equation (ii)]

$\Rightarrow$ $3C = 2B = 2 \times 60 = 120$

$\Rightarrow$ $C = 40$

Also, $A = 2C = 2 \times 40 = 80$

Hence, the required angles of a triangle are 80°, 60° and 40°.

14. If the n^{th} terms of the two APs: 9, 7, 5, ... and 24, 21, 18, ... are the same, find the value of n. Also find that term.

Ans.

It is given that first AP is 9, 7, 5, ...

Here, first term = 9

and common difference = 7 – 9 = –2

Also, it is given that second AP is 24, 21, 18, ...

Here, first term = 24

and common difference, $d = 21 - 24 = -3$

Let n^{th} term be the same for above two AP's

We know that $a_n = a + (n - 1)d$

n^{th} term of first AP $= 9 + (n - 1)(-2)$

n^{th} term of second AP $= 24 + (n - 1)(-3)$

By given condition

$9 + (n - 1)(-2) = 24 + (n - 1)(-3)$

$9 - 2n + 2 = 24 - 3n + 3$

$3n - 2n = 24 + 3 - 9 - 2$

$n = 27 - 11 = 16$

Hence, the 16^{th} term is the same for both AP's.

16^{th} term = $a_{16} = 9 + (16 - 1)(-2)$

$= 9 + (15)(-2)$

$= 9 - 30 = -21$

Thus, the value of n is 16 and the n^{th} term is –21.

15. If the sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term. [CBSE 2012]

Ans.

Let a be the first term and d be the common difference of the AP.

We know that $a_n = a + (n - 1)d$

$\Rightarrow$ $a_3 = a + (3 - 1)d = a + 2d$

$a_7 = a + (7 - 1)d = a + 6d$

$a_8 = a + (8 - 1)d = a + 7d$

$a_{14} = a + (14 - 1)d = a + 13d$

According to the given condition

$a_3 + a_8 = 7$ and $a_7 + a_{14} = -3$

$\Rightarrow$ $a + 2d + a + 7d = 7$ and $a + 6d + a + 13d = -3$

$\Rightarrow$ $2a + 9d = 7$ ...(i)

and $2a + 19d = -3$ ...(ii)

Subtracting equation (i) from equation (ii)

$(2a + 19d) - (2a + 9d) = -3 - 7$

$\Rightarrow$ $2a + 19d - 2a - 9d = -10$

$\Rightarrow$ $10d = -10 \Rightarrow d = -1$

Putting value of d in equation (i)

$2a + 9d = 7$

$\Rightarrow$ $2a + 9(-1) = 7$

$\Rightarrow$ $2a - 9 = 7$

$\Rightarrow$ $2a = 16 \Rightarrow a = 8$

$\Rightarrow 10^{th}$ term = $a_{10} = a + (10 - 1)d$

$= 8 + (9)(-1) = 8 - 9 = -1$

16. Find the 12th term from the end of the AP: –2, –4, –6, ..., –100. [CBSE 2016]

Ans.

The given AP is –2, –4, –6, ..., –100

Here, first term, $a = -2$

Common difference,

$d = -4 - (-2) = -4 + 2 = -2$

Last term, L = –100

We know that n^{th} term of an AP from end

$a_n = L - (n - 1)d$

So, 12^{th} term from the end will be

$a_{12} = -100 - (12 - 1)(-2)$

$= -100 + (11)(-2)$

$= -100 + 22 = -78$

Hence, 12^{th} term from the end is –78.

17. Which term of the AP: 53, 48, 43, ... is the first negative term? [CBSE 2017]

Ans.

The given AP is 53, 48, 43, ...

Here, first term, $a = 53$

and common difference, $d = 48 - 53 = -5$

Let n^{th} term of the AP be the first negative term i.e., $a_n < 0$

We know that $a_n = a + (n - 1)d$

For $a_n < o$

$\Rightarrow$ $a + (n - 1)d < 0$

$\Rightarrow$ $53 + (n - 1)(-5) < 0$

$\Rightarrow$ $53 - 5n + 5 < 0$

$\Rightarrow$ $5n > 58$

$\Rightarrow$ $n > 11.6 \Rightarrow n = 12$

That is, 12^{th} term is the first negative term.

$\therefore$ $a_{12} = a + (12 - 1)d$

$= 53 + 11(-5)$

$= 53 - 55 = -2 < 0$

Hence, 12^{th} term is the first negative term.

18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3? [CBSE 2014, 11]

Ans.

Numbers that lie between 10 and 300 are 11, 12, ... 299.

Numbers between 10 and 300 which when divided by 4 leave a remainder 3 and are 11, 15, 19, ...299

Here, first number, $a = 11$

and common difference, $d = 4$

number of terms, $n = ?$

We know that $a_n = a + (n - 1)d$

$a + (n - 1)d = 299$

$\Rightarrow$ $11 + (n - 1)(4) = 299$

$\Rightarrow$ $11 + 4n - 4 = 299$

$\Rightarrow$ $4n = 292 \Rightarrow n = 73$

19. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, ..., 4\frac{1}{3}$. [CBSE 2010]

Ans.

The given AP is $\frac{-4}{3}, -1, \frac{-2}{3}, ...4\frac{1}{3}$

Here, first term, $a = \frac{-4}{3}$

Common difference, $d = -1 - \left(\frac{-4}{3}\right) = -1 + \frac{4}{3} = \frac{1}{3}$

Last term, L = $4\frac{1}{3} = \frac{13}{3}$

We know that $a_n = a + (n-1)d$

If a_n is the last term, then

$$L = a + (n-1)d$$

$$\frac{13}{3} = -\frac{4}{3} + (n-1)\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad 13 = -4 + (n-1)$$

$$\Rightarrow \quad (n-1) = 17$$

$$\Rightarrow \quad n = 18$$

So, the two middle most terms are $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ as numbers of terms are even.

The two middle most terms are $\left(\frac{18}{2}\right)^{th}$ and $\left(\frac{18}{2}+1\right)^{th}$

i.e., 9^{th} and 10^{th} term

$$a_9 = a + (9-1)d = a + 8d$$

$$= -\frac{4}{3} + 8\left(\frac{1}{3}\right) = \frac{(-4+8)}{3} = \frac{4}{3}$$

$$a_{10} = a + (10-1)d = a + 9d$$

$$= -\frac{4}{3} + 9\left(\frac{1}{3}\right) = \frac{(-4+9)}{3} = \frac{5}{3}$$

So, the sum of the two middle most terms = $a_9 + a_{10}$

$$= \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$$

20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference. [CBSE 2019, 17, 14, 10]

Ans.

Let a be the first term, d be the common difference and n be the number of terms.

It is given that:

first term, $a = -5$

Last term, $L = 45$

We know that, if the last term of *an* AP is known, then the sum of n terms of *an* AP is

$$S_n = \frac{n}{2}(a+l)$$

$$120 = \frac{n}{2}(-5+45)$$

$$120 \times 2 = 40 \times n$$

$$\Rightarrow \quad n = 6$$

$$L = a + (n-1)d$$

$$\Rightarrow \quad 45 = -5 + (6-1)d$$

$$\Rightarrow \quad 50 = 5d \Rightarrow d = 10$$

Hence, number of terms = 6

and common difference = 10

21. Find the sum:

(i) 1 + (–2) + (–5) + (–8) + ... + (–236) **[CBSE 2020]**

(ii) $4 - \frac{1}{n} + 4 - \frac{2}{n} + 4 - \frac{3}{n} +$ ... upto n terms **[CBSE 2017]**

(iii) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} +$ ... upto 11 terms

Ans.

(i) 1 + (–2) + (–5) ... (–236)

Here, $a = 1$

and common difference, $d = (-2) - 1 = -3$

$\therefore$ Sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Also, last term, $L = a + (n-1)d$.

where n is the total no. of terms

$$L = a + (n-1)d$$

$$-236 = 1 + (n-1)(-3)$$

$$-237 = (n-1)(-3)$$

$$\Rightarrow \quad (n-1) = 79$$

$$\Rightarrow \quad n = 80$$

$\therefore$ Sum of n terms of an AP

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{80}{2}[2 \times 1 + (80-1)(-3)]$$

$$= 40[2 - (79 \times 3)]$$

$$= 40[2 - 237]$$

$$= 40[-235] = -9400$$

Hence, the required sum is –9400.

(ii) $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{9}\right) +$ ... up to n terms

Here, first term,

$$a = 4 - \frac{1}{n}$$

and common difference,

$$d = \left(4 - \frac{2}{n}\right) - \left(4 - \frac{1}{n}\right)$$

$$= 4 - \frac{2}{n} - 4 + \frac{1}{n} = \frac{-2}{n} + \frac{1}{n} = \frac{-1}{n}$$

Sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}\left[2\left(4 - \frac{1}{n}\right) + (n-1)\left(\frac{-1}{n}\right)\right]$$

$$= \frac{n}{2}\left[8 - \frac{2}{n} - 1 + \frac{1}{n}\right] = \frac{n}{2}\left[7 - \frac{1}{n}\right] = \frac{(7n-1)}{2}$$

Hence, the required sum is $\frac{(7n-1)}{2}$.

(iii) $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+$ ...+ 11 terms

Here, first term, A = $\frac{a-b}{a+b}$

and common difference,

$$D = \frac{3a-2b}{a+b}-\frac{(a-b)}{a+b}=\frac{2a-b}{a+b}$$

Sum of n terms

$$S_n = \frac{n}{2}[2A + (n-1)D]$$

$$= \frac{n}{2}\left[\frac{2(a-b)}{(a+b)}+\frac{(n-1)(2a-b)}{(a+b)}\right]$$

$$= \frac{n}{2}\left[\frac{2a-2b+2an-2a-nb+b}{(a+b)}\right]$$

$$= \frac{n}{2}\left[\frac{2an-b-nb}{(a+b)}\right]$$

It is given that no. of terms, $n = 11$

$$S_{11} = \frac{11}{2}\left[\frac{2a(11)-b-11b}{(a+b)}\right]=\frac{11}{2}\left[\frac{22a-12b}{(a+b)}\right]$$

$$= \frac{11\times 2}{2}\left[\frac{(11a-6b)}{(a+b)}\right]=\frac{11(11a-6b)}{(a+b)}$$

22. Which term of the AP: –2, –7, –12, ... will be –77? Find the sum of this AP upto the term –77. [CBSE 2012]

Ans.

The given AP is –2, –7, –12, ...

Here, first term, $a = -2$

Common difference,

$$d = -7 - (-2) = -7 + 2 = -5$$

Let n^{th} term of the AP be –77.

We know that $a_n = a + (n-1)d$

$$-77 = -2 + (n-1)(-5)$$

$$\Rightarrow \quad -75 = -(n-1)5$$

$$\Rightarrow \quad (n-1) = 15 \Rightarrow n = 16$$

Hence, 16^{th} term of the given AP will be –77.

Now, the sum of n terms of an AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

So, the sum of 16 term i.e., upto the term (–77)

i.e., $S_{16} = \frac{16}{2}[2 \times (-2) + (n-1)(-5)]$

$$= 8[-4 + (16-1)(-5)]$$

$$= 8[-4 - 75]$$

$$= 8 \times [-79] = -632$$

Hence, the sum of this AP upto 16 terms is –632.

23. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an AP. Also find S_{20}. [CBSE 2013]

Ans.

It is given that

$$a_n = 3 - 4n$$

For $n = 1$, $a_1 = 3 - 4(1) = 3 - 4 = -1$

$n = 2$, $a_2 = 3 - 4(2) = 3 - 8 = -5$

$n = 3$, $a_3 = 3 - 4(3) = 3 - 12 = -9$

$n = 4$, $a_4 = 3 - 4(4) = 3 - 16 = -13$

So, the series becomes –1, –5, –9, –13, ...

Difference between two consecutive terms

$$a_2 - a_1 = -5 - (-1) = -5 + 1 = -4$$

$$a_3 - a_2 = -9 - (-5) = -9 + 5 = -4$$

$$a_4 - a_3 = -13 - (-9) = -13 + 9 = -4$$

Clearly, each successive term has the same difference, so it forms an AP.

Also, we know that sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)]$$

$$\Rightarrow \quad S_{20} = 10[-2 + (19)(-4)]$$

$$= 10[-2 - 76]$$

$$= 10[-78] = -780$$

Hence, the required sum of 20 terms

i.e., $S_{20} = -780$

24. In an AP, if $S_n = n(4n + 1)$, find the AP. [CBSE 2019, 14, 12]

Ans.

It is given that

$$S_n = n(4n + 1)$$

We know that n^{th} term of an AP

$$a_n = S_n - S_{n-1}$$

$$S_n = 4n^2 + n$$

$$\Rightarrow \quad S_{n-1} = 4(n-1)^2 + (n-1)$$

$$= 4(n^2 + 1 - 2n) + (n-1)$$

$$= 4n^2 + 4 - 8n + n - 1$$

$$= 4n^2 - 7n + 3$$

$$\Rightarrow \quad a_n = (4n^2 + n) - (4n^2 - 7n + 3)$$

$$= 4n^2 + n - 4n^2 + 7n - 3 = 8n - 3$$

For $n = 1$, $a_1 = 8(1) - 3 = 8 - 3 = 5$

$n = 2$, $a_2 = 8(2) - 3 = 16 - 3 = 13$

$n = 3$, $a_3 = 8(3) - 3 = 24 - 3 = 21$

Hence, the required AP is 5, 13, 21, ...

25. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

Ans.

It is given that

$$S_n = 3n^2 + 5n$$

$$a_k = 164$$

We know that n^{th} term of an AP

$a_n = S_n - S_{n-1}$

$S_n = 3n^2 + 5n$

$\Rightarrow \quad S_{n-1} = 3(n-1)^2 + 5(n-1)$

$= 3(n^2 + 1 - 2n) + 5(n-1)$

$= 3n^2 + 3 - 6n + 5n - 5$

$= 3n^2 - n - 2$

$a_n = S_n - S_{n-1}$

$= (3n^2 + 5n) - (3n^2 - n - 2)$

$= 3n^2 + 5n - 3n^2 + n + 2 = 6n + 2$

$a_n = 6n + 2$

or $\quad a_k = 6k + 2 \qquad$ [As $a_k = 164$]

$164 = 6k + 2 \Rightarrow 6k = 162 \Rightarrow k = 27$

Hence, $k = 27$.

26. If S_n denotes the sum of first n terms of an AP, prove that

$S_{12} = 3(S_8 - S_4)$ [CBSE 2015]

Ans.

We know that sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_4 = \frac{4}{2}[2a + (4-1)d]$$

$$= 2[2a + 3d] = 4a + 6d$$

$$S_8 = \frac{8}{2}[2a + (8-1)d]$$

$$= 4[2a + 7d] = 8a + 28d$$

$(S_8 - S_4) = (8a + 28d) - (4a + 6d)$

$= 8a + 28d - 4a - 6d$

$= 4a + 22d \qquad$ ...(i)

$$S_{12} = \frac{12}{2}[2a + (12-1)d]$$

$= 6[2a + 11d] = 12a + 66d$

$= 3[4a + 22d]$

$S_{12} = 3[S_8 - S_4] \qquad$ [using equation (i)]

Hence proved.

27. Find the sum of the first 17 terms of an AP whose 4^{th} and 9^{th} terms are –15 and –30 respectively. [CBSE 2014]

Ans.

It is given that

4^{th} term = $a_4 = -15$ and 9^{th} term = $a_9 = -30$

Let a be the first term, d be the common difference and n be the number of terms in an AP.

We know that,

$a_n = a + (n-1)d$

$\Rightarrow \quad a_4 = a + (4-1)d$

$\Rightarrow \quad -15 = a + 3d \qquad$ ...(i)

[Given $a_4 = -15$]

Again, $\quad a_n = a + (n-1)d$

$\Rightarrow \quad a_9 = a + (9-1)d$

$\Rightarrow \quad -30 = a + 8d \qquad$ ...(ii)

[Given $a_9 = -30$]

Subtracting equation (i) from equation (ii) we get

$(a + 8d) - (a + 3d) = -30 - (-15)$

$a + 8d - a - 3d = -30 + 15$

$5d = -15$

$d = -3$

Putting the value of d in equation (i), we get

$-15 = a + 3(-3)$

$-15 = a - 9$

$\Rightarrow \quad a = -15 + 9 = -6$

We know that sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of first 17 terms of an AP,

$$S_{17} = \frac{17}{2}[2(-6) + (17-1)(-3)]$$

$$= \frac{17}{2}[-12 + (16)(-3)] = \frac{17}{2}[-17 - 48]$$

$$= \frac{17}{2} \times (-60) = -\frac{17}{1} \times (30) = -510$$

Hence, the required sum is –510.

28. If the sum of the first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms.

[CBSE 2016, 13, 12]

Ans.

It is given that

$S_6 = 36$ and $S_{16} = 256$

Let a be the first term and d be the common difference of an AP.

We know that sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_6 = 36$(Given)

$$\Rightarrow \quad \frac{6}{2}[2a + (6-1)d] = 36$$

$\Rightarrow \quad 3[2a + (6-1)d = 36$

$\Rightarrow \quad 2a + 5d = 12 \qquad$ ...(i)

Also, $\quad S_{16} = 256$

$$\Rightarrow \quad \frac{16}{2}[2a + (16-1)d] = 256$$

$\Rightarrow \quad 8[2a + 15d] = 256$

$\Rightarrow \quad 2a + 15d = 32 \qquad$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$(2a + 15d) - (2a + 5d) = 32 - 12$

$\Rightarrow \quad 2a + 15 - 2a - 5d = 20$

$\Rightarrow \quad 10d = 20$

$\Rightarrow \quad d = 2$

Putting the value of d in equation (i), we get

$2a + 5d = 12$

$2a + 5(2) = 12$

$\Rightarrow \quad 2a + 10 = 12$

$\Rightarrow \quad 2a = 2 \quad \Rightarrow \quad a = 1$

We have to find sum of first 10 terms, $S_{10} = ?$

$$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$$

$$= 5[2(1) + 9(2)]$$

$$= 5[2 + 18]$$

$$= 5 \times 20$$

$$S_{10} = 100$$

Hence, the required sum of the first 10 terms is 100.

29. Find the sum of all the 11 terms of an AP whose middle most term is 30.

Ans.

It is given that

total number of terms, $n = 11$

$\Rightarrow$ Middle most term $= \frac{(n+1)^{th}}{2}$ term

[as n is odd]

$$= \frac{(11+1)^{th}}{2} \text{ term} = 6^{th} \text{ term}$$

It is given that $a_6 = 30$

We know that $a_n = a + (n - 1)d$

$a_6 = 30$

$\Rightarrow \quad a + (6 - 1)d = 30$

$\Rightarrow \quad a + 5d = 30$ ...(i)

Sum of n terms of AP,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{11} = \frac{11}{2}[2a + (11 - 1)d]$$

$$= \frac{11}{2}[2a + 10d]$$

$$S_{11} = \frac{11}{2}[2(a + 5d)] = \frac{11}{2}[2 \times 30]$$

[Using equation (i)]

$$= \frac{11}{2} \times 2 \times 30 = 330$$

Hence, the sum of 11 terms of the AP is 330.

30. Find the sum of last 10 terms of the AP: 8, 10, 12, ..., 126. [CBSE 2015]

Ans.

The given AP is 8, 10, 12, ...126

For finding sum of last 10 terms, we write the given AP in reverse order i.e., 126, 124, 122, ...10, 8.

Here, first term, $a = 126$

and common difference, $d = 124 - 126 = -2$

$\therefore \quad S_{10} = ?$

We know that sum of n terms of AP

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2(126) + (10 - 1)(-2)]$$

$$= 5[252 + 9(-2)] = 5[252 - 18]$$

$$= 5 \times 234 = 1170$$

31. Find the sum of first 7 numbers which are multiples of 2 as well as of 9.
[Hint: Take the LCM of 2 and 9]

Ans.

Numbers which are multiples of 2 as well as 9 will be multiples of their L.C.M. We know that L.C.M. of 2 and 9 = 18.

Multiples of 18 are 18, 36, 54, ...

Series becomes 18, 36, 54, ...

Here, first term, $a = 18$

and common difference, $d = 36 - 18 = 18$

We know that sum of n terms

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Sum of first seven numbers

$$S_7 = \frac{7}{2}[2(18) + (7 - 1)18]$$

$$= \frac{7}{2}[36 + 6 \times 18]$$

$$= 7[18 + 54] = 7 \times 72 = 504$$

Hence, the sum of first 7 numbers which are multiples of 2 as well as 9 is 504.

32. How many terms of the AP: –15, –13, –11, ... are needed to make the sum –55? Explain the reason for double answer. [CBSE 2020, 16]

Ans.

The given series is $-15, -13, -11, \ldots$

Here, first term, $a = -15$

common difference,

$d = -13 - (-15) = -13 + 15 = 2$

Let n' be the number of terms needed to make the sum –55.

We know that sum of first n terms

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\Rightarrow \quad -55 = \frac{n}{2}[2(-15) + (n - 1)(2)]$

$\Rightarrow \quad -55 = n[-15 + (n - 1)]$

$= n[-15 + n - 1]$

$-55 = n[n - 16]$

$\Rightarrow \quad n^2 - 16n + 55 = 0$

Splitting the middle term

$\Rightarrow n^2 - 11n - 5n + 55 = 0$

$\Rightarrow n - (n - 11) - 5(n - 11) = 0$

$\Rightarrow (n - 5)(n - 11) = 0 \Rightarrow n = 5, 11$

Hence, either 5 or 11 terms are needed to make the sum –55

When $n = 5$, AP will be –15, –13, –11, –9, –7

So, resulting sum = –55 as all terms are negative when $n = 11$, AP will be –15, –13, –11, –9, –7, –5, –3, –1, 1, 3, 5

So, resulting sum = –55 as the sum of 6^{th} to 11^{th} terms is 0. This means that the sum of 11 terms will be equal to the sum of 5 terms i.e., –55

33. The sum of the first *n* terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2n terms of another AP whose first term is –30 and the common difference is 8. Find *n*.

Ans.

It is given that

first term of first AP, $a = 8$

and common difference of first AP, $d = 20$

Let n be the number of terms in first AP.

We know that sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 \times 8 + (n - 1)20]$$

$$= \frac{n}{2}[16 + 20n - 20]$$

$$= \frac{n}{2}[20n - 4] = n[10n - 2]$$

$$\Rightarrow S_n = n[10n - 2] \quad ...(i)$$

Now, first term of second AP (a') = –30

Common difference of secod (d') = 8

$\therefore$ Sum of first 2n terms of second AP

$$S_{2n} = \frac{2n}{2}[2a' + (2n - 1)d']$$

$$= n[2(-30) + (2n - 1)8]$$

$$= n[-60 + 16n - 8]$$

$$S_{2n} = n[16n - 68] \quad ...(ii)$$

By given condition,

Sum of first n terms of first AP

= sum of first 2n terms of second AP

$$\Rightarrow S_n = S_{2n}$$

Using equation (i) and equation (ii), we get

$$\Rightarrow n(10n - 2) = n(16n - 68)$$

$$\Rightarrow 10n - 2 - 16n + 68 = 0$$

$$\Rightarrow -6n + 66 = 0$$

$$\Rightarrow -6(n - 11) = 0$$

$$\Rightarrow n = 11$$

Hence, the required value of n is 11.

EXERCISE 5.4

1. The sum of the first 5 terms of an AP and the sum of the first 7 terms of the same AP is 167. If the sum of the first 10 terms of this AP is 235, find the sum of its first 20 terms.

Ans.

Let a be the first term, d be the common difference and n be the number of terms of an AP.

We know that

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Sum of first five terms, S_5

$$S_5 = \frac{5}{2}[2a + (5 - 1)d] = \frac{5}{2}[2a + 4d]$$

$$S_5 = 5[a + 2d]$$

$$S_5 = 5a + 10d \quad ...(i)$$

Sum of first seven terms, S_7

$$S_7 = \frac{7}{2}[2a + (7 - 1)d]$$

$$= \frac{7}{2}[2a + 6d]$$

$$S_7 = 7[a + 3d]$$

$$S_7 = 7a + 21d \quad ...(ii)$$

Now, by given condition

$$S_5 + S_7 = 167$$

$$\Rightarrow 5a + 10d + 7a + 21d = 167$$

[using equation (i) & equation (ii)]

$$\Rightarrow 12a + 31d = 167 \quad ...(iii)$$

Also, it is given that sum of first 10 terms of this AP is 235.

$$\Rightarrow S_{10} = 235$$

$$\Rightarrow \frac{10}{2}[2a + (10 - 1)d] = 235$$

$$\Rightarrow 5[2a + 9d] = 235$$

$$\Rightarrow 2a + 9d = \frac{235}{5} = 47$$

$$\Rightarrow 2a + 9d = 47 \quad ...(iv)$$

Multiplying equation (iv) by 6 and subtracting it from equation (iii), we get

$$(12a + 31d) - (12a + 54d) = 167 - 282$$

$$\Rightarrow 23d = 115 \Rightarrow d = 5$$

Putting the value of d in equation (4), we get

$$2a + 9d = 47$$

$$\Rightarrow 2a + 9(5) = 47$$

$$\Rightarrow 2a + 45 = 47$$

$$\Rightarrow 2a = 2 \Rightarrow a = 1$$

Sum of first 20 terms of this AP,

$$S_{20} = \frac{20}{2}[2a + (20 - 1)d]$$
$$= 10[2(1) + 19(5)]$$
$$= 10[2 + 95]$$
$$= 10 \times 97$$
$$= 970$$

Hence, the required sum of first 20 terms is 970.

2. Find the:

(i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5. **[CBSE 2014]**

(ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

(iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[Hint (iii): These numbers will be: multiple of 2 + multiple of 5 – multiples of 2 as well as of 5]

Ans.

(i) Multiples of 2 as

as 5 will be multiples of LCM of 2 and 5.
LCM of (2, 5) = 10

Multiples of 2 as well as 5 between 1 and 500 is 10, 20, 30, 40, ...490

This forms an AP with first term, a= 10

Common difference, d = 20 – 10 = 10

Last term, L = 490

We know that sum of n terms between 1 and 500

$$S_n = \frac{1}{2}[a + L] \quad ...(i)$$

Also, $L = a + (n - 1)\,d$

$\Rightarrow$ $490 = 10 + (n - 1)10$

$\Rightarrow$ $480 = (n - 1)10$

$\Rightarrow$ $(n - 1) = 48$

$\Rightarrow$ $n = 49$

Putting this value in equation (i), we get

$$S_{49} = \frac{49}{2}[10 + 490]$$
$$= \frac{49}{2} \times 500 = \frac{49}{2} \times 250 = 12250$$

$\Rightarrow$ $S_{49} = 12250$

(ii) Here, multiples of 2 as well as 5 from 1 to 500 are 10, 20, 30, ...500

Here, first term, a = 10

common difference, d = 20 – 10 = 10

Last term, L = 500

We know that $a_n = a + (n - 1)d$

$L = a + (n - 1)d$

[Where n is total no. of terms]

$500 = 10 + (n - 1)d$

$490 = (n - 1)10$

$\Rightarrow$ $(n - 1) = 49 \Rightarrow n = 50$

Also we know that $S_n = \frac{n}{2}(a + L)$

$\Rightarrow$ $S_{50} = \frac{50}{2}(10 + 500)$

$= 25 \times 510 = 12750$

Hence, $S_{50} = 12750$

(iii) Multiples of 2 or 5 = Multiples of 2 + multiples of 5 – [Multiples of 2 and 5] ...(i)

Multiples of 2 [2, 4, 6, ...500]

Multiples of 5 [5, 10, 15, ...500]

Multiples 2 and of 5 [10, 20, 30, ...500]

1st list of multiples of 2 [2, 4, 6, ...500]

Here, first term, $a_1 = 2$

and common difference $d_1 = 2$

Let number of terms be n_1

Then last term, $L = a + (n_1 - 1)d$

$500 = 2 + (n_1 - 1)(2)$

$498 = (n_1 - 1)2$

$\Rightarrow$ $(n_1 - 1) = 249 \Rightarrow n_1 = 250$

Sum of [2, 4, 6, ... 500]

$$S_{n_1} = \frac{n_1}{2}[a + l]$$
$$= \frac{250}{2}[2 + 500]$$
$$S_{n1} = 225 \times 502 = 62750$$

2nd list of multiples of 5 [5, 10, 15, ...500]

Here, first term, a' = 5,

common difference, d' = 5

Last term, L' = 500

Let n_2 be the number of terms of second list. We know that $a_n = a + (n - 1)d$

$L' = a' + (n_2 - 1)\,d'$

$500 = 5 + (n_2 - 1)5$

$\Rightarrow$ $495 = (n_2 - 1)5$

$\Rightarrow$ $n_2 - 1 = 99$

$\Rightarrow$ $n_2 = 100$

3rd list of multiples of 2 as well as 5

[10, 20, 30, ...500]

Here, first term, a'' = 10

Common difference, d'' = 10

Last term , L'' = 500

Let n_3 be the number of terms, then

$L'' = a'' + (n_3 - 1)\,d''$

$500 = 10 + (n_3 - 1)(10)$

$490 = (n_3 - 1)10$

$\Rightarrow$ $n_3 - 1 = 49$

$\Rightarrow$ $n_3 = 50$

Sum of 2nd List,

$$S_n = \frac{n}{2}(a + L)$$

$$S_{n2} = \frac{n_2}{2}(5 + 500)$$

$$= \frac{100}{2} \times 505 = 25250$$

Sum of 3rd List,

$$S_{n3} = \frac{n_3}{2}(10 + 500)$$

$$= \frac{50}{2} \times 510 = 12750$$

$\Rightarrow$ Sum of multiples of 2 or 5

$$= S_{n_1} + S_{n_2} - S_{n_3}$$
$$= 62750 + 25250 - 12750$$
$$= 88000 - 12750 = 75250$$

3. The 8th term of an AP is half its second term and the 11th term exceeds one third of its 4th term by 1. Find the 15th term.

[CBSE 2016, 12]

Ans.

Let a be the first term and d be the common difference of given AP.

It is given that

$$a_8 = \frac{1}{2}(a_2) \quad \text{...(i)}$$

$$a_{11} = \frac{1}{3}a_4 + 1 \quad \text{...(ii)}$$

We know that $a_n = a + (n - 1)d$

$$a_8 = a + 7d$$

$$a_2 = a + d$$

$$a_{11} = a + 10d \text{ and } a_4 = a + 3d$$

Putting these values in equation (i) and equation (ii), we get

$$a8 = \frac{1}{2}(a2)$$

$$\Rightarrow \quad a + 7d = \frac{1}{2}(a + d)$$

$$\Rightarrow \quad 2a + 14d = a + d$$

$$\Rightarrow \quad a + 13d = 0 \quad \text{...(iii)}$$

Also, $$a_{11} = \frac{a_4}{3} + 1$$

$$\Rightarrow \quad a + 10d = \frac{a+3d}{3} + 1 = \frac{a+3d+3}{3}$$

$$\Rightarrow \quad 3a + 30d = a + 3d + 3$$

$$\Rightarrow \quad 2a + 27d = 3 \quad \text{...(iv)}$$

From equation (iii) and equation (iv), we get

$$2(-13d) + 27d = 3$$

$$-26d + 27d = 3 \Rightarrow d = 3$$

From equation (iii),

$$a + 13d = 0$$

$$a + 13(3) = 0$$

$$a + 39 = 0$$

$$a = -39, d = 3$$

$$a_{15} = a + (15 - 1)d$$

$$= -39 + (14)(3)$$

$$a_{15} = -39 + 42 = 3$$

Hence, the 15th term is 3.

4. An AP consists of 37 terms. The sum of the 3 middle most terms is 225 and the sum of the last 3 is 429. Find the AP.

Ans.

It is given that

Total number of terms, $n = 37$

Since n is odd, therefore

middle most term = $\left(\frac{n+1}{2}\right)^{th}$ term = 19th term

3 middle most terms = 18th, 19th and 20th term by given condition

$$a_{18} + a_{19} + a_{20} = 225$$

We know that $a_n = a + (n - 1)d$

$$\Rightarrow \quad (a + 17d) + (a + 18d) + (a + 19d) = 225$$

$$\Rightarrow \quad 3a + 54d = 225$$

$$\Rightarrow \quad a + 18d = 75 \quad \text{...(i)}$$

Also, it is given that sum of last 3 terms = 429

$$\Rightarrow \quad a_{35} + a_{36} + a_{37} = 429$$

$$\Rightarrow (a + 34d) + (a + 35d) + (a + 36d) = 429$$

$$\Rightarrow \quad 3a + 105d = 429$$

$$\Rightarrow \quad a + 35d = 143 \quad \text{...(ii)}$$

Subtracting equation (i) from equation (ii), we get

$$(a + 35d) - (a + 18d) = 143 - 75$$

$$a + 35d - a - 18d = 68$$

$$\Rightarrow \quad 17d = 68 \Rightarrow d = 4$$

Putting value of d in equation (i), we get

$$a + 18(4) = 75$$

$$\Rightarrow \quad a + 72 = 75$$

$$\Rightarrow \quad a = 3, \ d = 4$$

Required AP is

$a, a + d, a + 2d, a + 3d, \ldots$

$3, 3 + 4, 3 + 2(4), 3 + 3(4), \ldots$

$3, 7, 11, 15, \ldots$

5. Find the sum of the integers between 100 and 200 that are:

(i) divisible by 9 **[CBSE 2012, 11]**

(ii) not divisible by 9

[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9]

Ans.

(i) Integers between 100 and 200 that are divisible by 9 are 108, 117, 126, ...198

Let n be the number of terms between 100 and 200

Here, first term, $a = 108$

Common difference, $d = 117 - 108 = 9$

Last term, L = 198

We know that $a_n = a + (n-1)d$

$$L = a + (n-1)d$$

$$\Rightarrow \quad 198 = 108 + (n-1)9$$

$$\Rightarrow \quad 198 - 108 = (n-1)9$$

$$\Rightarrow \quad 90 = (n-1)9$$

$$\Rightarrow \quad (n-1) = 10 \Rightarrow n = 11$$

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ Sum of 11 terms between 100 and 200 which is divisible by 9 is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad S_{11} = \frac{11}{2}[2(108) + (11-1)(9)]$$

$$= \frac{11}{2}[2(108) + 10 \times 9]$$

$$= 11\,[108 + 5 \times 9]$$

$$= 11\,[108 + 45]$$

$$= 11 \times 153 = 1683$$

Hence, the required sum of integers is 1683.

(ii) Sum of integers between 100 and 200 which is not divisible by 9

= sum of total no. between 100 and 200 – sum of no. between 100 and 200 which are divisible by 9 ...(i)

From part (i), we know that sum of no. between 100 and 200 divisible by 9 = 1683

Total numbers between 100 and 200 is 101, 102, 103, ... 199.

Here, first term, $a = 101$

Common difference, $d = 102 - 101 = 1$

Last term, L = 199

Let n be total no. of terms.

We know that $a_n = a + (n-1)d$

$$\Rightarrow \quad L = a + (n-1)d$$

$$199 = 101 + (n-1)(1)$$

$$199 - 101 = (n-1)$$

$$n = 99$$

$\therefore$ Sum of 99 terms between 100 and 200

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{99} = \frac{99}{2}[2(101) + (99-1)(1)]$$

$$= \frac{99}{2}[202 + 98]$$

$$= \frac{99}{2} \times 300$$

$$S_{99} = 99 \times 150 = 14850$$

Putting this value in equation (i), we get sum of no. between 100 and 200 not divisible by 9

$$= 14850 - 1683$$

$$= 13167$$

Hence, the required sum is 13167.

6. The ratio of the 11th term to the 18th term of an AP is 2:3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first 5 terms to the sum of the first 21 terms. [CBSE 2017, 16, 14]

Ans.

Let a be the first term and d be the common difference of the AP.

It is given that $a_{11} : a_{18} = 2:3$

We know that $a_n = a + (n-1)d$

$$a_{11} = a + (11-1)d = a + 10d$$

$$a_{18} = a + (18-1)d = a + 17d$$

$$\frac{a+10d}{a+17d} = \frac{2}{3}$$

$$\Rightarrow \quad 3(a + 10d) = 2(a + 17d)$$

$$\Rightarrow \quad 3a + 30d = 2a + 34d$$

$$\Rightarrow \quad 3a - 2a = 34d - 30d$$

$$\Rightarrow \quad a = 4d \qquad \ldots(i)$$

Now, $a_5 = a + (5-1)d = a + 4d$

$$= 4d + 4d = 8d$$

[using equation (i)]

$$a_{21} = a + (21-1)d = a + 20d$$

$$= 4d + 20d = 24d$$

[using equation (i)]

$$\therefore \quad a_5 : a_{21} = 8d : 24d = 1 : 3$$

Also, we know that

Sum of first n terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of first 5 terms,

$$S_5 = \frac{5}{2}[2(4d) + (5-1)(d)]$$

$$\Rightarrow \quad S_5 = \frac{5}{2}[8d + 4d)] = \frac{5 \times 12d}{2}$$

$$\Rightarrow \quad S_5 = 30d$$

Sum of first 21 terms,

$$S_{21} = \frac{21}{2}[2(4d) + (21-1)(d)]$$

$$\Rightarrow \quad S_{21} = \frac{21}{2}[8d + 20d] = \frac{21 \times 28d}{2}$$

$$\Rightarrow \quad S_{21} = 294d$$

So, ratio of sum of first five terms to the sum of first 21 terms is

$$S_5 : S_{21} = 30d : 294d$$

$$= 5:49$$

7. Show that the sum of an AP whose 1st term is a, the 2nd b and the last term c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Ans.

It is given that

First term, $= a$

Second term $= b$

Last term, L $= c$

Common difference $= b - a$

AP is $a, b, \ldots c$

Let n be the number of terms of the given AP.

We know that $a_n = a + (n-1)d$

$\Rightarrow$ $L = a + (n-1)(b-a)$

$\Rightarrow$ $c = a + (n-1)(b-a)$

$\Rightarrow$ $(c-a) = (n-1)(b-a)$

$\Rightarrow$ $(n-1) = \frac{(c-a)}{(b-a)}$ ...(i)

$\Rightarrow$ $n = \frac{c-a}{b-a} + 1 = \frac{(c-a)+(b-a)}{b-a}$

$= \frac{c+b-2a}{b-a}$ ...(ii)

Now, we know that sum of an AP

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{b+c-2a}{2(b-a)}\left[2a + \frac{c-a}{b-a}(b-a)\right]$

[using equation (i) and equation (ii)]

$= \frac{(b+c-2a)}{2(b-a)}[2a+c-a]$

$= \frac{(b+c-2a)(a+c)}{2(b-a)}$

Hence, proved

8. Solve the equation $-4 + (-1) + 2 + \ldots + x = 437$.

Ans.

The given equation is $-4 + (-1) + 2 + \ldots + x = 437$

Here, in the given AP

first term, $a = -4$

and common difference,

$d = (-1) - (-4) = -1 + 4 = 3$

Last term, $L = x$

We know that $a_n = a + (n-1)d$

$\Rightarrow$ $L = x = a + (n-1)(d)$

$\Rightarrow$ $x = -4 + (n-1)(3)$

$\Rightarrow$ $x = -4 + 3n - 3$

$\Rightarrow$ $n = \frac{x+7}{3}$

Also, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_n = \frac{x+7}{2\times 3}\left[2(-4) + \left(\frac{x+7}{3} - 1\right)(3)\right]$

$= \frac{x+7}{6}[-8 + x + 4] = \frac{(x+7)(x-4)}{6}$

The given equation is $(-4) + (-1) + 2 \ldots + x = 437$

$\Rightarrow$ $S_n = 437$

$\Rightarrow$ $\frac{(x+7)(x-4)}{6} = 437$

$\Rightarrow$ $x^2 - 4x + 7x - 28 = 437 \times 6$

$\Rightarrow$ $x^2 + 3x - 28 = 2622$

$\Rightarrow$ $x^2 + 3x - 2650 = 0$

By quadratic formula

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-2650)}}{2(1)} = \frac{-3 \pm \sqrt{10609}}{2}$

$= \frac{-3 \pm 103}{2}$

$\Rightarrow x = \frac{-3+103}{2}, \frac{-3-103}{2} = \frac{100}{2}, \frac{-106}{2} = 50, -53$

But x can not be negative. so $x = 50$.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. The common difference between the terms of two AP's is same. If the difference between their 50th terms is 100, what is the difference between their 100th terms?

Ans.

Let a_1 and a_2 be the first terms of two AP's and d be their common difference.

$(a_1 + 49d) - (a_2 + 49d) = 100$

$a_1 - a_2 = 100$ ...(i)

Then, difference between their 100th terms is

$(a_1 + 99d) - (a_2 + 99d) = a_1 - a_2$ [Using (i)]

$= 100$

The difference between their 100th terms is 100 i.e., same as difference in 50th terms.

2. Two AP's have the same common difference. The difference their between their 100th terms is 100, what is the difference between 1000th terms.

Ans.

Let a_1 and a_2 are two AP's and their common difference be d.

According to question,

$[a_1 + (100 - 1)d] - [a_2 + (100 - 1)d] = 100$

$[a_n = a + (n - 1)d]$

$[a_1 + 99d] - [a_2 + 99d] = 100$

$a_1 - a_2 = 100$ ...(i)

Now, to find the difference between their 1000^{th} terms

$[a_1 + (1000 - 1)d] - [a_2 + (1000 - 1)d]$

$= a_1 + 999d - a_2 - 999d$

$= a_1 - a_2$

By equation (i), we get

$[a_1 + (1000 - 1)d] - [a_2 + (1000 - 1)d]$

$= a_1 - a_2 = 100$

Therefore, difference between their 1000^{th} terms would be equal to 100.

3. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

Ans.

Let a be first term and d be the common difference of the A.P. Then

$a_n = a + (n - 1)d$

$a_4 = a + (4 - 1)d = a + 3d$

$\Rightarrow$ $a = -3d$ [Given $a_4 = 0$]

$a_{25} = a + (25 - 1)d$

$= a + 24d$

$= -3d + 24d$ [Putting $a = -3d$]

$= 21d$ ...(i)

And $a_{11} = a + (11 - 1)d = -3d + 10d = 7d$ ...(ii)

Hence, $a_{25} = 3 \times a_{11}$. [From (i) and (ii)]

4. Divide 56 in four parts in A.P such that the ratio of the product of their extremes (1st and 4th) to the product of means (2nd and 3rd) is 5 : 6.

Ans.

Let the four parts of the A.P. are $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$

Now, $a - 3d + a - d + a + d + a + 3d = 56$

$4a = 56$

$a = 14$

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$ [Putting $a = 14$]

$$\frac{196-9d^2}{196-d^2} = \frac{5}{6}$$

$1176 - 54d^2 = 980 - 5d^2$

$\Rightarrow$ $49d^2 = 196$

$\Rightarrow$ $d^2 = 4 \Rightarrow d = \pm 2$

when,

$a = 14$ and $d = 2$

The 4 parts are $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$, *i.e.*, 8, 12, 16, 20.

when, $a = 14$ and d = -2

The 4 parts are 20, 16, 12 and 8.

5. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans.

There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I

= number of sections × 1

= 3 × 1 = 3

The number of trees planted by class II

= number of sections × 2

= 3 × 2 = 6

The number of trees planted by class III

= number of sections × 3

= 3 × 3 = 9

The number of trees planted by class XII

= number of sections × 12

= 3 × 12 = 36

Total number of trees planted = 3 + 6 + 9 +....+ 36

There are 12 terms of an AP.

Where first term, $a = 3$, last term, $l = 36$

So, sum = $\frac{n}{2}(a + l) = \frac{12}{2}(3 + 36) = 234$

6. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

Ans.

Let the required numbers in A.P. are $(a - d)$, a, $(a + d)$ respectively.

Now, $a - d + a + a + d = 15$

$3a = 15 \Rightarrow a = 5$

According to question, number is

$100(a - d) + 10a + a + d = 111a - 99d$

Number on reversing the digits is

$100(a + d) + 10a + a - d = 111a + 99d$

Now, as per given condition in question,

$(111a - 99d) - (111a + 99d) = 594$

$-198d = 594 \Rightarrow d = -3$

So, Digits of number are $[5 - (-3), 5, (5 + (-3)]$

$= 8, 5, 2$

Required number is $111 \times (5) - 99 - 3$

$= 555 + 297 = 852$

The number is 852.

7. Among the natural numbers 1 to 49, find a number *x*, such that the sum of numbers preceding it is equal to sum of numbers preceding it.

Ans.

1, 2, 3, 4, ..., $x - 1$, x, $x + 1$, ... 49

$1 + 2 + 3 + 4 + ... + x - 1 = x + 1 + ... + 49$

$$S_{x-1} = S_{50} - S_x$$

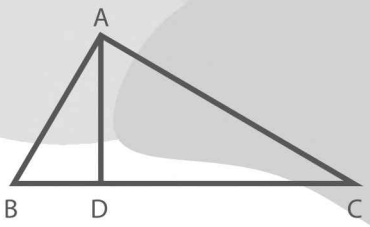
$$\frac{(x-1)x}{2} = \frac{49 \times 50}{2} - \frac{(x+1)x}{2}$$

$$x^2 - x = 2450 - x^2 - x$$

$$x^2 = 1225$$

$x = \pm 35$ (–35 is not between 1 to 49 therefore rejected)

$x = 35$.



6 Triangles

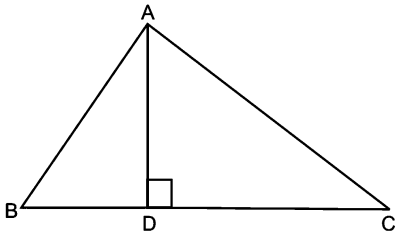
EXERCISE 6.1

Choose the correct option from the given four options in the following questions:

1. In the figure, if ∠BAC = 90° and AD⊥BC, then:

(A) BD.CD = BC² **(B) AB.AC = BC²**

(C) BD.CD = AD² **(D) AB.AC = AD²**



Ans. (C)

Explanation:

In ΔABC

⇒ ∠BAC + ∠B + ∠C = 180°

[Angle sum property]

It is given that ∠BAC = 90°

⇒ 90° + ∠B + ∠C = 180°

⇒ ∠B = 180° − (90° + ∠C)

⇒ ∠B = 90° − ∠C

Similarly, in ΔADC

⇒ ∠DAC = 90° − ∠C

In ΔADB and ΔADC,

∠BDA = ∠CDA = 90° [∵ AD ⊥ BC]

∠DBA = ∠DAC [Each angle = 90° − ∠C]

Using AA similarity criterion,

ΔADB ~ ΔCDA

$\Rightarrow \frac{BD}{AD} = \frac{AD}{CD}$

⇒ $BD.CD = AD^2$

2. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. then, the length of the side of the rhombus is:

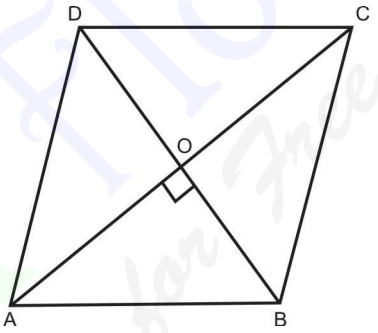
(A) 9 cm **(B) 10 cm**

(C) 8 cm **(D) 20 cm**

Ans. (B)

Explanation: 10 cm

A rhombus is a simple quadrilateral whose four sides are of the same length and diagonals are perpendicular bisectors of each other.



It is given that

AC = 16 cm and BD = 12 cm

∠AOB = 90°

Since AC and BD bisect each other

$\Rightarrow AO = \frac{1}{2}AC$ and $BO = \frac{1}{2}BD$

⇒ AO = 8 cm and BO = 6 cm

In right angled ΔAOB,

$AB^2 = AO^2 + OB^2$

[Using the Pythagoras theorem]

$AB^2 = 8^2 + 6^2 = 64 + 36 = 100$

⇒ $AB = \sqrt{100} = 10$ cm

3. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?

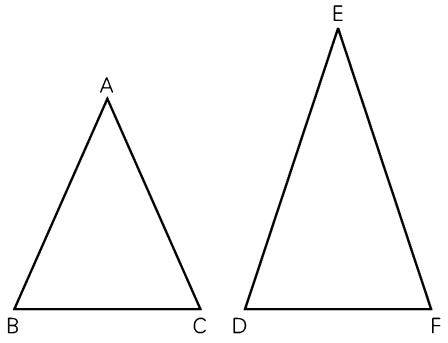
(A) BC.EF = AC.FD **(B) AB.EF = AC.DE**

(C) BC.DE = AB.EF **(D) BC.DE = AB.FD**

Ans. (C)

Explanation: We know that

If sides of one triangle are proportional to the side of the other triangle, and the corresponding angles are also equal, then the triangles are similar by SSS similarity.



It is given that

$$\Delta ABC \sim \Delta EDF$$

$$\Rightarrow \quad \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

Also $\Delta ABC \times \Delta DEF$

$$\frac{AB}{DE} \neq \frac{BC}{EF}$$

i.e., $AB.EF = BC.DE$

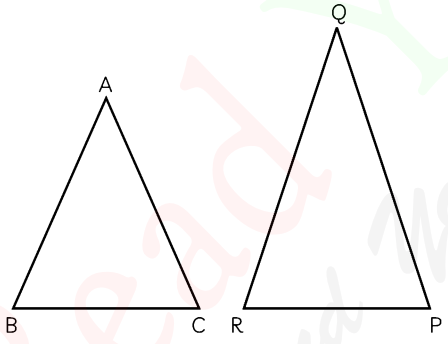
Hence (C) is not true.

4. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then:

(A) $\Delta PQR \sim \Delta CAB$ **(B) $\Delta PQR \sim \Delta ABC$**
(C) $\Delta CBA \sim \Delta PQR$ **(D) $\Delta BCA \sim \Delta PQR$**

Ans. (A)

Explanation:



It is given that in ΔABC and ΔPQR,

$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$

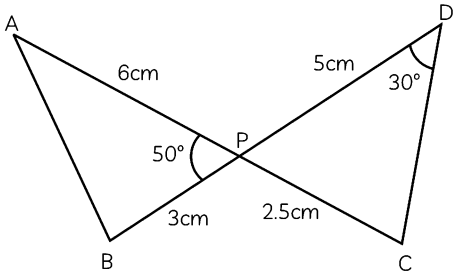
This shows that the sides of one triangle are proportional to the side of the other triangle, thus their corresponding angles are also equal.

i.e., $\angle A = \angle Q$, $\angle B = \angle R$ and $\angle C = \angle P$

Thus, $\Delta PQR \sim \Delta CAB$

5. In the given figure, two line segments AC and BD intersect each other at point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to:

(A) 50° **(B) 30°**
(C) 60° **(D) 100°**



Ans. (D)

Explanation:

Considering ΔAPB and ΔCPD,

$\angle APB = \angle CPD = 50°$

[Vertically opposite angles]

$$\frac{AP}{PD} = \frac{6}{5} \quad \text{...(i)}$$

Also, $\frac{BP}{CP} = \frac{3}{2.5}$

Or $\frac{BP}{CP} = \frac{6}{5}$ ...(ii)

From equations (i) and (ii), we get

$AP/PD = BP/CP$

$\Rightarrow \quad \Delta APB \sim \Delta DPC$

[using SAS similarity criterion]

$\Rightarrow \quad \angle A = \angle D = 30°$

[Corresponding angles of similar triangles]

Since, the sum of angles of a triangle = 180°,

in ΔAPB,

$\angle A + \angle B + \angle APB = 180°$ [Angle sum property]

$\Rightarrow \quad 30° + \angle B + 50° = 180°$

$\Rightarrow \quad \angle B = 180° - (50° + 30°)$

$\angle B = 180 - 80° = 100°$

Therefore, $\angle PBA = 100°$

6. If in two triangles ΔDEF and ΔPQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ **(B) $\frac{DE}{PQ} = \frac{EF}{RP}$**
(C) $\frac{DE}{QR} = \frac{DF}{PQ}$ **(D) $\frac{EF}{RP} = \frac{DE}{QR}$**

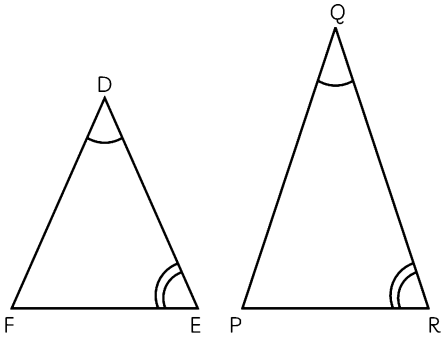
Ans. (B)

Explanation:

It is given that in ΔDEF and ΔPQR,

$\angle D = \angle Q$ and $\angle R = \angle E$

We know that if two corresponding angles of two triangles are congruent, then both the triangles are similar because if two angle pairs are equal, then the third angle must also be equal.



$\Rightarrow$ $\Delta DEF \sim \Delta QRP$ [By AA similarity criterion]

$\Rightarrow$ $\angle F = \angle P$ [Corresponding angles of similar triangles]

$$\frac{DF}{PQ} = \frac{ED}{RQ} = \frac{FE}{PR}$$

Hence, except option (B), all are true.

7. In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are:

(A) congruent but not similar

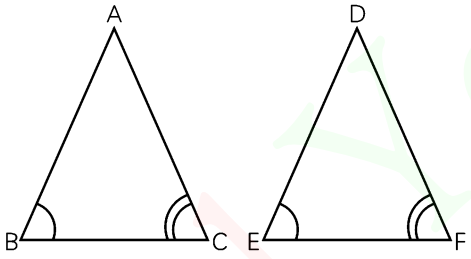
(B) similar but not congruent

(C) neither congruent nor similar

(D) congruent as well as similar

Ans. (B)

Explanation:



In ΔABC and ΔDEF

$\angle B = \angle E$ [Given]

$\angle F = \angle C$ [Given]

$\Rightarrow$ $\Delta ABC \sim \Delta DEF$ [By AA similarity criterion]

AB and DE sides are corresponding sides.

But $AB = 3DE$ [Given]

We know that two triangles are congruent if they have the same shape and size and satisfy the rule of congruency. But ΔABC and ΔDEF do not satisfy any rule of congruency, which are SAS, ASA, AAA and SSS, so both are not congruent.

$\Rightarrow$ ΔABC cannot be congruent to ΔDEF.

Hence, Δ's are similar but not congruent.

8. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{3}$, then $\left\{\frac{ar(PRQ)}{ar(BCA)}\right\}$ is equal to:

(A) 9 **(B) 3**

(C) $\frac{1}{3}$ **(D) $\frac{1}{9}$** **[CBSE 2018]**

Ans. (A)

Explanation:

It is given that

$\Delta ABC \sim \Delta PQR$ ...(i)

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\left\{\frac{ar(PRQ)}{ar(BCA)}\right\} = \frac{QR^2}{BC^2} = \left(\frac{QR}{BC}\right)^2$$

$$\left\{\frac{ar(PRQ)}{ar(BCA)}\right\} = \frac{3^2}{1^2} = \frac{9}{1}$$ [Using eq. (i)]

Thus, the area of ΔPRQ = 9 times the area of ΔBCA.

Trick Applied

Ratio of the area of two similar triangles is equal to the ratio of the squares of their corresponding sides.

9. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Which of the following is true:

(A) $DE = 12$ cm, $\angle F = 50°$

(B) $DE = 12$ cm, $\angle F = 100°$

(C) $EF = 12$ cm, $\angle D = 100°$

(D) $EF = 12$ cm, $\angle D = 30°$ **[CBSE 2016, 10]**

Ans. (B)

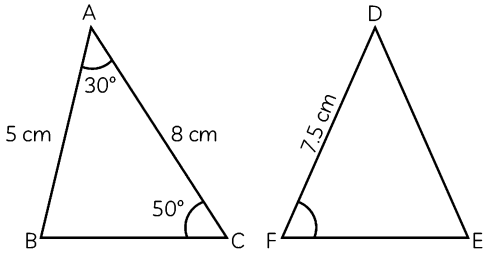
Explanation:

It is given that

$\Delta ABC \sim \Delta DFE$

$\angle A = 30°$

$\angle C = 50°$



$\Rightarrow$ We have,

$\angle A = \angle D = 30°$ [Corresponding angles of similar triangles are equal]

$\angle C = \angle E = 50°$ [Corresponding angles of similar triangles are equal]

We know that sum of the angles of a triangle = 180°

$\Rightarrow$ $\angle B = \angle F = 180° - (30° + 50°) = 100°$

Also, since $\Delta ABC \sim \Delta DFE$

and $AB = 5$ cm $AC = 8$ cm $DF = 7.5$ cm

$\Rightarrow \frac{AB}{DF} = \frac{AC}{DE}$

$\Rightarrow \frac{5}{7.5} = \frac{8}{DE}$

$DE = \frac{8 \times 7.5}{5} = 8 \times 1.5 = 12$ cm

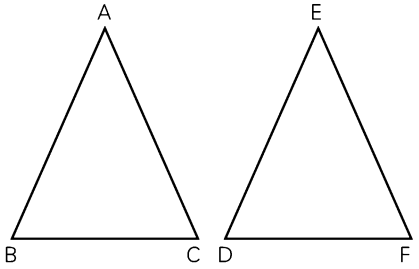
10. If in triangles ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar when:

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$

(C) $\angle B = \angle D$ (D) $\angle A = \angle F$

Ans. (C)

Explanation:



In ΔABC and ΔDEF,

$\frac{AB}{DE} = \frac{BC}{FD}$

Angle formed by AB and BC is $\angle B$.

Angle formed by DE and FD is $\angle D$.

$\Rightarrow \quad \angle B = \angle D$

$\therefore \quad \Delta ABC \sim \Delta EDF$ [By SAS similarity criterion]

Hence, (C) is the correct answer.

11. If ΔABC ~ ΔQRP, $\left\{\frac{ar(ABC)}{ar(PQR)}\right\} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then PR is equal to:

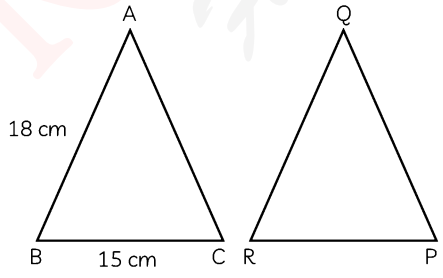
(A) 10 cm (B) 12 cm

(C) $\frac{20}{3}$ cm (D) 8 cm

Ans. (A)

Explanation:

It is given that ΔABC ~ ΔQRP



By similar triangles area property, the ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

$\Rightarrow \quad \left\{\frac{ar(ABC)}{ar(PQR)}\right\} = \frac{BC^2}{PR^2}$,

$\Rightarrow \quad \left\{\frac{ar(ABC)}{ar(PQR)}\right\} = \frac{9}{4}$ [Given]

$\Rightarrow \quad \frac{BC^2}{PR^2} = \frac{9}{4}$

It is given that AB = 18 cm and BC = 15 cm

$\Rightarrow \quad \frac{15^2}{PR^2} = \frac{9}{4}$

$\Rightarrow \quad PR^2 = \frac{225 \times 4}{9} = 100$

$\Rightarrow \quad PR = 10$ cm

12. If S is a point on side PQ of a ΔPQR such that PS = QS = RS, then:

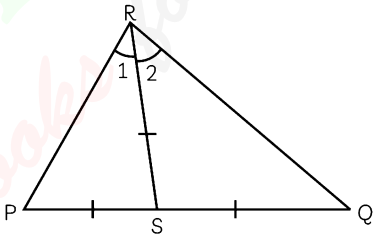
(A) $PR.QR = RS^2$ (B) $QS^2 + RS^2 = QR^2$

(C) $PR^2 + QR^2 = PQ^2$ (D) $PS^2 + RS^2 = PR^2$

Ans. (C)

Explanation:

In ΔPQR, PS = QS = RS [Given]



Let $\angle PRS = \angle 1$ and $\angle SRQ = \angle 2$

In ΔPSR, PS = RS [Given]

$\Rightarrow \quad \angle 1 = \angle P$ [Angles opposite to equal sides in a triangle are equal]

Similarly, in ΔSRQ

RS = SQ [Given]

$\Rightarrow \quad \angle Q = \angle 2$

Now, in ΔPQR,

$\angle P + \angle Q + \angle PRQ = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle 1 + \angle 2 + (\angle 1 + \angle 2) = 180°$

[As $\angle 1 = \angle P$ and $\angle Q = \angle 2$]

$\Rightarrow \quad 2(\angle 1 + \angle 2) = 180°$

$\Rightarrow \quad \angle 1 + \angle 2 = 90°$

$\Rightarrow \quad \angle PRQ = 90°$

By Pythagoras theorem, we have

$PQ^2 = PR^2 + RQ^2$

EXERCISE 6.2

1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reasons for your answer. [CBSE 2014]

Ans. No.

According to the question,

$a = 25$ cm $\quad b = 5$ cm $\quad c = 24$ cm

By Pythagoras theorem, we have,

$$a^2 = b^2 + c^2$$
$$= 5^2 + 24^2$$
$$= 25 + 576 = 601$$

But $a^2 = 625$

$625 \neq 601$

Since the sides do not satisfy the property of Pythagoras theorem, we can say that a triangle with sides 25cm, 5cm and 24cm is not a right triangle.

2. It is given that ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠F = ∠P ? Why?

Ans. No.

Explanation:

We know that when ΔDEF ~ ΔRPQ, each angle of triangle ΔDEF is equal to the corresponding angle of ΔRPQ. So,

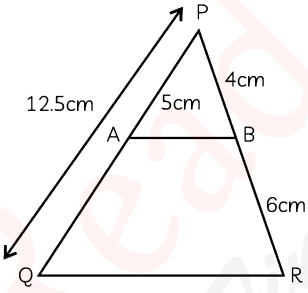
$\angle D = \angle R; \angle E = \angle P; \angle F = \angle Q$

$\Rightarrow \angle D = \angle R$ is true but $\angle F \neq \angle P$.

Hence, it is not true that $\angle D = \angle R$ and $\angle F = \angle P$.

3. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reasons for your answer.

Ans. Yes.



According to the question,

PQ = 12.5 cm $\quad$ PA = 5 cm

BR = 6 cm $\quad$ PB = 4 cm

$\Rightarrow$ QA = QP − PA = 12.5 − 5 = 7.5 cm

$$\Rightarrow \frac{PA}{AQ} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3} \quad \text{...(i)}$$

$$\Rightarrow \frac{PB}{BR} = \frac{4}{6} = \frac{2}{3} \quad \text{...(ii)}$$

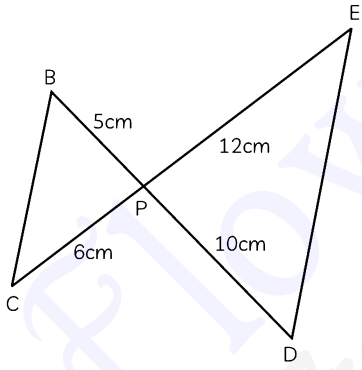
From eqn. (i) and (ii), we get

$$\frac{PA}{AQ} = \frac{PB}{BR}$$

According to the converse of basic proportionality theorem, if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Hence, AB || QR.

4. In figure, BD and CE intersect each other at point P. Is ΔPBC ~ ΔPDE? Why?



[CBSE 2012]

Ans. Yes.

In ΔPBC and ΔPDE,

$\angle BPC = \angle EPD$ [Vertically opposite angles]

$$\frac{PB}{PD} = \frac{5}{10} = \frac{1}{2} \quad \text{...(i)}$$

$$\frac{PC}{PE} = \frac{6}{12} = \frac{1}{2} \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\frac{PB}{PD} = \frac{PC}{PE}$$

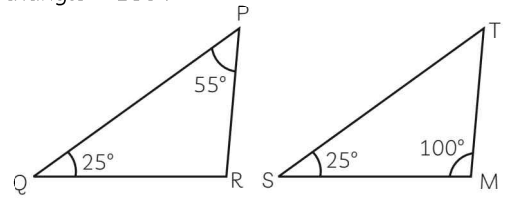
Since, ∠BPC of ΔPBC = ∠EPD of ΔPDE and the sides including these angles are proportional, both the triangles are similar.

Hence, ΔPBC ~ ΔPDE, by SAS similarity criterion.

5. In triangles ΔPQR and ΔMST, ∠P = 55°, ∠Q = 25°, ∠M = 100° and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

Ans. No.

We know that the sum of the three angles of a triangle = 180°.



In ΔPQR

$\angle P + \angle Q + \angle R = 180°$

$\Rightarrow 55° + 25° + \angle R = 180°$

$\Rightarrow \angle R = 180° - (55° + 25°) = 180° - 80° = 100°$

Similarly, in ΔTSM

$\angle T + \angle S + \angle M = 180°$

$\Rightarrow \angle T + 25° + 100° = 180°$

$\Rightarrow \angle T = 180° - (25° + 100°)$

$\Rightarrow \angle T = 180° - 125° = 55°$

Since, $\angle Q \neq \angle T, \angle P \neq \angle S$,

$\therefore \Delta QPR \sim \Delta TSM$

6. Is the following statement true? Why?
"Two quadrilaterals are similar, if their corresponding angles are equal".

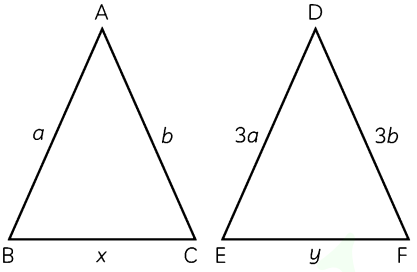
Ans. No.

Two quadrilaterals will be similar, if their corresponding angles are equal and ratio of corresponding sides must also be proportional.

eq. : rectangle, square

7. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Ans. Yes.



Let the sides of the first triangle be a, b and x.

Then, the corresponding two sides of the other triangle will be 3a, 3b and let the third side be y respectively.

Perimeter of the first triangle, $P_1 = a + b + x$

Perimeter of the second triangle, $P_2 = 3a + 3b + y$

It is given that $P_2 = 3P_1$

$\Rightarrow 3a + 3b + y = 3(a + b + x)$

$\Rightarrow y = 3x$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{3}$

Hence, $\Delta ABC \sim \Delta DEF$, by SSS similarity criterion.

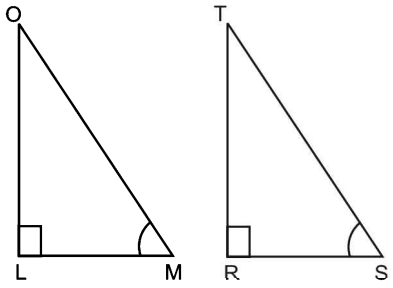
8. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that two triangles will be similar? Why?

Ans. Yes.

As the acute angles of both triangles are equal and one of the angles of both the triangles is right angle i.e., of 90°.

Therefore, by angle sum property, the third angle will also be equal. Hence, by AAA similarity criterion, both the triangles will be similar.

Alternate solution:



Let two right angled triangles be ΔLMO and ΔRST in which

$\angle L = \angle R = 90°$ and $\angle M = \angle S$

[Equal acute angles]

By angle sum property of triangle, the sum of interior angles of a triangle is 180°.

Therefore, $\angle O$ of the first triangle must be equal to $\angle T$ of the second triangle.

$\Rightarrow \angle O = \angle T$

Hence, by AAA similarity criterion,

$\Delta LMO \sim \Delta RST$

9. The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that the ratio of their areas is $\frac{6}{5}$? Why?

[CBSE 2011]

Ans. No.

It is given that the ratio of altitudes of similar triangles is $\frac{3}{5}$.

Using the property of area of two similar triangles, we have

$$\frac{\text{Area 1}}{\text{Area 2}} = \frac{\text{Altitude 1}^2}{\text{Altitude 2}^2} = \left(\frac{\text{Altitude1}}{\text{Altitude2}}\right)^2$$

$$\frac{\text{Area 1}}{\text{Area 2}} = \left(\frac{3}{5}\right)^2$$

$$\frac{\text{Area 1}}{\text{Area 2}} = \frac{9}{25}$$

$$\frac{9}{25} \neq \frac{6}{5}$$

Hence, the given statement is not correct because it does not satisfy the criteria.

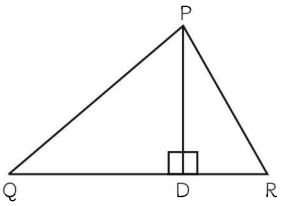
10. D is a point on side QR of ΔPQR such that $PD \perp QR$. Will it be correct to say that $\Delta PQD \sim \Delta RPD$? Why?

Ans. No.

We know that two triangles are similar to each other if any of the following criteria holds true:

1. **AA similarity:** Two angles of one triangle are congruent (equal) to two angles of the other triangle.

2. **SSS similarity:** Three sets of corresponding sides of two triangles are in proportion.
3. **SAS similarity:** Two sets of corresponding sides are in proportion and the angles they include are congruent.



According to the question :

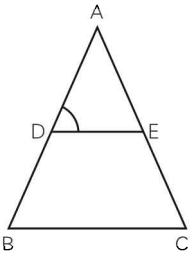
$\angle QDP = \angle RDP = 90°$

[As given PD perpendicular to QR]

And apart from that, we can't find any more relations in triangles PQD and RPD,

Hence, we can't say that $\Delta PQD \sim \Delta RPD$.

11. In the given figure, if $\angle D = \angle C$, then it is true that $\Delta ADE \sim \Delta ACB$? Why?



Ans. Yes.

In ΔADE and ΔACB,

$\angle D = \angle C$ [Given]

$\angle A = \angle A$ [Common angle]

$\Rightarrow \Delta ADE \sim \Delta ACB$ [By AA similarity criterion]

12. Is it true to say that if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, then the triangles are similar? Give reason for your answer.

Ans. No. The given statement is not correct.

We know that by SAS similarity criteria, if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Here, an angle of one triangle is equal to an angle of another triangle and two sides are proportional, even then the triangles are not similar because these sides do not include an angle.

EXERCISE 6.3

1. In a ΔPQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that $QM \perp PR$.

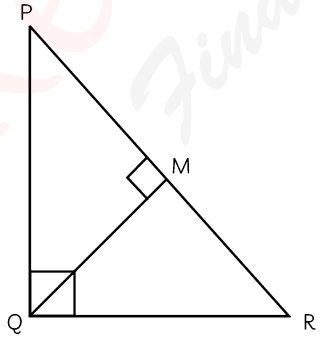
Prove that $QM^2 = PM \times MR$. [CBSE 2019, 15]

Ans.

Given: In a ΔPQR, $PR^2 - PQ^2 = QR^2$ and $QM \perp PR$

To prove: $QM^2 = PM \times MR$

Proof:



It is given that

$PR^2 - PQ^2 = QR^2$

$PR^2 = PQ^2 + QR^2$

$\Rightarrow$ ΔPQR is right angle triangle, right angled at Q. [By Converse of Pythagoras Theorem]

In ΔPQR

$\angle P + \angle Q + \angle R = 180°$

$\angle P + 90° + \angle R = 180°$

$\Rightarrow \angle P = 180° - (90° + R)$

$\Rightarrow \angle P = 90° - \angle R$ ...(i)

Also, it is given that QM is perpendicular to PR

In ΔQMR

$\angle RQM + \angle QMR + \angle R = 180°$

$\Rightarrow \angle RQM + 90° + \angle R = 180°$

$\Rightarrow \angle RQM = 180° - (90° + R)$

$\Rightarrow \angle RQM = 90° - \angle R$

$\Rightarrow \angle RQM = \angle P$ [Using eq. (i)]

Now, in ΔQMP and ΔQMR,

$\angle QMP = \angle QMR = 90°$ [As $QM \perp PR$]

$\Rightarrow \angle RQM = \angle P$

$\Rightarrow \Delta QMP \sim \Delta RMQ$

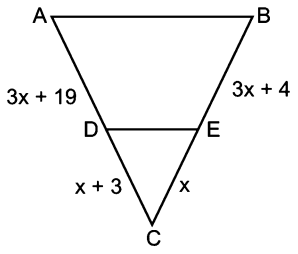
[By AA similarity criterion]

$\Rightarrow \frac{QM}{RM} = \frac{PM}{QM}$ [Corresponding sides are proportional of similar triangles]

$\Rightarrow$ $QM^2 = PM \times RM$

Hence, proved.

2. Find the value of x for which DE || AB in the given figure.



[CBSE 2017, 15]

Ans. In ΔABC,

DE || AB

Using basic proportionality theorem,

$\Rightarrow \quad \frac{CD}{AD} = \frac{CE}{BE}$

$\Rightarrow \quad \frac{x+3}{3x+19} = \frac{x}{3x+4}$

$\Rightarrow \quad (x + 3)(3x + 4) = x(3x + 19)$

$\Rightarrow 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow \quad 19x - 13x = 12$

$\Rightarrow \quad 6x = 12$

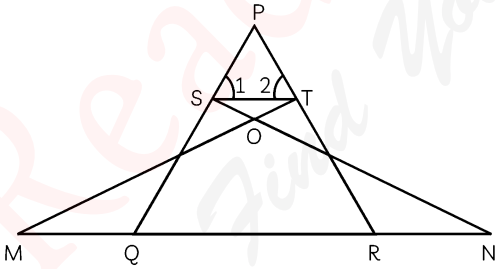
$\Rightarrow \quad x = \frac{12}{6} = 2$

Hence, the required value of x is 12.

Trick Applied

- *If a Line is drawn parallel to one side of a triangle to intersect the other two sides, then these two sides are divided in the same ratio (Basic Proportionality Theorem)*

3. In the given figure, if ∠1 = ∠2 and ΔNSQ ≅ ΔMTR, then prove that ΔPTS ~ ΔPRQ.



[CBSE 2013]

Ans.

Given: ΔNSQ ≅ ΔMTR

To prove: ΔPTS ~ ΔPRQ

Proof: According to the given condition,

ΔNSQ ≅ ΔMTR

$\angle 1 = \angle 2$

Since ΔNSQ ≅ ΔMTR,

SQ = TR ....(i)

Also, $\angle 1 = \angle 2$

$\Rightarrow$ PS = PT ....(ii)

[Sides opposite to equal angles are also equal]

From eq. (i) and (ii)

$\frac{PS}{SQ} = \frac{PT}{TR}$

$\Rightarrow$ ST || QR

[By converse of basic proportionality theorem]

$\Rightarrow \quad \angle 1 = \angle PQR$ [Corresponding angles]

And

$\Rightarrow \quad \angle 2 = \angle PRQ$ [Corresponding angles]

In ΔPTS and ΔPRQ

$\angle P = \angle P$ [Common angles]

$\angle 1 = \angle PQR$ [proved above]

$\angle 2 = \angle PRQ$ [proved above]

$\Rightarrow$ ΔPTS ~ ΔPRQ [By AAA similarity criterion]

Hence, Proved.

Trick Applied

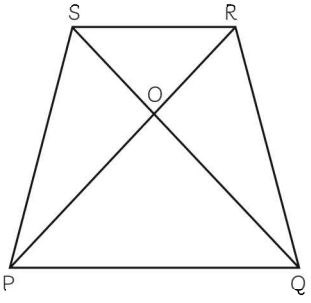
- *Step 1. Prove that ST || QR*
- *Step 2. Use AA similarity criterion to show that ΔPTS ~ ΔPRQ.*

4. Diagonals of a trapezium PQRS intersect each other at the point O, PQ || RS and PQ = 3 RS. Find the ratio of the areas of ΔPOQ and ΔROS.

Ans.

Given: PQRS is a trapezium in which PQ || RS and PQ = 3RS

$\Rightarrow \quad \frac{PQ}{RS} = \frac{3}{1} = 3$ ...(i)



In ΔPOQ and ΔROS

$\angle QOP = \angle SOR$ [vertically opposite angles]

$\angle RPQ = \angle SRP$ [alternate angles]

$\Rightarrow$ ΔPOQ ~ ΔROS [by AA similarity criterion]

By property of area of similar triangle,

$\frac{\text{Area 1}}{\text{Area 2}} = \frac{PQ^2}{RS^2} = \frac{3^2}{1^2} = \frac{9}{1}$

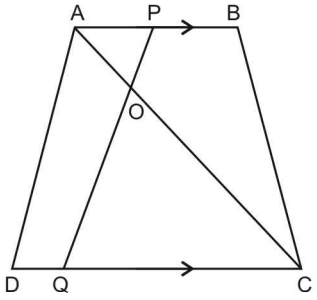
$\frac{\text{Area 1}}{\text{Area 2}} = 9$

Hence, the required ratio = 9:1.

Trick Applied

Use the property of area of similar triangle to get required ratio.

5. In the figure, if AB || DC and AC, PQ intersect each other at the point O, prove that OA.CQ = OC.AP



Ans.

Given: ABCD is a quadrilateral in which AC and PQ intersect each other at the point O and AB || DC

To Prove: OA.CQ = OC.AP

Proof: In ΔAOP and ΔCOQ

$\angle AOP = \angle COQ$ [Vertically opposite angles]

$\angle APO = \angle CQO$ [AB || DC and PQ is transversal, equal alternate angle]

$\Rightarrow \quad \Delta AOP \sim \Delta COQ$ [Using AAA similarity criterion]

$\Rightarrow \quad \frac{OA}{OC} = \frac{AP}{CQ}$ [Corresponding sides of two similar triangles are proportional]

$OA \times CQ = OC \times AP$

Hence, Proved

6. Find the altitude of an equilateral triangle of side 8 cm. [CBSE 2016, 15]

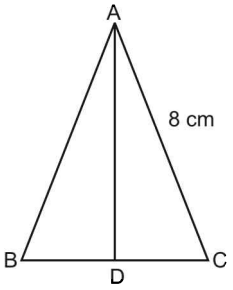
Ans.

Given: ABC is an equilateral triangle of side 8 cm

Construction: Draw altitude AD which is perpendicular to BC.

AB = BC = CA = 8 cm

[all sides of an equilateral triangle are equal]



Since AD is the altitude of equilateral triangle,

$BD = CD = \frac{1}{2}BC = 4$ cm.

By Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad (8)^2 = AD^2 + (4)^2$

$\Rightarrow \quad 64 = AD^2 + 16$

$\Rightarrow \quad AD = \sqrt{48} = 4\sqrt{3}$ cm.

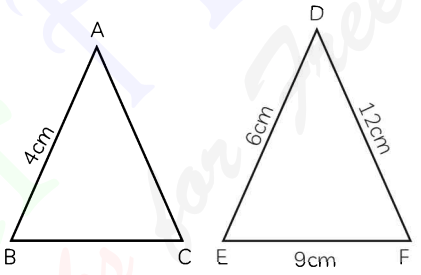
Hence, the altitude of an equilateral triangle is $4\sqrt{3}$ cm.

7. If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm, then find the perimeter of ΔABC

Ans.

Given: AB = 4 cm, DE = 6 cm, EF = 9 cm FD = 12 cm and ΔABC ~ ΔDEF

To Find: Perimeter of ΔABC



Since $\Delta ABC \sim \Delta DEF$, [Given]

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$$

Taking first two terms, we have

$$\frac{4}{6} = \frac{BC}{9}$$

$$\Rightarrow \quad BC = \frac{(4 \times 9)}{6}$$

$\Rightarrow \quad BC = 6$ cm

Taking last two terms, we have

$$\frac{BC}{9} = \frac{AC}{12}$$

$$\frac{6}{9} = \frac{AC}{12}$$

$$\Rightarrow \quad AC = \frac{(6 \times 12)}{9}$$

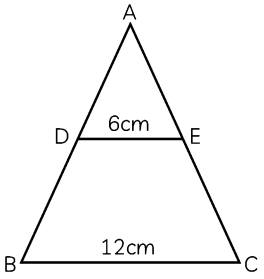
$\Rightarrow \quad AC = 8$ cm

Perimeter of ΔABC = AB + BC + AC

= 4 + 6 + 8 = 18 cm

Thus, the perimeter of the triangle is 18 cm.

8. In the given figure, if DE || BC, then find the ratio of ar (ΔADE) and ar(DECB). [CBSE 2013]



Ans.

Given: ΔABC with DE || BC, DE = 6cm and BC =12cm

In ΔABC and ΔADE

$\angle ABC = \angle ADE$ [Corresponding angles]

$\angle ACB = \angle AED$ [Corresponding angles]

$\Rightarrow$ ΔABC ~ ΔADE [By AA similarity criterion]

$\Rightarrow$ By property of area of similar triangle,

$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta ABC} = \frac{DE^2}{BC^2} = \frac{6^2}{12^2} = \frac{36}{144} = \frac{1}{4}$$

Let Area (ΔADE) = k,

$\Rightarrow$ Area (ΔABC) = $4k$

$\Rightarrow$ Area (DECB) = Area (ABC) – Area (ADE)

$= 4k - k = 3k$

$\Rightarrow$ Required ratio = Area (ADE):Area (DECB)

$= k:3k = 1:3$

Hence, area (ΔADE):area (DECB) = 1:3

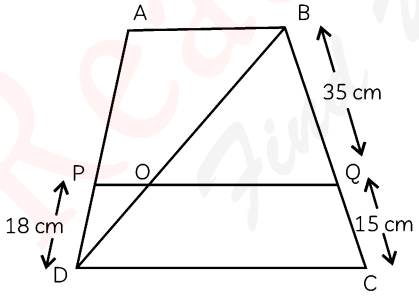
9. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively, such that PQ || DC, if PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.

Ans.

Given: A trapezium, ABCD with AB || DC,

P and Q are points on AD and BC respectively with PQ || DC

PD = 18 cm, BQ = 35 cm and QC = 15 cm



In trapezium ABCD

AB || CD [Given]

PQ || DC [Given]

$\Rightarrow$ AB || CD || PQ ...(i)

In ΔBCD,

OQ || CD [From eq. (i)]

$\Rightarrow$ By basic proportionality theorem,

$$\frac{BQ}{QC} = \frac{OB}{OD}$$

$$\frac{QC}{BQ} = \frac{OD}{OB} \quad \text{...(ii)}$$

In ΔABD, OP || AB [From eq. (i)]

By basic proportionality theorem,

$$\frac{DP}{AP} = \frac{DO}{OB} \quad \text{...(iii)}$$

$\Rightarrow$

From equation (ii) and (iii)

$$\frac{DP}{AP} = \frac{QC}{BQ}$$

$$\frac{18}{AP} = \frac{15}{35}$$

$\Rightarrow$ $AP = \frac{(18 \times 35)}{15}$

$\Rightarrow$ $AP = 42$

Now, $AD = AP + DP$

$\Rightarrow$ $AD = 42 + 18 = 60$

Hence, AD = 60 cm.

10. Corresponding sides of two similar triangles are in the ratio of 2:3. If the area of the smaller triangle is 48 cm^2, then find the area of the larger triangle. [CBSE 2016, 10]

Ans.

Let the two triangles be ABC and PQR, where ΔABC is smaller of the two.

Given: ΔABC ~ ΔPQR with AB:PQ = 2:3

Area of smaller ΔABC = 48cm^2

According to the question,

ΔABC ~ ΔDEF

$\Rightarrow$ By property of area of similar triangle,

Ratio of area of both triangles = (Ratio of their corresponding sides)^2

$$\Rightarrow \frac{\text{Area } \Delta ABC}{\text{Area } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{2^2}{3^2} = \frac{4}{9}$$

$$\Rightarrow \frac{48}{\text{Area } \Delta PQR} = \frac{4}{9}$$

$$\Rightarrow \text{Area } \Delta PQR = \frac{48 \times 9}{4}$$

$\Rightarrow$ Area ΔPQR = 108 cm^2

Trick Applied

Ratio of the area of two similar triangles is equal to the ratio of the squares of their corresponding sides.

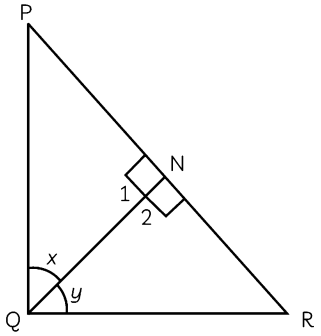
11. In a ΔPQR, N is a point on PR, such that QN ⊥ PR. If PN.NR = QN^2, then prove that ∠PQR = 90°. [CBSE 2019, 15]

Ans.

Given: ΔPQR in which QN ⊥ PR and PN × NR = QN^2

To prove: $\angle PQR = 90°$

Proof:



In ΔQNP and ΔQNR

$QN \perp PR$ [Given]

$\angle 1 = \angle 2 = 90°$

$QN^2 = NR \times NP$ [Given]

$QN \times QN = NR \times NP$

$\Rightarrow \quad \frac{QN}{NP} = \frac{NR}{QN}$

$\Rightarrow \quad \Delta PNQ \sim \Delta QNR$

[By SAS similarity criterion]

Let $\angle RQN = y$ ...(i)

And $\angle PQN = x$ ...(ii)

In ΔPQR, we have

$\angle P + \angle PQR + \angle R = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad x + x + y + y = 180°$ [Using eq. (i) and (ii)]

$\Rightarrow \quad 2x + 2y = 180°$

$\Rightarrow \quad x + y = 90°$

$\Rightarrow \quad \angle PQR = 90°$ Hence, proved.

Trick Applied

- *Step 1. Prove $\Delta QNP \sim \Delta RNQ$*
- *Step 2. Angle sum property.*

12. Areas of two similar triangles are 36 cm^2 and 100 cm^2. If the length of a side of the larger triangle is 20cm. Find the length of the corresponding side of the smaller triangle.

[CBSE 2014]

Ans.

Let the two triangles be ΔABC and ΔDEF, where ΔABC is the smaller of the two and

area of $\Delta ABC = 36cm^2$ and area $\Delta DEF = 100\ cm^2$.

Also, $DE = 20cm$

Let length of the corresponding side of ΔABC, $AB = x$ cm

As $\Delta ABC \sim \Delta DEF$, by property of area of similar triangle :

Ratio of area of both triangles = (Ratio of their corresponding sides)^2

$\Rightarrow \quad \frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \frac{AB^2}{DE^2}$

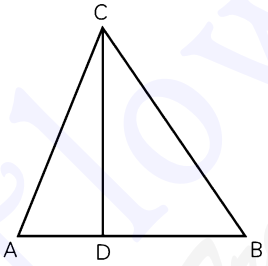
$\Rightarrow \quad \frac{36}{100} = \frac{x^2}{400}$

$\Rightarrow \quad x^2 = \frac{36 \times 400}{100} = 144$

$\Rightarrow \quad x = \sqrt{144} = 12$

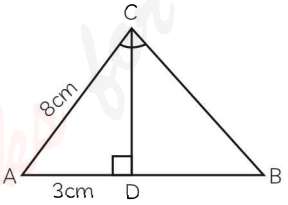
Hence, the length of corresponding side of the smaller triangle is 12 cm.

13. In the given figure, if $\angle ACB = \angle CDA$, AC = 8cm and AD = 3 cm, then find BD.



Ans.

Given: AC = 8 cm, AD = 3 cm and $\angle ACB = \angle CDA$



In ΔACD and ΔACB, we have

$\angle CDA = \angle ACB$ [Given]

$\angle A = \angle A$ [Common]

$\Rightarrow \quad \Delta ACD \sim \Delta ABC$ [By AA similarity criterion]

$\Rightarrow \quad \frac{AC}{AB} = \frac{DC}{BC} = \frac{AD}{AC}$

[Corresponding sides of two similar triangles are proportional]

$\Rightarrow \quad \frac{8}{AB} = \frac{DC}{BC} = \frac{3}{8}$

Using first and last terms

$\Rightarrow \quad \frac{8}{AB} = \frac{3}{8}$

$\Rightarrow \quad AB = \frac{(8 \times 8)}{3} = \frac{64}{3}$

$\Rightarrow \quad BD = AB - AD$

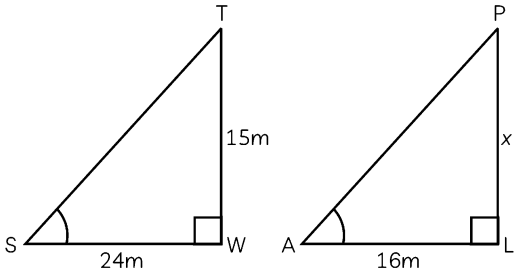
$= \frac{64}{3} - 3 = \frac{64 - 9}{3} = \frac{55}{3}$ cm.

14. A 15 metres high tower casts a shadow 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.

Ans.

Let TW = 15 m be the height of the tower and SW = 24 m be its shadow's length.

Also, let PL = x metres be the height of the telephone pole and AL = 16 m be its shadow's length.



In ΔTWS and ΔPLA

$\angle W = \angle L = 90°$

$\angle S = \angle A$

[Since both the figures occur at the same time, hence will have equal angle of elevation]

$\Rightarrow$ ΔTWS ~ ΔPLA [By AA similarity criterion]

$\Rightarrow \quad \frac{TW}{PL} = \frac{WS}{LA} = \frac{TS}{PA}$

[Corresponding sides of two similar triangles are proportional]

$\Rightarrow \quad \frac{15}{x} = \frac{24}{16}$

$\Rightarrow \quad x = (15 \times 16)/24$

$\Rightarrow \quad x = 10$ m

Hence, the height of the telephone pole is 10 m.

Trick Applied

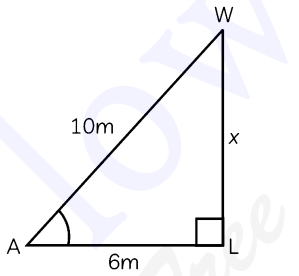
Prove both the triangles to be similar. Then, use ratio of sides to calculate required length.

15. Foot of a 10m long ladder leaning against a vertical wall is 6m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

[CBSE 2015]

Ans.

Let WL be a vertical wall of height x metres and AW be a ladder of length 10 m kept against the wall at a distance of 6 m from the base.



By Pythagoras theorem, in right angled triangle WLA

$WL^2 + AL^2 = WA^2$

$x^2 + 6^2 = 10^2$

$x^2 = 10^2 - 6^2 = 100 - 36$

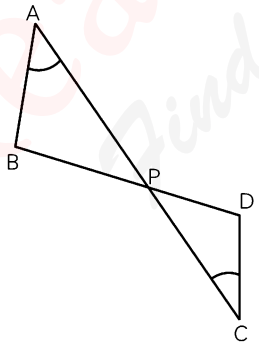
$x^2 = 64$

$\Rightarrow \quad x = 8$

Hence, the height of the point on the wall where the top of the ladder reaches is 8 m.

EXERCISE 6.4

1. In the given figure, if $\angle A = \angle C$, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.



[CBSE 2012]

Ans.

Given: $\angle A = \angle C$, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm.

To find: lengths of PD and CD

In ΔAPB and ΔCPD,

$\angle A = \angle C$ [Common]

$\angle APB = \angle CPD$ [vertically opposite angles]

$\Rightarrow$ ΔAPB ~ ΔCPD [By AA similarity criterion]

$\Rightarrow \quad \frac{AB}{CD} = \frac{AP}{CP} = \frac{PB}{PD}$ [Corresponding sides of two similar triangles are proportional]

$\Rightarrow \quad \frac{6}{CD} = \frac{12}{4} = \frac{15}{PD}$

Considering $\frac{AP}{CP} = \frac{PB}{PD}$, we get

$\frac{12}{4} = \frac{15}{PD}$

$PD = \frac{15 \times 4}{12} = \frac{60}{12} = 5$ cm

Considering $\frac{AP}{CP} = \frac{AB}{CD}$, we get

$\Rightarrow \quad CD = \frac{(6 \times 4)}{12} = 2$ cm

Hence, length of PD = 5 cm and length of CD = 2 cm.

2. It is given that Δ ABC ~ Δ EDF such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles. [CBSE 2016]

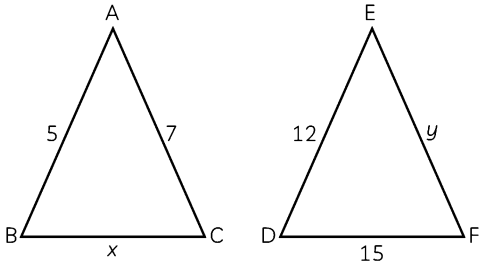
Ans.

Given: AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm

To find: lengths of BC and EF

According to the question

$\Delta ABC \sim \Delta EDF$



$$\Rightarrow \quad \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$$

[Corresponding sides of two similar triangles are proportional]

$$\Rightarrow \quad \frac{5}{12} = \frac{7}{y} = \frac{x}{15}$$

Taking first two parts, we get

$$\Rightarrow \quad \frac{5}{12} = \frac{7}{y}$$

$$\Rightarrow \quad y = (12 \times 7)/5$$

$$\Rightarrow \quad y = 16.8 \text{ cm}$$

Taking first and third part, we get

$$\Rightarrow \quad \frac{5}{12} = \frac{x}{15}$$

$$\Rightarrow \quad x = (5 \times 15)/12$$

$$\Rightarrow \quad x = 6.25 \text{ cm}$$

Hence, lengths of the remaining sides of the triangles are EF = 16.8 cm and BC = 6.25 cm.

Trick Applied

Corresponding sides of similar triangles are in same ratio.

3. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio. [CBSE 2015, 13, 12]

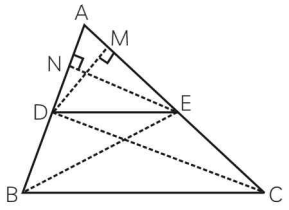
Ans.

It is the statement of "Basic Proportionality theorem".

Given: A triangle ABC in which a line parallel to side BC intersects the other two sides AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.



Proof:

Now, $\text{area of } \Delta ADE = \frac{1}{2}(\text{base} \times \text{height})$

$$= \frac{1}{2}(AD \times EN)$$

So, $$ar(ADE) = \frac{1}{2}(AD \times EN)$$

Similarly, $$ar(BDE) = \frac{1}{2}DB \times EN$$

$$ar(ADE) = \frac{1}{2}AE \times DM$$

and $$ar(DEC) = \frac{1}{2}EC \times DM$$

Therefore, $$\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2}AD \times EN}{\frac{1}{2}DB \times EN} = \frac{AD}{DB} \quad ...(i)$$

and $$\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2}AE \times DM}{\frac{1}{2}EC \times DM} = \frac{AE}{EC} \quad ...(ii)$$

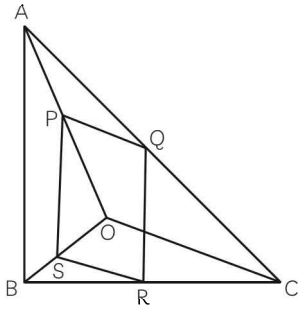
Note that ΔBDE and DEC are on the same base DE and between the same parallels BC and DE.

So, $$ar(BDE) = ar(DEC) \quad ...(iii)$$

Therefore, from (i), (ii) and (iii), we have:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

4. In the given figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR.



Ans.

Given: In ΔABC, PQRS is a parallelogram and AB || PS.

To prove: OC || SR

Proof: According to the question,PQRS is a parallelogram,

$\Rightarrow$ PQ || SR and PS || QR

Also given, AB || PS

In ΔOAB and ΔOPS

PS || AB [Given]

$\angle POS = \angle AOB$ [Common angle]

$\angle OSP = \angle OBA$ [Corresponding angles]

$\Delta OPS \sim \Delta OAB$ [By AA similarity criterion]

$\Rightarrow \quad \frac{PS}{AB} = \frac{OS}{OB}$ ...(i)

QR || PS || AB

In ΔCQR and ΔCAB,

$\angle QCR = \angle ACB$ [Common angle]

$\angle CRQ = \angle CBA$ [Corresponding angles]

$\Delta CQR \sim \Delta CAB$ [By AA similarity criteria]

$\Rightarrow \quad \frac{QR}{AB} = \frac{CR}{CB}$

Since PQRS is a parallelogram, PS = QR

$$\frac{PC}{AB} = \frac{CR}{CB} \quad ...(ii)$$

From equation (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

$$\frac{OB}{OS} = \frac{CB}{CR}$$

Subtracting 1 from L.H.S and R.H.S, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\frac{OB - OS}{OS} = \frac{(CB - CR)}{CR}$$

$$\frac{BS}{OS} = \frac{BR}{CR}$$

$\Rightarrow$ SR || OC

[By converse of basic proportionality theorem]

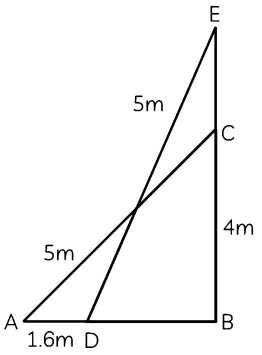
Hence, Proved.

5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Ans.

Let AC be the ladder of length 5 m.

and BC = 4 m be the height of the wall, against which the ladder is placed.



In right angled ΔABC,

Using the Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$(5)^2 = (AB)^2 + (4)^2$

$25 = (AB)^2 + 16$

$(AB)^2 = 25 - 16 = 9$

$AB = \sqrt{9} = 3$ cm

AD = 1.6 cm [Given]

BD = AB – AD = 3 – 1.6 = 1.4 cm ...(i)

Now, in right angled ΔEBD,

Using the Pythagoras theorem,

$ED^2 = EB^2 + BD^2$

$(5)^2 = (EB)^2 + (1.4)^2$ [From eq. (i) BD = 1.4]

$25 = (EB)^2 + 1.96$

$(EB)^2 = 25 - 1.96 = 23.04$

$EB = \sqrt{23.04} = 4.8$ cm

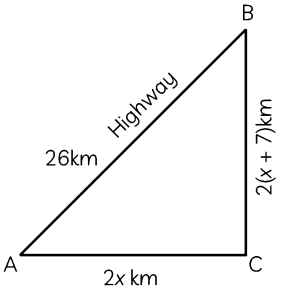
Now, we have,

EC = EB – BC = 4.8 – 4 = 0.8m

Hence, the top of the ladder would slide upwards on the wall by a distance of 0.8 m.

6. For going to city B from city A, there is a route via city C such that AC ⊥ CB, AC = 2*x* km and CB = 2 (*x* + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

Ans.



Given: AC ⊥ CB, AB = 26 km, AC = 2*x* km and CB = 2(*x* + 7) km

In ΔACB,

Using Pythagoras theorem,

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow (26)^2 = (2x)^2 + \{2(x + 7)\}^2$$

$$\Rightarrow 676 = 4x^2 + 4(x^2 + 49 + 14x)$$

$$\Rightarrow 676 = 4x^2 + 4x^2 + 196 + 56x$$

$$\Rightarrow 676 = 8x^2 + 56x + 196$$

$$\Rightarrow 8x^2 + 56x - 480 = 0$$

Dividing the equation by 8, we get,

$$\Rightarrow x^2 + 7x - 60 = 0$$

$$\Rightarrow x^2 + 12x - 5x - 60 = 0$$

$$\Rightarrow x(x + 12) - 5(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x = -12 \text{ or } x = 5$$

Since the distance can't be negative, we neglect $x = -12$

$$\Rightarrow x = 5$$

Now, $AC = 2x = 10$ km

$BC = 2(x + 7)$

$= 2(5 + 7) = 24$ km

Thus, the distance of city B from city A via city

$C = AC + BC$

$= 10 + 24 = 34$ km

Distance of city B from city A through the highway = BA = 26 km

Therefore, the distance saved = 34 – 26 = 8 km.

Hence, the distance saved by highway is 8 km.

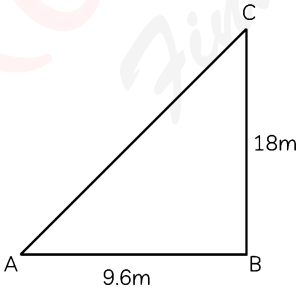
Trick Applied

- *Step 1. Using Pythagoras theorem calculate the value of x.*
- *Step 2. Saved distance will be equal to difference of the two.*

7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Ans.

As pole is vertical so ∠ABC = 90°



Let BC = 18 m be the flag pole and its shadow be AB = 9.6 m.

The required distance of the top of the pole, C from the far end, A of the shadow is AC.

In right angled ΔABC,

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$AC^2 = (9.6)^2 + (18)^2$

$AC^2 = 92.16 + 324$

$AC^2 = 416.16$

$AC = \sqrt{416.16} = 20.4$ m

Hence, the required distance is 20.4 m

8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3m, find how far she is away from the base of the pole.

[CBSE 2016]

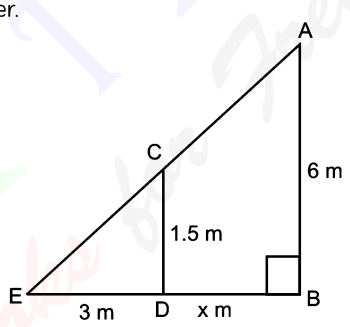
Ans.

Let A be the position of the street bulb fixed on a pole AB = 6 m.

CD = 1.5 m be the height of the woman

and ED = 3 m shadow of the woman

Let distance between pole and woman be x meter.



Here, woman and pole both are standing vertical.

$$\Rightarrow CD \parallel AB$$

In ΔCDE and ΔABE

$$\angle E = \angle E \quad \text{[Common angle]}$$

$$\Rightarrow \angle ABE = \angle CDE \quad \text{[Each equal to 90°]}$$

$$\Rightarrow \Delta CDE \sim \Delta ABE \quad \text{[by AA similarity criterion]}$$

$$\Rightarrow \frac{ED}{EB} = \frac{CD}{AB} \quad \text{[corresponding sides of similar triangle are proportional]}$$

$$\Rightarrow \frac{3}{3+x} = \frac{1.5}{6}$$

$$\Rightarrow 1.5(3 + x) = 3 \times 6$$

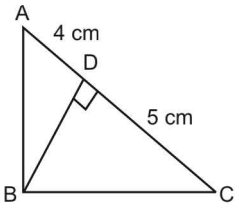
$$\Rightarrow 4.5 + 1.5x = 18$$

$$\Rightarrow 1.5x = 18 - 4.5$$

$$\Rightarrow x = \frac{13.5}{1.5} = 9 \text{ m}$$

Hence, the woman is 9 m away from the pole.

9. In the given figure, ABC is a triangle right angled at B and BD ⊥ AC. If AD = 4 cm, and CD = 5 cm, find BD and AB.



Ans.

Given: ΔABC in which ∠B = 90° and BD ⊥ AC

Also, AD = 4 cm and CD = 5 cm

To find: BD and AB

In ΔADB and ΔCDB,

∠ADB = ∠CDB [each equal to 90°]

∠BAD = ∠DBC [Each equal to 90°– ∠C]

⇒ ΔDBA ~ ΔDCB [By AA similarity criteria]

⇒ $\frac{DA}{DB} = \frac{DB}{DC}$ [Corresponding sides of similar triangle are proportional]

⇒ $DB^2 = DA \times DC$

⇒ $DB^2 = 4 \times 5$

⇒ $DB = 2\sqrt{5}$ cm

In right angled ΔBDA,

BD ⊥ AC [Given]

⇒ ∠BDA = 90°

⇒ $AB^2 = AD^2 + BD^2$

[By Pythagoras therorem]

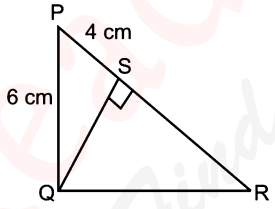
$= 4^2 + (2\sqrt{5})^2$

$= 16 + 20 = 36$

⇒ AB = 6 cm

Hence, BD = $2\sqrt{5}$ cm and AB = 6 cm.

10. In the given figure, PQR is a right triangle right angled at Q and QS ⊥ PR . If PQ = 6 cm and PS = 4 cm, find QS, RS and QR.



Ans.

Given: ΔPQR is right triangle with ∠Q = 90°,

QS ⊥ PR, PS = 4 cm and PQ = 6 cm.

To find: QS, RS and QR

In ΔSQP and ΔSRQ,

∠PSQ = ∠RSQ [Each equal to 90°]

∠SPQ = ∠SQR [Each equal to 90°–∠R]

⇒ ΔSQP ~ ΔSRQ [By AA similarity criteria]

⇒ $\frac{PS}{SQ} = \frac{SQ}{SR}$

⇒ $SQ^2 = PS \times SR$ ...(i)

In right angled ΔPSQ,

$PQ^2 = PS^2 + QS^2$

[By Pythagoras theorem]

$(6)^2 = (4)^2 + QS^2$

$36 = 16 + QS^2$

$QS^2 = 36 - 16 = 20$

⇒ $QS = \sqrt{20} = 2\sqrt{5}$ cm

Putting the value of QS in eq. (i), we get

⇒ $SQ^2 = PS \times SR$

⇒ $(2\sqrt{5})^2 = 4 \times SR$

⇒ $SR = \frac{20}{4} = 5$ cm

In right angled ΔQSR,

⇒ $QR^2 = QS^2 + SR^2$

⇒ $QR^2 = (2\sqrt{5})^2 + (5)^2$

⇒ $QR^2 = 20 + 25$

⇒ $QR = \sqrt{45} = 3\sqrt{5}$cm

Hence, QS = $2\sqrt{5}$ cm, RS = 5 cm and

QR = $3\sqrt{5}$ cm.

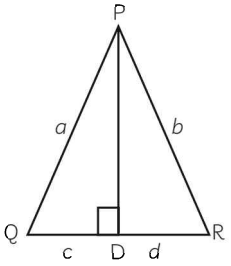
11. In Δ PQR, PD ⊥ QR such that D lies on QR. If PQ = *a*, PR = *b*, QD = *c* and DR = *d*, prove that (*a* + *b*)(*a* – *b*) = (*c* + *d*)(*c* – *d*). [CBSE 2014]

Ans.

Given: In ΔPQR, PD ⊥ QR and PQ = *a*, PR = *b*, QD = *c* and DR = *d*.

To prove: (*a* + *b*)(*a* – *b*) = (*c* + *d*)(*c* – *d*)

Proof:



In right angled ΔPDQ, using pythagoras theorem

$PQ^2 = PD^2 + QD^2$

⇒ $a^2 = PD^2 + c^2$

⇒ $PD^2 = a^2 - c^2$ ...(i)

In right angled ΔPDR, using pythagoras theorem

$PR^2 = PD^2 + DR^2$

⇒ $b^2 = PD^2 + d^2$

⇒ $PD^2 = b^2 - d^2$ ...(ii)

From equation (i) and (ii), we get

$a^2 - c^2 = b^2 - d^2$

⇒ $a^2 - b^2 = c^2 - d^2$

⇒ $(a - b)(a + b) = (c - d)(c + d)$ Hence, proved

12. **In a quadrilateral ABCD, $\angle A + \angle D = 90°$. Prove that $AC^2 + BD^2 = AD^2 + BC^2$**

[Hint: Produce AB and DC to meet at E.]

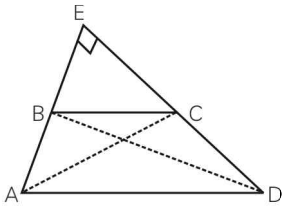
Ans.

Given: A quadrilateral ABCD in which $\angle A + \angle D = 90°$

To prove: $AC^2 + BD^2 = AD^2 + BC^2$

Construction: Produce AB and DC to meet at E. Also, join AC and BD.

Proof:



In ΔAED,

$\angle EAD + \angle EDA = 90°$ [Given]

$\Rightarrow \quad \angle E = 180° - (\angle EAD + \angle EDA) = 90°$

[Angle sum property]

By Pythagoras theorem,

$AD^2 = AE^2 + DE^2$ ...(i)

In ΔBEC,

$BC^2 = BE^2 + CE^2$ .....(ii)

[By Pythagoras theorem]

Adding equations (i) and (ii), we get

$AD^2 + BC^2 = AE^2 + DE^2 + BE^2 + CE^2$ .....(iii)

In ΔAEC,

$AC^2 = AE^2 + CE^2$ [By Pythagoras theorem] ...(iv)

And in ΔBED,

$BD^2 = BE^2 + DE^2$ [By Pythagoras theorem] ...(v)

Adding equations (iii) and (iv), we get

$AC^2 + BD^2 = AE^2 + CE^2 + BE^2 + DE^2$ ...(vi)

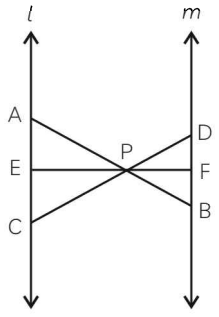
From equation (iii) and (vi), we get

$AC^2 + BD^2 = AD^2 + BC^2$

Hence, proved.

13. **In the given figure, l || m and line segments AB, CD and EF are concurrent at point P.**

Prove that $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$



Ans.

Given: l || m and line segments AB, CD and EF are concurrent at P.

Points A, E and C are on line l

Points D, F and B are on line m.

To prove: $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$

Proof:

In ΔAPC and ΔBPD

$\angle APC = \angle BPD$ [Vertically opposite angles]

$\angle PAC = \angle PBD$ [Alternate angles]

$\Rightarrow \quad \Delta APC \sim \Delta BPD$ [By AA similarity criterion]

$\Rightarrow \quad \frac{AP}{BP} = \frac{AC}{BD} = \frac{PC}{PD}$ ...(i)

In ΔAPE and ΔBPF,

$\angle APE = \angle BPF$ [Vertically opposite angles]

$\angle PAE = \angle PBF$ [Alternate angles]

$\Rightarrow \quad \Delta APE \sim \Delta BPF$ [By AA similarity criterion]

$\Rightarrow \quad \frac{AP}{BP} = \frac{AE}{BF} = \frac{PE}{PF}$ ...(ii)

In ΔPEC and ΔPFD,

$\angle EPC = \angle FPD$ [Vertically opposite angles]

$\angle PCE = \angle PDF$ [Alternate angles]

$\Rightarrow \quad \Delta PEC \sim \Delta PFD$ [By AA similarity criterion]

$\Rightarrow \quad \frac{PE}{PF} = \frac{EC}{FD} = \frac{PC}{PD}$ ...(iii)

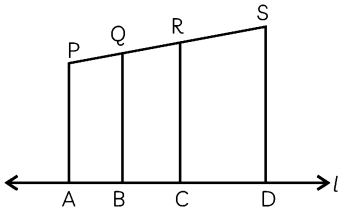
From eqns. (i), (ii) and (iii), we get

$\Rightarrow \quad \frac{AP}{BP} = \frac{AC}{BD} = \frac{PC}{PD} = \frac{AE}{BF} = \frac{PE}{PF} = \frac{EC}{FD}$

$\Rightarrow \quad \frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$

Hence, proved.

14. **In the given figure, PA, QB, RC and SD are all perpendiculars to a line 'l'. AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.**



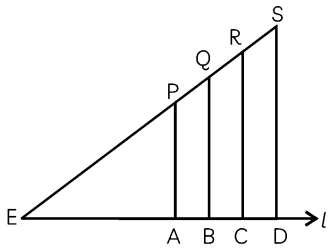
Ans.

Given: PA, QB, RC and SD are perpendicular on line l.

AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm.

To find: PQ, QR and RS

Construction: Produce SP and l to meet each other at E.



Proof:

In ΔEDS,

AP || BQ || DS || CR [Given]

⇒ PQ:QR:RS = AB:BC:CD

⇒ PQ:QR:RS = 6:9:12

Let PQ = 6x, QR = 9x, and RS = 12x.

PQ + QR + RS = 36 cm

⇒ 6x + 9x + 12x = 36

⇒ 27x = 36

⇒ $x = \frac{36}{27} = \frac{4}{3}$

⇒ PQ = 6 × x = 8 cm

⇒ QR = 9 × x = 12 cm

⇒ RS = 12 × x = 16 cm

Hence, the length of PQ = 8 cm, QR = 12 cm and RS = 16 cm.

15. 'O' is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || DC. Through 'O', a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO. [CBSE 2012]

Ans.

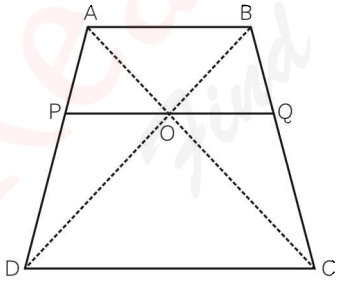
Given: ABCD is a trapezium with diagonals AC and BD intersecting at O

PQ || AB meeting AD in P and BC in Q

To Prove: PO = QO

Construction: Join AC and BD

Proof:



In ΔABD and ΔPOD

PO || AB [As PQ || AB]

∠D = ∠D [Common angle]

∠ABD = ∠POD [Corresponding angles]

⇒ ΔABD ~ ΔPOD [By AA similarity criterion]

⇒ $\frac{AB}{OP} = \frac{AD}{PD}$

[By basic proportionality theorem]

⇒ $\frac{OP}{AB} = \frac{PD}{AD}$ ...(i)

Similarly, In ΔABC and ΔOQC

OQ || AB [As PQ || AB]

∠C = ∠C [Common angle]

∠BAC = ∠QOC [Corresponding angle]

⇒ ΔABC ~ ΔOQC [By AA similarity criterion]

⇒ $\frac{AB}{OQ} = \frac{BC}{QC}$

[By basic proportionality theorem]

⇒ $\frac{OQ}{AB} = \frac{QC}{BC}$ ...(ii)

Also in ΔADC, OP || DC

⇒ $\frac{AP}{PD} = \frac{OA}{OC}$

[By basic proportionality theorem] ...(iii)

Similarly ΔABC, OQ || AB

⇒ $\frac{BQ}{QC} = \frac{OA}{OC}$

[By basic proportionality theorem] ...(iv)

From equation (iii) and (iv),

$\frac{AP}{PD} = \frac{BQ}{QC}$

⇒ $\frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1$

[Adding 1 on both sides, we get,]

⇒ $\frac{(AP + PD)}{PD} = \frac{(BQ + QC)}{QC}$

⇒ $\frac{AD}{PD} = \frac{BC}{QC}$

⇒ $\frac{PD}{AD} = \frac{QC}{BC}$

[From equation (i) and (ii)]

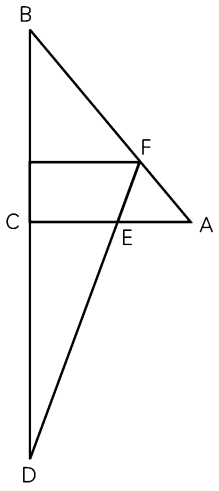
⇒ $\frac{OP}{AB} = \frac{OQ}{AB}$

⇒ OP = OQ

Hence, proved.

16. In the given figure, line segment DF intersects side AC of a triangle ABC at point E such that E is the midpoint of CA and ∠AEF = ∠AFE. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$

[Hint: Take point G on AB such that CG || DF.]

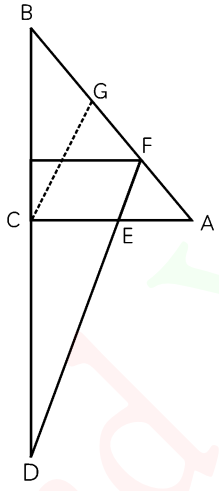


Ans.

Given: In the given figure of ΔABC, EA = EC and ∠AEF = ∠AFE

To prove: $\frac{BD}{CD}=\frac{BF}{CE}$

Construction: Draw CG || EF



Proof:

Since E is the midpoint of CA

$\Rightarrow$ CE = AE

In ΔACG, CG || EF and E is the midpoint of AC

$\Rightarrow$ F will be the midpoint of AG.

$\Rightarrow$ FG = FA

In ΔACG

∠AEF = ∠AFE [Sides corresponding to equal angles are equal]

$\Rightarrow$ AE = AF

But, AE = AF = EC [Given]

FG = FA = EA = EC ....(i)

In ΔBDF, CG || DF [By construction]

$\Rightarrow \quad \frac{BC}{CD}=\frac{BG}{GF}$

[Using Basic proportionality theorem]

$\Rightarrow \quad \frac{BC}{CD}=\frac{BF-GF}{GF}$

$\Rightarrow \quad \frac{BC}{CD}=\frac{BF}{GF}-1$

$\Rightarrow \quad \frac{BC}{CD}+1=\frac{BF}{GF}$

$\Rightarrow \quad \frac{BC}{CD}+1=\frac{BF}{CE}$ [using eqn. (i)]

$\Rightarrow \quad \frac{BC+CD}{CD}=\frac{BF}{CE}$

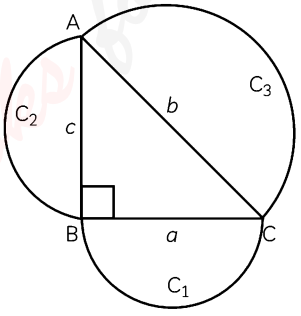
$\Rightarrow \quad \frac{BD}{CD}=\frac{BF}{CE}$

Hence, Proved.

17. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.

Ans.

Given: In the given figure, ΔABC is right angled triangled at B. Three semicircles C_1, C_2 and C_3 are drawn taking the sides BC, AB and AC of the triangle ABC as diameters.



To prove: Area of semicircles (C_1 + C_2) = Area of semicircle C_3

Proof:

Let AB = *c*, BC = *a* and AC = *b*.

In ΔABC,

∠B = 90°

$\Rightarrow$ $BC^2 + AB^2 = AC^2$ [By Pythagoras theorem]

$\Rightarrow \quad a^2 + c^2 = b^2$ ...(i)

We know that area of a semicircle = $\frac{\pi r^2}{2}$

Area of semicircle drawn on AC, C_3

$$=\frac{1}{2}\times\pi\times\left(\frac{b}{2}\right)^2$$

Area of semicircle drawn on BC, C_1

$$=\frac{1}{2}\times\pi\times\left(\frac{a}{2}\right)^2$$

Area of semicircle drawn on AB, C_2

$$= \frac{1}{2} \times \pi \times \left(\frac{c}{2}\right)^2$$

ar (semicircle C_1) + ar (semicircle C_2) = $C_1 + C_2$

$$= \frac{1}{2} \times \pi \times \left(\frac{a}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{c}{2}\right)^2$$

$$= \pi \times \frac{a^2 + c^2}{8}$$

[Using eq. (i) $a^2 + c^2 = b^2$]

$$= \pi \times \frac{b^2}{8}$$

$\Rightarrow$ $C_1 + C_2 = C_3$

Hence, proved.

Trick Applied

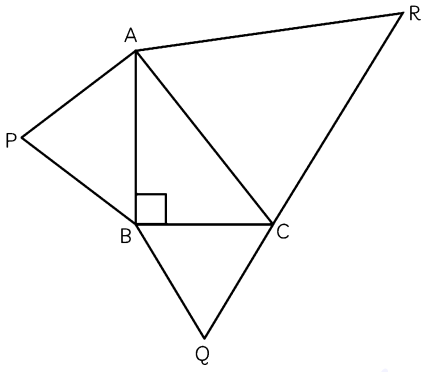
- *Step 1. Draw 3 semicircles on 3 sides of right triangle.*
- *Step 2. Find area of each semicircle.*
- *Step 3. Prove the desired result.*

18. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles drawn on the other two sides of the triangle.

Ans.

Given: A right angled triangle ABC right angled at B. Equilateral triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively

To prove: Area (ΔPAB) + Area (ΔQBC) = Area (ΔRAC)



Proof: Let AB = c, BC = a and AC = b.

In ΔABC, $\angle B = 90°$

$\Rightarrow$ $BC^2 + AB^2 = AC^2$ [By Pythagoras theorem]

$\Rightarrow$ $a^2 + c^2 = b^2$ ...(i)

We know that area of equilateral triangle with side $a = \frac{\sqrt{3}}{4}a^2$

Area of equilateral triangle RAC = $\frac{\sqrt{3}}{4}b^2$

Area of equilateral triangle PAB = $\frac{\sqrt{3}}{4}c^2$

Area of equilateral triangle QBC = $\frac{\sqrt{3}}{4}a^2$

Area (ΔPAB) + Area (ΔQBC) = $\frac{\sqrt{3}}{4}c^2 + \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4}(c^2 + a^2)$$

$$= \frac{\sqrt{3}}{4}b^2$$ [Using eq. (i)]

= Area (ΔRAC) Hence, proved.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

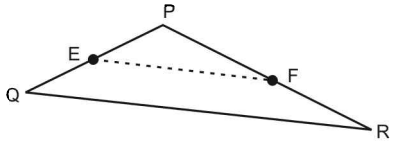
Ans.

Since, the perimeters and two sides of triangles are proportional

Then, the third side will also be proportional to the corresponding third side.

Hence, the two triangles will be similar by SSS criterion.

2. Kitchen garden of Ms. Sanjana is in the form of a triangle as shown. She wants to divide it in two parts; one triangle and one trapezium.



She takes PE = 4 m, QE = 4.5 m, PF = 8 m and RF = 9 m.

Is EF || QR? Justify your answer.

Ans.

$\frac{PE}{QE} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{PF}{RF} = \frac{8}{9}$

Since, $\frac{PE}{QE} = \frac{PF}{RF} = \frac{8}{9}$

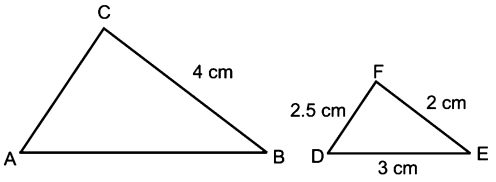
$\therefore$ By converse of thalles theorem

EF || QR.

3. $\Delta ABC \sim \Delta DEF$ such that DE = 3 cm, EF = 2 cm, DF = 2.5 cm and BC = 4 cm. Find perimeter of ΔABC.

Ans.

DE = 3 cm, EF = 2 cm, DF = 2.5 and BC = 4 cm



Since, $\Delta ABC \sim \Delta DEF$

$$\frac{AC}{DF}=\frac{AB}{DE}=\frac{BC}{EF}=\frac{4}{2}=\frac{2}{1}$$

AC = 5 cm and AB = 6 cm

Perimeter of ΔABC = AB + BC + AC

= 4 + 6 + 5 = 15 cm

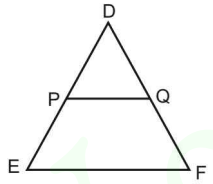
So, perimeter of ΔABC is 15 cm.

4. P and Q are the points on the sides DE and DF of a triangle DEF such that DP = 5 cm, DE = 15 cm, DQ = 6 cm and QF = 18 cm. Is PQ || EF? Give reasons for your answer.

Ans.

$$\frac{DP}{PE}=\frac{5}{15-5}=\frac{1}{2}$$

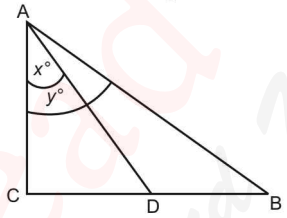
and $$\frac{DQ}{QF}=\frac{6}{18}=\frac{1}{3}$$



Since, $\frac{DP}{PE} \neq \frac{DQ}{QF}$

Hence, PQ is not parallel to EF.

5. In the figure given below, if D is mid point of BC, find the value of $\frac{\tan x^\circ}{\tan y^\circ}$.



Ans.

In this figure,

$$\frac{\tan x^\circ}{\tan y^\circ}=\frac{\frac{CD}{AC}}{\frac{BC}{AC}}=\frac{CD}{BC} \quad ...(i)\left[\because \tan\theta=\frac{P}{B}\right]$$

As, D is the mid point of BC. So, BC = 2 CD

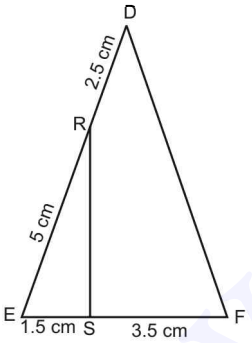
Putting the value of BC in equation (i), we get,

$$\frac{\tan x^\circ}{\tan y^\circ}=\frac{CD}{2CD}=\frac{1}{2}.$$

6. R and S are points on the sides DE and EF respectively of a ΔDEF such that ER = 5 cm, RD = 2.5 cm, SE = 1.5 cm and FS = 3.5 cm. Find whether RS || DF or not.

Ans.

Given: In ΔDEF and R and S points are on sides DE and EF.



Construction: Join RS

To find: RS II DF or not

Proof: We have,

RE = 5cm and RD = 2.5 cm

$$\frac{RE}{RD}=\frac{5}{2.5}=\frac{2}{1}$$

Similarly, we have,

ES = 1.5 cm and SF = 3.5 cm

$$\frac{ES}{SF}=\frac{3.5}{1.5}=\frac{7}{3}$$

Here, we can see that,

$$\frac{RE}{RD} \neq \frac{ES}{SF}$$

So, RS is not parallel to DF.

7. D, E and F are respectively the mid points of the sides AB, BC and CA of triangle ABC respectively. Find the ratio of areas of triangle DEF and triangle ABC.

Ans.

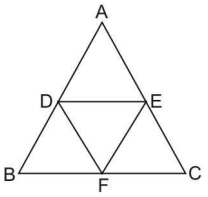
Since D and E are the mid-point of ΔABC

$$\therefore \quad \frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{2}$$

By thales theorem DEHBC and $DE=\frac{1}{2}BC$

Similarly, $EF=\frac{1}{2}AB$

$$DF=\frac{1}{2}AC$$



So, $$\frac{DE}{BC}=\frac{EF}{AB}=\frac{DF}{AC}=\frac{1}{2}$$

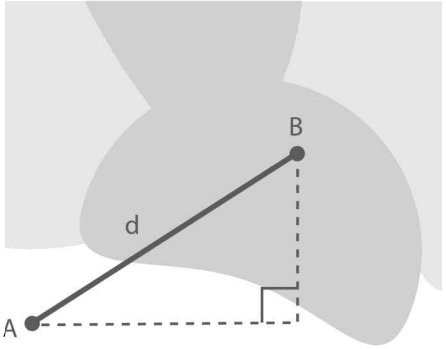
By SSS criteria, $\Delta EFD \sim \Delta ABC$

$$\frac{ar(DEF)}{ar(ABC)}=\left(\frac{DF}{BC}\right)^2$$

$$=\left(\frac{1}{2}\right)^2$$

$$=\frac{1}{4}$$

7 Coordinate Geometry



EXERCISE 7.1

Choose the correct option from the given four options in the following questions:

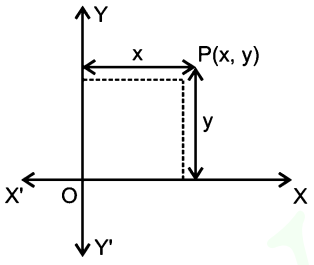
1. The distance of the point P (2, 3) from the x-axis is:

(A) 2 (B) 3

(C) 1 (D) 5 **[CBSE 2012]**

Ans. (B)

Explanation: We know that P (*x*, *y*) any point on the cartesian plane is represented as,



where

x = perpendicular distance from y

y = perpendicular distance from x

⇒ distance of the point P (2, 3) from x-axis = 2, (B).

2. The distance between the points A (0, 6) and B (0, –2) is:

(A) 6 (B) 8

(C) 4 (D) 2

[CBSE 2020, 10]

Ans. (B)

Explanation: Distance between two points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For distance between A (0, 6), B (0, –2)

$x_1 = 0$ $x_2 = 0$

$y_1 = 6$ $y_2 = -2$

$$\therefore\ d = \sqrt{(0-0)^2 + (-2-6)^2} = \sqrt{(-8)^2} = \sqrt{8^2} = 8$$

Trick Applied

The distance between two points P (x_1, y_1) and Q (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. The distance of the p oint P (–6, 8) from the origin is:

(A) 8 (B) $2\sqrt{7}$

(C) 10 (D) 6 **[CBSE 2018]**

Ans. (C)

Explanation: Distance between two points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For distance between P (–6, 8) and origin

$x_1 = -6$ $x_2 = 0$

$y_1 = 8$ $y_2 = 0$

∴ Distance,

$$d = \sqrt{[0-(-6)]^2 + (0-8)^2} = \sqrt{6^2 + (-8)^2}$$

$$= \sqrt{36+64} = \sqrt{100} = 10$$

Trick Applied

Co-ordinate of Origin is (0, 0).

4. The distance between the points (0, 5) and (–5, 0) is:

(A) 5 (B) $5\sqrt{2}$

(C) $2\sqrt{5}$ (D) 10

[CBSE 2020, 10]

Ans. (B)

Explanation: Distance between (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For distance between (0, 5) and (–5, 0)

$x_1 = 0$ $x_2 = -5$

$y_1 = 5$ $y_2 = 0$

∴ Distance,

$$d = \sqrt{(-5-0)^2 + (0-5)^2} = \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

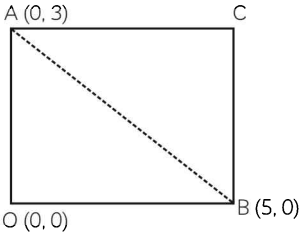
5. AOBC is a rectangle whose three vertices are A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is:

(A) 5 (B) 3

(C) $\sqrt{34}$ (D) 4 **[CBSE 2014]**

Ans. (C)

Explanation:



It is given that AOBC is a rectangle.

Length of diagonal AB = Distance between points A (0, 3) and B (5, 0).

Distance between points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For distance between A (0, 3) and B (5, 0)

$x_1 = 0$ $x_2 = 5$

$y_1 = 3$ $y_2 = 0$

∴ Distance,

$$d = \sqrt{(5-0)^2 + (0-3)^2} = \sqrt{25+9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is:

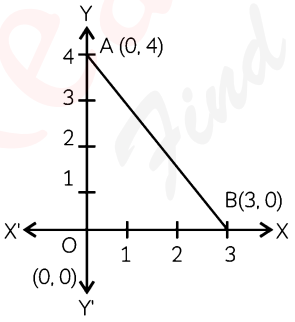
(A) 5 (B) 12

(C) 11 (D) $7+\sqrt{5}$

[CBSE 2014, 11]

Ans. (B)

Explanation: We plot the vertices of a triangle AOB (0, 4), (0, 0), and (3, 0) as shown below:



Perimeter of ΔAOB = sum of length of all sides = AO + OB + AB

AO, d_1= distance between A (0, 4) and O (0, 0)

BO, d_2= distance between O (0, 0) and B (3, 0)

AB, d_3= distance between A (0, 4) and B (3, 0)

We know that distance between two points (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

∴ Perimeter of ΔAOB = $d_1 + d_2 + d_3$

$$= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{(3-0)^2 + (0-0)^2} + \sqrt{(3-0)^2 + (0-4)^2}$$

$$= \sqrt{0+16} + \sqrt{9+0} + \sqrt{9+16}$$

$$= 4+3+\sqrt{25} = 7 + 5 = 12$$

Trick Applied

- *Step 1. Plot the points on graph and join them.*
- *Step 2. Determine length of each side using distance formula.*
- *Step 3. Add all side lengths to get the perimeter.*

7. The points (–4, 0), (4, 0), (0, 3) are the vertices of a:

(A) right triangle

(B) isosceles triangle

(C) equilateral triangle

(D) scalene triangle **[CBSE 2016]**

Ans. (B)

Explanation: Let A (–4, 0), B (4, 0), C (0, 3) be the vertices of a ΔABC.

We know that distance between two points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between A (–4, 0) and B (4, 0),

$$AB = \sqrt{(4-(-4))^2 + (0-0)^2} = \sqrt{(8)^2} = 8$$

Distance between B (4, 0) and C (0, 3)

$$BC = \sqrt{(0-4)^2 + (3-0)^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

Distance between A (–4, 0) and C (0, 3)

$$AC = \sqrt{(0+4)^2 + (3-0)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

Since BC = AC, ΔABC is an isosceles triangles.

8. The point which divides the line segment joining the points (7, –6) and (3, 4) in the ratio 1:2 internally, lies in the:

(A) I quadrant (B) II quadrant

(C) III quadrant (D) IV quadrant

[CBSE 2011]

Ans. (D)

Explanation: We know that if P (x, y) divides the line segment joining A (x_1, y_1) and B (x_2, y_2) internally in the ratio m:n, then

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Given that $x_1 = 7$, $y_1 = -6$, $x_2 = 3$, $y_2 = 4$, $m = 1$, $n = 2$

$\therefore \quad x = \dfrac{1(3)+2(7)}{1+2} = \dfrac{3+14}{3} = \dfrac{17}{3}$

$y = \dfrac{1(4)+2(-6)}{1+2} = \dfrac{4-12}{3} = \dfrac{-8}{3}$

As x-coordinate is positive and y-coordinate is negative:

$\therefore$ $(x, y) = \left(\dfrac{17}{3}, \dfrac{-8}{3}\right)$ lies is the IV quadrant.

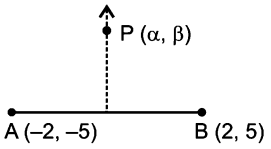
9. The point which lies on the perpendicular bisector of the line segment joining point A (–2, –5) and B (2, 5) is:

(A) (0, 0) (B) (0, –1)

(C) (–1, 0) (D) (1, 0)

Ans. (A)

Explanation: Let P (α, β) be a point which lies on the perpendicular bisector of the line segment AB.



Then, PA = PB or $PA^2 = PB^2$

$\Rightarrow (\alpha + 2)^2 + (\beta + 5)^2 = (\alpha - 2)^2 + (\beta - 5)^2$

$\Rightarrow \alpha^2 + 4\alpha + 4 + \beta^2 + 10\beta + 25$

$= \alpha^2 - 4\alpha + 4 + \beta^2 - 10\beta + 25$

$\Rightarrow \quad 8\alpha + 20\beta = 0$

This is true only when $\alpha = 0$ and $\beta = 0$

[From the given four options]

So, (0, 0) lies on the perpendicular bisector.

10. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is:

(A) (0, 1) (B) (0, –1)

(C) (–1, 0) (D) (1, 0)

[CBSE 2010]

Ans. (B)

Explanation: It is given that ABCD is a parallelogram with vertices A (–2, 3), B (6, 7) and C (8, 3). Let fourth vertex be D (x, y). We know that diagonals AC and BD will bisect each other.

Midpoint of diagonal AC (x_1, y_1)

$= \left(\dfrac{-2+8}{2}, \dfrac{3+3}{2}\right) = \left(\dfrac{6}{2}, \dfrac{6}{2}\right) = (3, 3)$

Midpoint of diagonal BD $(x_2, y_2) = \left(\dfrac{x+6}{2}, \dfrac{y+7}{2}\right)$

But the two midpoints are the same. So,

$\dfrac{x+6}{2} = 3$ and $\dfrac{y+7}{2} = 3$

$\Rightarrow \quad x + 6 = 6$ and $y + 7 = 6$

$\Rightarrow \quad x = 0$ and $y = -1$

Hence, the fourth vertex D $(x, y) = (0, -1)$.

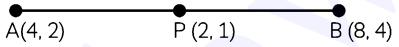
11. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then:

(A) $AP = \dfrac{1}{3}AB$ (A) AP = PB

(C) $PB = \dfrac{1}{3}AB$ (D) $AP = \dfrac{1}{2}AB$

Ans. (D)

Explanation: It is given that P (2, 1) lies on the line segment joining the points A (4, 2) and B (8, 4).



We know that distance between two points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between A (4, 2) and P (2, 1)

$= \sqrt{(2-4)^2 + (1-2)^2}$

$AP = \sqrt{(-2)^2 + (-1)^2} = \sqrt{2^2 + 1} = \sqrt{4+1} = \sqrt{5}$

Distance between P (2, 1) and B (8, 4)

$= \sqrt{(8-2)^2 + (4-1)^2}$

$PB = \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

Distance between A (4, 2) and B (8, 4)

$= \sqrt{(8-4)^2 + (4-2)^2}$

$AB = \sqrt{(4)^2 + (2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$\therefore$ $AB = 2\sqrt{5} = 2(AP) \Rightarrow AP = \dfrac{AB}{2}$.

12. If $P\left(\dfrac{a}{3}, 4\right)$ is the midpoint of the line segment joining the points Q (–6, 5) and R (–2, 3), then the value of *a* is:

(A) –4 (B) –12

(C) 12 (D) –6 **[CBSE 2010]**

Ans. (B)

Explanation: It is given that $P\left(\dfrac{a}{3}, 4\right)$ is the midpoint of the line segment joining the points Q (–6, 5) and R (–2, 3).

We know that midpoint (x, y) =

$$\left\{\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right\}$$

$\therefore$ Midpoint of QR = $P\left(\dfrac{-6-2}{2}, \dfrac{5+3}{2}\right) = \left(\dfrac{-8}{2}, \dfrac{8}{2}\right)$

= P (–4, 4)

But mid-point is given as $P\left(\frac{a}{3}, 4\right)$ comparing the two, we get

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

$$\Rightarrow \quad \frac{a}{3} = -4 \Rightarrow a = -12$$

Hence, the required value of a is -12.

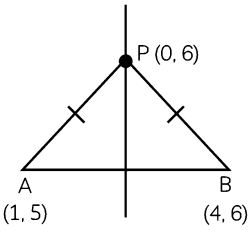
13. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the y-axis at:

(A) (0, 13) (B) (0, –13)

(C) (0, 12) (D) (13, 0)

Ans. (A)

Explanation: Let the perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cut the y-axis at P.



Then point P will be of the form $(0, b)$ as any point on the y-axis will have x-coordinates 0.

We know that AP = PB

$$\sqrt{(0-1)^2+(b-5)^2} = \sqrt{(4-0)^2+(6-b)^2}$$

$$\sqrt{1+(b-5)^2} = \sqrt{16+(6-b)^2}$$

$$1 + (b-5)^2 = 16 + (6-b)^2$$

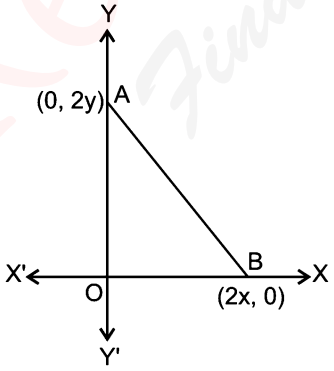
$$1 + b^2 + 25 - 10b = 16 + 36 + b^2 - 12b$$

$$12b - 10b = 52 - 26$$

$$2b = 26 \Rightarrow b = 13$$

Hence, point P is (0, 13).

14. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is:



(A) (x, y) (B) (y, x)

(C) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (D) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Ans. (A)

Explanation: Let P (h, k) be a point equidistant from the three vertices of ΔAOB: A $(0, 2y)$, B $(2x, 0)$ and O $(0, 0)$.

Then, PO = PA = PB

$$\Rightarrow \quad (PO)^2 = (PA)^2 = (PB)^2 \qquad \text{...(i)}$$

By distance formula

$$(PO)^2 = \left(\sqrt{(h-0)^2+(k-0)^2}\right)^2 = h^2 + k^2$$

$$(PA)^2 = \left(\sqrt{(h-0)^2+(k-2y)^2}\right)^2 = h^2 + (k-2y)^2$$

$$(PB)^2 = \left(\sqrt{(h-2x)^2+(k)^2}\right)^2 = (h-2x)^2 + (k)^2$$

Putting value in eqn (i), we get

$$h^2 + k^2 = h^2 + (k-2y)^2$$

$$\Rightarrow \quad h^2 + k^2 = h^2 + k^2 + 4y^2 - 4ky$$

$$\Rightarrow \quad 4y^2 - 4ky = 0$$

$$\Rightarrow \quad 4y(y-k) = 0 \Rightarrow y = k \qquad [\because y \neq 0]$$

Also, $h^2 + k^2 = (h-2x)^2 + k^2$

$$\Rightarrow \quad h^2 + k^2 = h^2 + 4x^2 - 4hx + k^2$$

$$\Rightarrow \quad 4x^2 - 4hx = 0$$

$$\Rightarrow \quad 4x(x-h) = 0$$

$$\Rightarrow \quad x = h \qquad [\because x \neq 0]$$

$\therefore$ Required point = $(h, k) = (x, y)$.

Alternative Method:

In any right triangle, the midpoint of the diagonal is equidistant from the three vertices.

Hence, the coordinate of the point are $\left(\frac{2x+0}{2}, \frac{0+2y}{2}\right)$ i.e. (x, y)

Trick Applied

- *Step 1. Consider new point to be P (h, k)*
- *Step 2. Determine PO, PA and PB using distance formula and equate them.*
- *Step 3. Solve 2 terms at a time to get required point.*

15. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is:

(A) $\left(\frac{-3}{4}, 1\right)$ (B) $\left(2, \frac{7}{3}\right)$

(C) $\left(5, \frac{-1}{2}\right)$ (D) $\left(-6, \frac{5}{2}\right)$

Ans. (D)

Explanation: It is given that centre of the circle is origin O (0, 0) and it pases through $\left(\frac{13}{2}, 0\right)$.

$\Rightarrow$ Radius of circle = Distance between (0, 0) and $\left(\frac{13}{2}, 0\right)$.

$= \sqrt{\left(\frac{13}{2}-0\right)^2 + (0-0)^2} = \sqrt{\left(\frac{13}{2}\right)^2}$

$= \frac{13}{2} = 6.5$

We know that the point which does not lie in the interior of circle will be at a distance greater than the radius from the centre

(A) Distance between (0, 0) and $\left(-\frac{3}{4}, 1\right)$

$= \sqrt{\left(-\frac{3}{4}-0\right)^2 + (1-0)^2} = \sqrt{\frac{9}{16}+1}$

$= \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25$

Clearly, 1.25 < 6.5

$\Rightarrow$ Point $\left(\frac{-3}{4}, 1\right)$ lies in the interior to the circle.

(B) Distance between (0, 0) and $\left(2, \frac{7}{3}\right)$

$= \sqrt{(2-0)^2 + \left(\frac{7}{3}-0\right)^2} = \sqrt{\frac{4+49}{9}}$

$= \sqrt{\frac{36+49}{9}} = \sqrt{\frac{85}{9}} = \frac{9.22}{3} = 3.1$

clearly 3.1 < 6.5

$\Rightarrow$ Point $\left(2, \frac{7}{3}\right)$ lies in the interior of the circle.

(C) Distance between (0, 0) and $\left(5, -\frac{1}{2}\right)$

$= \sqrt{(5-0)^2 + \left(\frac{-1}{2}-0\right)^2} = \sqrt{25+\frac{1}{4}}$

$= \sqrt{\frac{101}{4}} = \frac{10.04}{2} = 5.02$

Clearly, 5.02 < 6.5.

$\Rightarrow$ Point $\left(5, \frac{-1}{2}\right)$ lies in the interior of the circle.

(D) Distance between (0, 0) and $\left(-6, \frac{5}{2}\right)$

$= \sqrt{(-6,-0)^2 + \left(\frac{5}{2}-0\right)^2} = \sqrt{36+\frac{25}{4}}$

$= \sqrt{\frac{144+25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2}$

= 6.5 = radius

So the point $\left(-6, \frac{5}{2}\right)$ lies on the circle and not in the interior.

16. A line intersects the y-axis and x-axis at points P and Q, respectively. If (2, –5) is the midpoint of PQ, then the coordinates of P and Q, respectively are:

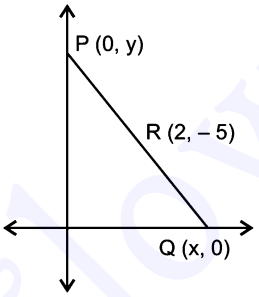
(A) (0, –5) and (2, 0) (B) (0, 10) and (–4, 0)

(C) (0, 4) and (–10, 0) (D) (0, –10) and (4, 0)

[CBSE 2017]

Ans. (D)

Explanation: Let coordinates of P be (0, *y*) and coordinates of Q be (*x*, 0).



Midpoint of P (0, *y*) and Q (*x*, 0) will be

$R\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$ [Mid point of line segment joining (x_1, y_1) and $x_2, y_2) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$]

But it is given that midpoint of PQ is (2, –5).

Comparing the two, we get $\frac{0+x}{2} = 2$ and $\frac{y+0}{2} = -5$

$\Rightarrow$ x = 4 and *y* = –10.

$\Rightarrow$ Coordinates of point P = (0, y) = (0, –10) and coordinates of point Q = (x, 0) = (4, 0).

17. If the distance between the points (4, *p*) and (1, 0) is 5, then the value of *p* is:

(A) 4 (B) ± 4

(C) –4 (D) 0

[CBSE 2020, 17, 11]

Ans. (B)

Explanation: It is given that distance between (4, *p*) and (1, 0) = 5

$\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$

$\Rightarrow \sqrt{(-3)^2 + (-p)^2} = 5 \Rightarrow \sqrt{9+p^2} = 5$

$\Rightarrow 9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4$

Hence, the required value of *p* is ± 4.

18. If the points A (1, 2), O (0, 0) and C (*a*, *b*) are collinear, then:

(A) *a* = *b* (B) *a* = 2*b*

(C) 2*a* = *b* (D) *a* = –*b*

[CBSE 2020, 15, 11]

Ans. (C)

Explanation: Let the given points A (1, 2), B (0, 0) and (a, b) be represented by (x_1, y_1), (x_2, y_2), (x_3, y_3). It is given that the points A, B, C are collinear.

We know that the area of triangle ABC, Δ

$$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$= \left|\frac{1}{2}[1(0 - b) + 0(b - 2) + a(2 - 0)]\right|$$

$$= \frac{1}{2}[1(-b) + 0 + a(2)] = \frac{1}{2}[-b + 2a]$$

Since points A, B and C are collinear

$\Rightarrow$ Area of $\Delta ABC = 0$

$$\Rightarrow \quad \frac{1}{2}(2a - b) = 0$$

$$\Rightarrow \quad 2a - b = 0$$

$$\Rightarrow \quad 2a = b$$

Hence, the required relation is $2a = b$.

EXERCISE 7.2

State whether the following statements are true or false. Justify.

1. ΔABC with vertices A (–2, 0), B (2, 0) and C (0, 2) is similar to ΔDEF with vertices D (–4 , 0), E (4, 0) and F (0, 4).

Ans. True.

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between A (–2, 0) and B (2, 0),

$AB = \sqrt{(2-(-2))^2 + (0-0)^2} = \sqrt{4^2} = 4$

Distance between B (2, 0) and C (0, 2),

$BC = \sqrt{(0-2)^2 + (2-0)^2} - \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Distance between C (0, 2) and A (–2, 0),

$CA = \sqrt{(-2-0)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Distance between D (–4, 0) and E (4, 0),

$DE = \sqrt{(4-(-4))^2 + (0-0)^2} = \sqrt{8^2} = 8$

Distance between E(4, 0) and F (0, 4),

$EF = \sqrt{(0-4)^2 + (4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Distance between F(0, 4) and D (–4, 0),

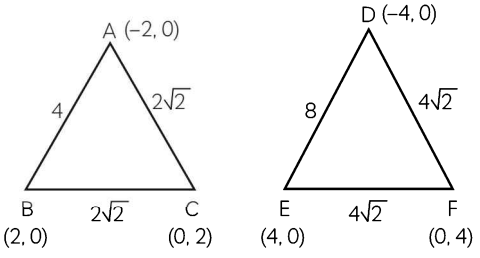
$FD = \sqrt{(-4-0)^2 + (0-4)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$

Now, $\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}$, $\frac{BC}{EF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$, $\frac{CA}{FD} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

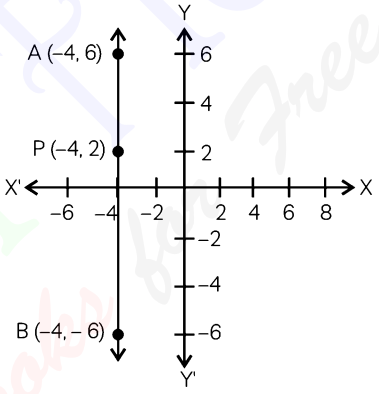
$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

We see that sides of ΔABC and ΔFDE are proportional, hence ΔABC is similar to ΔDEF.



2. Point P (–4, 2) lies on the line segment joining the points A (–4, 6) and B (–4, –6).

Ans. True.



3. Points P (0, 5), (0, –9) and (3, 6) are collinear.

Ans. False.

Let three given points be represented as $(x_1, y_1) = (0, 5)(x_2, y_2) = (0, -9)(x_3, y_3) = (3, 6)$. For the points to be collinear, the area of the triangle formed by them is zero.

Area of triangle,

$$\Delta = \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$= \frac{1}{2}[0(-9 - 6) + 0(6 - 5) + 3(5 + 9)]$$

$$= \frac{1}{2}[0 + 0 + 3(14)]$$

$$= 3 \times 7 = 21 \neq 0$$

Hence, the points are non-colliner.

4. Point P (0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points A (–1, 1) ad B (3, 3).

Ans. False.

(x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Also, we know that points lying on perpendicular bisector of the line segment

joining two points is equidistant from these two points.

$\therefore$ PA i.e., distance between P (0, 2) and A (–1, 1)

$= \sqrt{(-1-0)^2+(1-2)^2} = \sqrt{1+1} = \sqrt{2}$

PB i.e., distance between P (0, 2) and B (3, 3)

$= \sqrt{(3-0)^2+(3-2)^2} = \sqrt{9+1} = \sqrt{10}$

Clearly, PA ≠ PB.

Hence, point P does not lie on perpendicular bisector of AB.

5. Points A (3, 1), B (12, –2) and C (0, 2) cannot be the vertices of a triangle.

Ans. True.

Let three given points be represented as A (x_1, y_1) = (3, 1), B (x_2, y_2) = (12, –2) and C (x_3, y_3) = (0, 2).

Area of $\Delta ABC = \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$

$= \frac{1}{2}[3(-2-2) + 12(2-1) + 0(1-(-2))]$

$= \frac{1}{2}[3(-4) + 12(1) + 0(3)] = \frac{1}{2}[-12 + 12] = 0$

Since area of ΔABC = 0

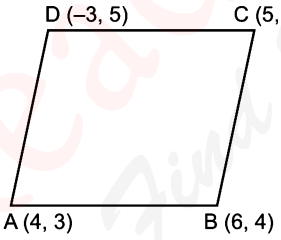
⇒ Points A (3, 1), and B (12, –2) and C (0, 2) are collinear. i.e., they cannot be the vertices of a triangle.

6. Points A (4, 3), B (6, 4), C (5, –6) and D (–3, 5) are the vertices of a parallelogram.

[CBSE 2012]

Ans. False.

We know that opposite sides of parallelogram are equal in length.



We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance between A (4, 3) and B (6, 4)

$AB = \sqrt{(6-4)^2+(4-3)^2}$

$= \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

Distance between B (6, 4) and C (5, –6)

$BC = \sqrt{(5-6)^2+(-6-4)^2}$

$= \sqrt{1^2+(-10)^2} = \sqrt{1+100} = 101$

Distance between C (5, –6) and D (–3, 5)

$CD = \sqrt{(5-0)^2+(0-0)^2} = \sqrt{5^2-0^2} = \sqrt{5^2} = 5$

$= \sqrt{25+1} = \sqrt{26}$

Distance between D (–3, 5) and A (4, 3)

$DA = \sqrt{(4+3)^2+(3-5)^2} = \sqrt{7^2+(-2)^2}$

$= \sqrt{49+4} = \sqrt{53}$

as AB ≠ CD and BC ≠ DA.

Hence, the given vertices are not vertices of a parallelogram.

7. A circle has its centre at the origin and a point P (5, 0) lies inside the circle. The point Q (6, 8) lies outside the circle.

Ans. True.

We know that

(x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance between centre O (0, 0) and point P (5, 0) that lies on circle will given us radius.

⇒ Radius = $\sqrt{(5-0)^2+(0-0)^2} = \sqrt{5^2-0^2} = \sqrt{5^2} = 5$

Distance between O (0, 0) and point Q (6, 8)

$OQ = \sqrt{(6-0)^2+(8-0)^2} = \sqrt{6^2+8^2} = \sqrt{36+64}$

$= \sqrt{100} = 10$

Clearly the distance OQ > radius.

Thus we can say that point Q (6, 8) lies outside the circle.

Trick Applied

Determine distance OP and OQ using distance formula and check if OQ is greater than OP.

8. The point A (2, 7) lies on the perpendicular bisector of line segment PQ joining the points P (6, 5) and Q (0, –4).

Ans. False.

We know that if A (2, 7) lies on the perpendicular bisector of B (6, 5) and Q (0, –4), then

AP = AQ

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AP = \sqrt{(6-2)^2+(5-7)^2} = \sqrt{4^2+(-2)^2}$

$= \sqrt{16+4} = \sqrt{20}$

$AQ = \sqrt{(0-2)^2+(-4-7)^2} = \sqrt{(-2)^2+(-11)^2}$

$= \sqrt{4+121} = \sqrt{125}$

Since AP ≠ AQ.

A (2, 7) does not lie on the perpendicular bisector of PQ.

9. Point P (5, –3) is one of the two points of trisection of the line segment joining points A (7, –2) and B (1, –5). [CBSE 2012]

Ans. True.

Let point P (5, –3) divide the line segment joining the points A (7, –2) and B (1, –5) in the ratio m:1 internally.

Using section formula,

Coordinate of point P will be

$$\left[\frac{m(1)+1(7)}{m+1}, \frac{m(-5)+1(-2)}{m+1}\right] = \left[\frac{m+7}{m+1}, \frac{-5m-2}{m+1}\right]$$

According to the question

$$(5, -3) = \left(\frac{m+7}{m+1}, \frac{-5m-2}{m+1}\right)$$

$$\Rightarrow 5 = \frac{m+7}{m+1} \text{ and } -3 = \frac{-5m-2}{m+1}$$

$\Rightarrow 5(m + 1) = (m + 7)$ and $3(m + 1) = 5m + 2$

$\Rightarrow 5m + 5 - m - 7 = 0$ and $(3m + 3 - 5m - 2 = 0$

$\Rightarrow 4m - 2 = 0$ and $-2m + 1 = 0$

$$\Rightarrow m = \frac{1}{2}$$

Hence, point P divides the line segment AB in the ratio 1:2. Thus point P is the point of trisection of AB.

10. Points A (–6, 10), B (–4, 6) and C (3, –8) are collinear such that AB = $\frac{2}{9}$AC.

Ans. True.

It is given that points A, B and C are collinear. This means area of ΔABC = 0.

Here, A (x_1, y_1) = (–6, 10), B (x_2, y_2) = (–4, 6), C (x_3, y_3) = (3, –8)

Area of triangles, Δ

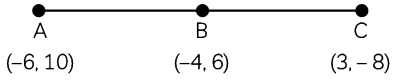
$$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$= \left|\frac{1}{2}[-6(6 - (-8) + (-4)(8 - 10) + 3(10 - 6)]\right|$$

$$= \left|\frac{1}{2}[-6(14) + 72 + 3(4)]\right|$$

$$= \left|\frac{1}{2}[-84 + 72 + 12]\right| = 0$$

This shows that the given points are collinear.



Distance between A (–6, 10) and B (–4, 6),

$$AB = \sqrt{(-4-(-6))^2 + (6-10)^2}$$

$$= \sqrt{(-4+6)^2 + (-4)^2} = \sqrt{2^2 + 4^2}$$

$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \quad \text{...(i)}$$

Distance between A (–6, 10) and C (3, –8),

$$AC = \sqrt{(3-(-6))^2 + (-8-10)^2}$$

$$= \sqrt{(3+6)^2 + (-18)^2} = \sqrt{9^2 + 18^2} = \sqrt{81+324}$$

$$= \sqrt{405} = \sqrt{81 \times 5} = 9\sqrt{5} \quad \text{...(ii)}$$

From eqn (i) and (ii),

$$AB = \frac{2}{9}AC$$

which is the required relation.

11. Point P (–2, 4) lies on a circle of radius 6 and centre C (3, 5). [CBSE 2014, 13]

Ans. False.

We know that if the distance between the centre and point P is equal to the radius, then the point lies on the circle. Distance between centre C (3, 5) and point P (–2, 4)

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$PC = \sqrt{(-2-3)^2 + (4-5)^2} = \sqrt{(-5)^2 + (-1)^2}$$

$$= \sqrt{25+1} = \sqrt{26}$$

Clearly, PC ≠ radius (6)

Hence, point P (–2, 4) does not lie on the circle with centre C (3, 5) and radius 6.

12. The points A (–1, –2), B (4, 3), C (2, 5) and D (–3, 0) in that order from a rectangle. [CBSE 2013]

Ans. True.

We know that opposite side of a rectangle are equal and also its diagonals are equal and bisect each other.

A (–1, –2), B (4, 3), C (2, 5) and D (–3, 0)

Distance between A (–1, –2) and B (4, 3)

AB =

$$\sqrt{(4-(-1))^2 + (3-(-2))^2} = \sqrt{(4+1)^2 + (3+2)^2}$$

$$= \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

Distance between B (4, 3) and C (2, 5)

$$BC = \sqrt{(2-4)^2 + (5-3)^2} = \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4} = 2\sqrt{2}$$

Distance between C (2, 5) and D (–3, 0)

$CD = \sqrt{(-3-2)^2+(0-5)^2} = \sqrt{(-5)^2+(-5)^2}$

$= \sqrt{5^2+5^2} = 5\sqrt{2}$

Distance between A (–1, –2) and D (–3, 0)

$AD = \sqrt{(-3+1)^2+(0+2)^2} = \sqrt{(-2)^2+2^2}$

$= \sqrt{4+4} = 2\sqrt{2}$

Distance between A (–1, –2) and C (2, 5)

$AC = \sqrt{(2-(-1))^2+(5-(-2))^2} = \sqrt{(2+1)^2+(5+2)^2}$

$= \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$

Distance between B (4, 3) and D (–3, 0)

$BD = \sqrt{(-3-4)^2+(0-3)^2} = \sqrt{(-7)^2+(-3)^2}$

$= \sqrt{49+9} = \sqrt{58}$

Clearly, AB = CD, AD = BC and AC = BD

i.e., opposite sides are equal and diagonals are also equal..

Hence, points A (–1, –2), B (4, 3), C (2, 5) and D (–3, 0) form a rectangle.

EXERCISE 7.3

1. Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).

Ans.

To find the type of triangle, first we determine the length of all three sides.

Using distance formula,

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Given points are

A (–5, 6), B (–4, –2) and C (7, 5)

$AB = \sqrt{(-4+5)^2+(-2-6)^2} = \sqrt{1^2+(-8)^2}$

$= \sqrt{1+64} = \sqrt{65}$

$BC = \sqrt{(7-(-4))^2+(5-(-2))^2}$

$= \sqrt{(7+4)^2+(5+2)^2}$

$= \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

$AC = \sqrt{(7+5)^2+(5-6)^2} = \sqrt{12^2+1^2}$

$= \sqrt{144+1} = \sqrt{145}$

We see that $AB \neq BC \neq AC$.

Also, $BC^2 \neq AB^2 + AC^2$.

Hence, it is not a right angled triangle.

Hence, the required triangle is scalene because all its sides are of different lengths.

Trick Applied

- *Step 1. Determine AB, BC and AC using distance formula*
- *Step 2. Check the conditions for type of triangle:*
  - *If two sides equal, then, isosceles triangle.*
  - *If sides satisfy Pythagoras theorem then right angled triangle.*
  - *If all sides equal, then equilateral triangle.*
  - *If none of the sides are equal, then scalene triangle.*

2. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there?

Ans.

We know that any point on x-axis is of the form (x, 0).

Let P (x, 0) be the point on x-axis having $2\sqrt{5}$ distance from the point Q (7, –4).

Distance between P (x, 0) and Q (7, –4) using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ = \sqrt{(7-x)^2+(-4-0)^2} = \sqrt{(7-x)^2+16}$

By given conditions

$PQ = 2\sqrt{5}$

$\Rightarrow (PQ)^2 = (2\sqrt{5})^2$

$\Rightarrow (7-x)^2 + 4^2 = (2\sqrt{5})^2$

$\Rightarrow 49 + x^2 - 14x + 16 = 20$

$\Rightarrow x^2 - 14x + 45 = 0$

$\Rightarrow x^2 - 9x - 5x + 45 = 0$

[using factorisation method]

$\Rightarrow x(x-9) - 5(x-9) = 0$

$\Rightarrow (x-9)(x-5) = 0$

$\Rightarrow x = 9, 5.$

Hence, there are two points that lie on x-axis, which are (5, 0) and (9, 0), having a distance of $2\sqrt{5}$ from the point (7, –4).

3. What type of a quadrilateral do the points A (2, –2), B (7, 3), C (11, –1) and D (6, –6), taken in that order, form?

Ans.

To find the type of quadrilateral, we will find the length of all four sides and the length of diagonals.

We know that distance between points (x_1, y_1) and (x_2, y_2)

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Here, A (2, –2), B (7, 3), C (11, –1) and D (6, –6)

$$AB = \sqrt{(7-2)^2+(3+2)^2} = \sqrt{5^2+5^2}$$
$$= \sqrt{25+25} = 5\sqrt{2}$$
$$BC = \sqrt{(11-7)^2+(-1-3)^2} = \sqrt{4^2+(-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$
$$CD = \sqrt{(6-11)^2+(-6+1)^2} = \sqrt{5^2+(-5)^2}$$
$$= \sqrt{25+25} = 5\sqrt{2}$$
$$AD = \sqrt{(6-2)^2+(-6+2)^2} = \sqrt{4^2+(-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Diogonal, AC $= \sqrt{(11-2)^2+(-1+2)^2} = \sqrt{9^2+1^2}$

$= \sqrt{81+1} = \sqrt{82}$

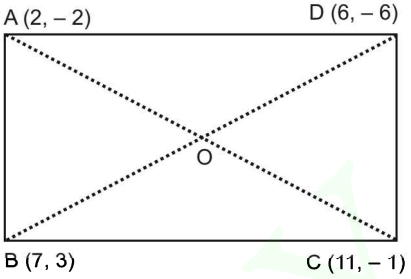
Diogonal BD $= \sqrt{(6-7)^2+(-6-3)^2} = \sqrt{1^2+9^2}$

$= \sqrt{1+81} = \sqrt{82}$

Here, we see that length of opposite sides

$AB = DC$

$AD = BC$



Also, length of diagonals

$AC = BD$

This shows that the given quadrilateral is a rectangle.

Trick Applied

- *Step 1. Determine AB, BC, CD and AC using distance formula*
- *Step 2. Check the conditions for type of quadrilateral*
  - *If all sides are equal and diagonals also equal then square.*
  - *If all sides are equal but diagonals not equal then rhombus.*
  - *If opposite sides are equal and diagonals also equal then rectangle.*

4. Find the value of *a* if the distance between the points A (–3, –14) and B (*a*, –5) is 9 units. [CBSE 2017, 11]

Ans.

It is given that distance between the points A (–3, –14) and B (a, –5) is 9 units.

We know that distance between 2 points (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

According to the question,

$AB = 9$

$$\sqrt{(a-3)^2+(-5+14)^2} = (9)$$

Squaring both sides, we get

$(a + 3)^2 + (9)^2 = (9)^2$

$\Rightarrow (a + 3)^2 = 0 \Rightarrow a = -3$

Hence, the required value of a is –3.

5. Find a point which is equidistant from the points A (–5, 4) and B (–1, 6)? How many such points are there?

Ans.

Let P (r, s) be the point which is equidistant from points A (–5, 4) and B (–1, 6)

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\therefore PA = PB \Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (-5 - r)^2 + (4 - s)^2 = (-1 - r)^2 + (6 - s)^2$

$\Rightarrow 25 + r^2 + 10r + 16 + s^2 - 8s$

$= 1 + r^2 + 2r + 36 + s^2 - 12s$

$\Rightarrow 25 + 10r + 16 - 8s = 1 + 2r + 36 - 12s$

$\Rightarrow 8r + 4s + 4 = 0$

$\Rightarrow 2r + s + 1 = 0$ ...(i)

Midpoint of AB $= \left(\frac{-5-1}{2}, \frac{4+6}{2}\right) = (-3, 5)$

At point (–3, 5) from eqn (i), we get

$\Rightarrow 2r + s = 2(-3) + 5 = -6 + 5 = -1$

$\Rightarrow 2r + s + 1 = 0$

Hence, midpoint of AB satisfies eqn (i). This implies that there are infinite number of points which satisfy eqn (i) are equidistant from points A and B.

Replacing r, s with x and y in the above eqn. we get

$2x + y + 1 = 0$

6. Find the coordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line segment joining the points A (–5, –2) and B (4, –2). Name the type of triangle formed by the points Q, A and B.

Ans.

Let Q $(x, 0)$ be the point on the x-axis which lies on the perpendicular bisector of AB

$\therefore QA = QB \Rightarrow (QA)^2 = (QB)^2$

$\Rightarrow (-5 - x)^2 + (-2 - 0)^2 = (4 - x)^2 + (-2 - 0)^2$

$\Rightarrow 25 + x^2 + 10x + 4 = 16 + x^2 - 8x + 4$

$\Rightarrow 10x + 8x = 16 - 25$

$\Rightarrow 18x = -9 \Rightarrow x = \frac{-1}{2}$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

Now $QA^2 = \left(-5+\frac{1}{2}\right)^2 + (-2-0)^2 = \left(\frac{-9}{2}\right)^2 + \frac{4}{1}$

$\Rightarrow \quad QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow \quad QA = \frac{\sqrt{97}}{2}$ units

Now $QB^2 = \left(4+\frac{1}{2}\right)^2 + (-2-0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$

$\Rightarrow \quad QB^2 = \frac{81}{4} + 4 = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow \quad QB = \frac{\sqrt{97}}{2}$ units

$AB = \sqrt{(4+5)^2 + (-2+2)^2} = \sqrt{9^2} = 9$ units

$\Rightarrow \quad AB = 9$ units and $QA = QB = \frac{\sqrt{97}}{2}$ units

Hence, ΔQAB is an isosceles Δ.

7. Find the value of m if the points (5, 1), (–2, –3) and (8, 2 m) are collinear. **[CBSE 2019, 15, 14, 10]**

Ans.

Let the given points be represented as A (x_1, y_1) = (5, 1), B (x_2, y_2) = (–2, –3) and C (x_3, y_3) = (8, 2 m)

Since, the points A, B and C are collinear.

$\Rightarrow$ Area of $\Delta ABC = 0$

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}[5(-3 - 2m) + (-2)(2m - 1) + 8(1 - (-3)] = 0$

$\Rightarrow \frac{1}{2}[-15 - 10m - 4m + 2 + 32] = 0$

$\Rightarrow \frac{1}{2}[-14m + 19] = 0$

$\Rightarrow \quad -14m + 19 = 0$

$\Rightarrow \quad m = \frac{19}{14}$

Hence, the required value of m is $\frac{19}{14}$.

Trick Applied

Collinearity Condition:
If three points A, B and C are collinear and 3 lies between A and C, then,

- *AB + BC = AC, AB, BC and AC can be calculated using the distance formula.*
- *The ratio in which B divides AC, calculated using section formula for both the x and y coordinates separately will be equal.*
- *Area of a triangle formed by the three points is zero.*

8. If the point A (2, –4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ. **[CBSE 2014, 12, 11, 10]**

Ans.

It is given that A (2, –4) is equidistant from P (3, 8) and Q (–10, y).

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow$ Distance between P (3, 8), A (2, –4) = Distance between A (2, –4) and Q (–10, y)

$\Rightarrow \sqrt{(2-3)^2 + (-4-8)^2} = \sqrt{(-10-2)^2 + (y+4)^2}$

$\Rightarrow \quad \sqrt{1^2 + 12^2} = \sqrt{12^2 + (y+4)^2}$

Squaring both sides, we get

$1^2 + 12^2 = 12^2 + (y + 4)^2$

$\Rightarrow \quad 1 + 144 = 144 + y^2 + 16 + 8y$

$\Rightarrow \quad y^2 + 8y + 15 = 0$

$\Rightarrow \quad y^2 + 5y + 3y + 15 = 0$

$\Rightarrow \quad y(y + 5) + 3(y + 5) = 0$

$\Rightarrow \quad (y + 5)(y + 3) = 0$

If $y + 5 = 0$ then $y = -5$

$y + 3 = 0$ then $y = -3$

Distance between P (3, 8) and Q (–10, y) when $y = -3$

$PQ = \sqrt{(-10-3)^3 + (-3-8)^2}$

$= \sqrt{(-13)^2 + (-11)^2} = \sqrt{169+121} = \sqrt{290}$

Distance between P (3, 8) and Q (–10, y) when $y = -5$

$= \sqrt{(-10-3)^2 + (-5-8)^2} = \sqrt{(-13)^2 + (-13)^2}$

$= \sqrt{169+169} = 2\sqrt{13}$

Hence, the values of y are –3 and –5 and the corresponding values of PQ are $\sqrt{290}$ and $13\sqrt{2}$.

9. In what ratio does the x-axis divide the line segment joining the points (–4, –6) and (–1, 7)? Find the coordinates of the point of division. **[CBSE 2020, 19, 12]**

Ans.

Let P be the point that divides the points (–4, –6) and (–1, 7) in the ratio m:1.

Coordinates of point P will be given by

$$\left(\frac{m(-1)+1(-4)}{m+1}, \frac{m(7)+1(-6)}{m+1}\right)$$

$$= \left(\frac{-m-4}{m+1}, \frac{7m-6}{m+1}\right) \quad \text{...(i)}$$

But according to the question, the line segment is divided by x-axis.

$\Rightarrow$ y coordinate of point P = 0

$\Rightarrow \frac{7m-6}{m+1} = 0 \Rightarrow 7m - 6 = 0 \Rightarrow m = \frac{6}{7}$

Hence, the required ratio is 6:7.

Coordinates of point P = $\left(\frac{-m-4}{m+1}, \frac{7m-6}{m+1}\right)$ [using eqn. (i)]

But $m = 6 : 7$

Also y coordinate = 0

$\Rightarrow$ coordinate of point P = $\left(\frac{\frac{-6}{7}-4}{+\frac{6}{7}+1}, 0\right)$

$= \left(\frac{\frac{-6-28}{7}}{+\frac{6}{7}+1}, 0\right) = \left(\frac{-34}{13}, 0\right)$

Trick Applied

Finding ratio given the points:
To find the ratio in which a given point P (x, y) divides the line segment joining A (x_1, y_1) and B (x_2, y_2),

- *Assume that the ratio is k:1*
- *Substitute the ratio in the section formula for any of the coordinates to get the value of k.*

$x = \frac{kx_2 + x_1}{k+1}$

Since, x_1, x_2 and x are known, k can be calculated. The same can be calculated from the y-coordinates also.

10. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B (2, –5). [CBSE 2015]

Ans.

Let the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divide the line segment joining points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B (2, –5) in the ratio $m:n$.

By section formula, coordinates of point P are given as:

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right) = \left(\frac{2m+\frac{n}{2}}{m+n}, \frac{-5m+\frac{3}{2}n}{m+n}\right)$

$\Rightarrow \left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{2m+\frac{n}{2}}{m+n}, \frac{-5m+\frac{3n}{2}}{m+n}\right)$

On equating the coordinates, we get:

$\frac{3}{4} = \frac{2m+\frac{n}{2}}{m+n}$ and $\frac{5}{12} = \frac{-5m+\frac{3n}{2}}{m+n}$

$\Rightarrow \frac{3}{4} = \frac{4m+n}{2(m+n)}$ and $\frac{5}{12} = \frac{-10m+3n}{2(m+n)}$

$\Rightarrow 3(m + n) = 2(4m + n)$ and $5(m + n) = 6(-10m + 3n)$

$\Rightarrow 3m + 3n = 8m + 2n$ and $5m + 5n = -60m + 18n$

$\Rightarrow 5m - n = 0$ and $65m - 13n = 0$

$\Rightarrow 5m = n$ and $13(5m - n) = 0$

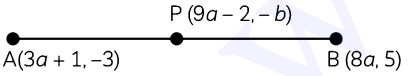
$\Rightarrow \frac{m}{n} = \frac{1}{5}$ and $\frac{m}{n} = \frac{1}{5}$

Hence, the required ratio is 1:5.

11. If P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3:1, find the values of a and b. [CBSE 2010]

Ans.

It is given that P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3:1.



By section formula, the coordinates of P are given as:

$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\frac{3(8a)+1(3a+1)}{3+1}, \frac{3(5)+1(-3)}{3+1}\right)$

$\Rightarrow (9a - 2, -b) = \left(\frac{24a+3a+1}{4}, \frac{15-3}{4}\right)$

$\Rightarrow 9a - 2 = \frac{27a+1}{4}$ and $-b = \frac{15-3}{4}$

$\Rightarrow 36a - 8 = 27a + 1$ and $-4b = 12$

$\Rightarrow 36a - 27a = 1 + 8$ and $b = \frac{12}{(-4)}$

$\Rightarrow 9a = 9$ and $b = -3$

$\Rightarrow a = 1$ and $b = -3$

Hence, the required values of a and b are 1 and –3.

12. If (a, b) is the midpoint of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, find the value of k and the distance AB. [CBSE 2012]

Ans.

It is given that (a, b) is the midpoint of line segment AB, where A (10, –6) and B (k, 4).

We know that midpoint of line segment joining points (x_1, y_1) and $(x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\Rightarrow (a, b) = \left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$

Equating coordinates on both sides, we get

$a = \frac{10+k}{2}$ and $b = \frac{-6+4}{2}$

$\Rightarrow a = \frac{10+k}{2}$ and $b = \frac{-2}{2} = -1$ ...(i)

Also, it is given that

$a - 2b = 18$ [As $b = -1$]

$\Rightarrow$ $a - 2(-1) = 18$

$\Rightarrow$ $a + 2 = 18 \Rightarrow a = 16$

Putting the value of a in eqn (i), we get

$$a = \frac{10+k}{2} = 16$$

$\Rightarrow$ $10 + k = 32 \Rightarrow k = 22$ ...(ii)

Hence, the required value of k is 22.

Distance between A (10, –6) and B (k, 4)

$= \sqrt{(k-10)^2+(4-(-6))^2}$

$= \sqrt{(22-10)^2+(4+6)^2}$ [From eqn (ii), $k = 22$]

$= \sqrt{(12)^2+(10)^2} = \sqrt{144+100} = \sqrt{244} = 2\sqrt{61}$

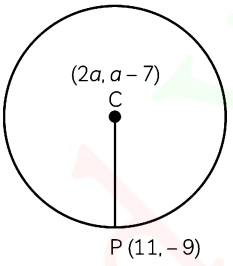
Hence, the required value of k is 22 and the required distance AB = $2\sqrt{61}$.

13. The centre of a circle is C ($2a$, $a - 7$). Find the values of a if the circle passes through the point P (11, –9) and has diameter $10\sqrt{2}$ units. [CBSE 2019, 14]

Ans.

It is given that C ($2a$, $a - 7$) is the centre of the circle and point P (11, –9) lies on the circle,

Diameter = $10\sqrt{2}$ units.



Diameter = $10\sqrt{2}$ [Given]

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

We know that distance between center C and point P will give us the radius of the circle.

$\therefore$ Radius of circle = Distance between P (11, –9) and C ($2a$, $a - 7$)

$\Rightarrow$ $5\sqrt{2} = \sqrt{(2a-11)^2+(a-7+9)^2}$

$\Rightarrow$ $5\sqrt{2} = \sqrt{(2a-11)^2+(a+2)^2}$

Squaring both sides, we get

$(5\sqrt{2})^2 = (2a - 11)^2 + (a + 2)^2$

$50 = 4a^2 + 121 - 44a + a^2 + 4 + 4a$

$50 = 5a^2 - 40a + 125$

$a^2 - 8a + 15 = 0$ [By factorisation method]

$a^2 - 5a - 3a + 15 = 0$

$A(a - 5) - 3(a - 5) = 0$

$(a - 5)(a - 3) = 0$

$\Rightarrow$ $a = 5, 3$

Hence, the required value of a = 3, 5.

14. The line segment joining points A (3, 2) and B (5, 1) is divided at point P in the ratio 1:2 and it lies on the line $3x - 18y + k = 0$. Find the value of k. [CBSE 2012, 10]

Ans.

It is given that line segment joining the points A (3, 2) and B (5, 1) is divided at point P in the ratio 1:2.

$\therefore$ Coordinates of points by section formula

$$= \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Coordinates of point P $= \left\{\frac{1(5)+2(3)}{1+2}, \frac{1(1)+2(2)}{1+2}\right\}$

$= \left(\frac{5+6}{3}, \frac{1+4}{3}\right) = \left(\frac{11}{3}, \frac{5}{3}\right)$

Also, it is given that P lies on line

$3x - 18y + k = 0$

$\Rightarrow$ $3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$

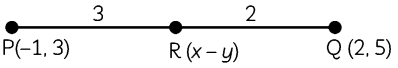
$\Rightarrow$ $11 - 30 + k = 0 \Rightarrow -19 + k = 0$

$\Rightarrow$ $k = 19$

Hence, the required value of k is 19.

15. Find the coordinates of point R on the line segment joining points P (–1, 3) and Q (2, 5) such that PR = $\frac{3}{5}$ PQ. [CBSE 2015, 12]

Ans.



It is given that $\frac{PR}{PQ} = \frac{3}{5} \Rightarrow \frac{PQ}{PR} = \frac{5}{3}$

$\Rightarrow$ $\frac{PR+RQ}{PR} = \frac{5}{3} \Rightarrow \frac{PR}{PR} + \frac{RQ}{PR} = \frac{5}{3}$

$\Rightarrow$ $\frac{PQ}{PR} = \frac{5}{3} - 1 \Rightarrow \frac{PQ}{PR} = \frac{5-3}{3} = \frac{2}{3}$

$\Rightarrow$ RQ:PR = 2:3 $\Rightarrow$ PR:RQ = 3:2

Let R (x, y) be the point which divides the line segment joining points P (–1, 3) and Q (2, 5) in the ratio 3:2.

By internal section formula

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$(x, y) = \left(\frac{3(2)+2(-1)}{3+2}, \frac{3(5)+2(3)}{3+2}\right)$$

$$= \left(\frac{6-2}{5}, \frac{15+6}{5}\right) = \left(\frac{4}{5}, \frac{21}{5}\right)$$

Hence, the required coordinates of point R are $\left(\frac{4}{5}, \frac{21}{5}\right)$.

16. Find the values of k if points A $(k + 1, 2k)$, B $(3k, 2k + 3)$ and C $(5k - 1, 5k)$ are collinear. [CBSE 2017, 15, 14]

Ans.

We know that if three points are collinear, then the area formed by these points is zero.

Since, points A, B and C are collinear.

$\therefore$ Area of ΔABC, $\Delta = 0$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0 \quad ...(i)$$

Here, A $(x_1, y_1) = (k + 1, 2k)$, B $(x_2, y_2) = (3k, 2k + 3)$ and C $(x_3, y_3) = (5k - 1, 5k)$

Putting theses value in eqn (i), we get

$$\frac{1}{2}[(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - 2k - 3)] = 0$$

$$\Rightarrow \frac{1}{2}[(k + 1)(-3k + 3) + 3k(3k) + (5k - 1)(-3)] = 0$$

$$\Rightarrow \frac{1}{2}[-3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3] = 0$$

$$\Rightarrow \frac{1}{2}[6k^2 - 15k + 6] = 0$$

$$\Rightarrow 6k^2 - 15k + 6 = 0$$

$$\Rightarrow 2k^2 - 5k + 2 = 0$$

$$2k^2 - 5k - 2 = 0$$

$$\Rightarrow 2k^2 - 4k - k + 2 = 0$$

$$\Rightarrow 2k(k - 2) - 1(k - 2) = 0$$

$$\Rightarrow (k - 2)(2k - 1) = 0$$

$$\Rightarrow k = 2, \frac{1}{2}$$

Hence, the required values of k are 2 and $\frac{1}{2}$.

17. Find the ratio in which line $2x + 3y - 5 = 0$ divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division. [CBSE 2010]

Ans.

Let the line $2x + 3y - 5 = 0$ divide the line segment joining the points A (8, –9) and B (2, 1) in the ratio m:1 at point P.

$\therefore$ Coordinates of P, using section formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{2m + 8}{m + 1}, \frac{m - 9}{m + 1}\right)$$

It is given that P lies on $2x + 3y - 5 = 0$

$$\Rightarrow 2\left(\frac{2m + 8}{m + 1}\right) + 3\left(\frac{m - 9}{m + 1}\right) - 5 = 0$$

$$\Rightarrow 2(2m + 8) + 3(m - 9) - 5(m + 1) = 0$$

$$\Rightarrow 4m + 16 + 3m - 27 - 5m - 5 = 0$$

$$\Rightarrow 2m - 16 = 0$$

$$\Rightarrow m = \frac{16}{2} = 8$$

$$\Rightarrow m:1 = 8:1$$

$\therefore$ Coordinates of point P $= \left(\frac{2m + 8}{m + 1}, \frac{m - 9}{m + 1}\right)$

$$= \left(\frac{16 + 8}{9}, \frac{8 - 9}{9}\right) = \left(\frac{24}{9}, \frac{-1}{9}\right)$$

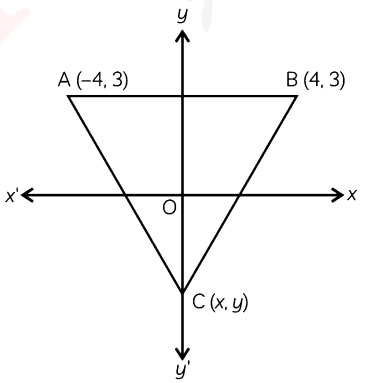
$$= \left(\frac{8}{3}, \frac{-1}{9}\right)$$

Hence, the required point of division is $\left(\frac{8}{3}, \frac{-1}{9}\right)$.

EXERCISE 7.4

1. If (–4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle. [CBSE 2011]

Ans.



Let the third vertex of the equilateral triangle be (x, y).

The vertices of the triangles will be

A (–4, 3), B (4, 3) and C (x, y)

We know that in an equilateral triangle

$$AB = BC = AC$$

$$\Rightarrow AB^2 = BC^2 = AC^2$$

Taking the first two parts

$$AB^2 = BC^2$$

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\Rightarrow (4 + 4)^2 + (3 - 3)^2 = (x - 4)^2 + (y - 3)^2$$

$$\Rightarrow 8^2 + 0^2 = x^2 + 16 - 8x + y^2 + 9 - 6y$$

$$\Rightarrow 64 = x^2 - 8x + y^2 - 6y + 25$$

$\Rightarrow x^2 + y^2 - 8x - 6y - 39 = 0$ ...(i)

Taking the first and third parts

$AB^2 = AC^2$

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow (4 + 4)^2 + (3 - 3)^2 = (x + 4)^2 + (y - 3)^2$

$\Rightarrow 8^2 + 0^2 = x^2 + 16 + 8x + y^2 + 9 - 6y$

$\Rightarrow x^2 + y^2 + 8x - 6y - 39 = 0$ ...(ii)

Subtracting eqn (i) from eqn (ii), we get

$(x^2 + y^2 + 8x - 6y - 39) - (x^2 + y^2 - 8x - 6y - 39) = 0$

$\Rightarrow 16x = 0 \Rightarrow x = 0$

Putting the value of x in eq^n (i), we get

$\Rightarrow 0 + y^2 - 8(0) - 6y - 39 = 0$

$\Rightarrow y^2 - 6y - 39 = 0$ ...(iii)

We know that solution of eq^n $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution of eqn (iii) will be

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-39)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 + 156}}{2}$$

$$\Rightarrow y = \frac{6\sqrt{192}}{2} = \frac{62\sqrt{48}}{2} = 3\sqrt{48}$$

$\Rightarrow y = 3 \pm 4\sqrt{3}$

$\Rightarrow y = 3 + 4\sqrt{3}$ or $y = 3 - 4\sqrt{3}$

It is given that origin lies in the interior of the triangles and the x-coordinate of the third vertex is zero.

The value of y should be negative.

$\therefore$ Third vertex, C $(x, y) = (0, 3 - 4\sqrt{3})$

2. Points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of point D. [CBSE 2010]

(ii) Find the coordinates of the point P on AD such that AP:PD = 2:1. [CBSE 2010]

(iii) Find the coordinates of points Q and R on medians BE and CF respectively, such that BQ:QE = 2:1 and CR:RF = 2:1.

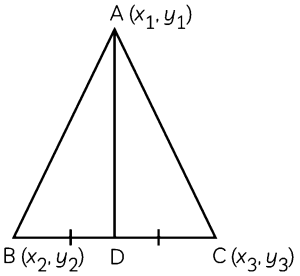
(iv) What are the coordinates of the centroid of the triangles ABC?

Ans.

It is given that A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of ΔABC.

(i) We know that median bisects the apposite side into two equal parts i.e., D is the midpoint of BC

$\Rightarrow$ BD = DC

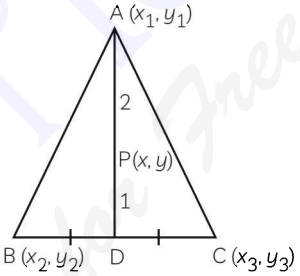


Coordinates of midpoint of BC i.e. D

$$= \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

(ii) Let the coordinates of point P be (x, y)



It is given that P (x, y) divides the line joining A (x_1, y_1) and D $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$ in the ratio 2 : 1.

By internal section formula, coordinates of point P

$$= \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

P (x, y)

$$= \left(\frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2 + 1}, \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2 + 1}\right)$$

$$= \left(\frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2 + 1}, \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2 + 1}\right)$$

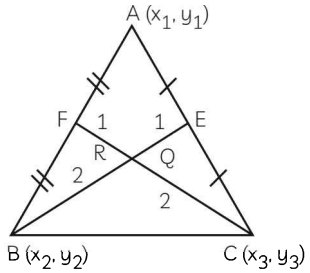
Hence, the coordinates of point P are

$$\left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{3}\right)$$

(iii) It is given that Q and R are points an median BE and CF such that.

BQ:QE = 2:1 and CR:RF = 2:1

Let the coordinates of point Q be (p, q)



BE is the median of AC

$\Rightarrow$ BE divides AC into two equal parts

$\therefore$ Midpoint of AC = Coordinate of E

$$\Rightarrow \quad E = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$$

$\Rightarrow$ Coordinate of point Q as BQ:QE = 2:1

$$= \left[\frac{2\left(\frac{x_1+x_3}{2}\right)+1(x_2)}{2+1}, \frac{2\left(\frac{y_1+y_3}{2}\right)+1(y_2)}{2+1}\right]$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_3+y_2}{3}\right)$$

Hence, the required coordinates of point Q

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_3+y_2}{3}\right)$$

Now, let coordinates of point E be (s, t).

Given that, point R (s, t) divides the line joining C (x_3, y_3) and F in the ratio (2:1).

Coordinates of point F $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

[CF is the median, F being the midpoint of AB]

$\therefore$ Coordinates of point R (s, t)

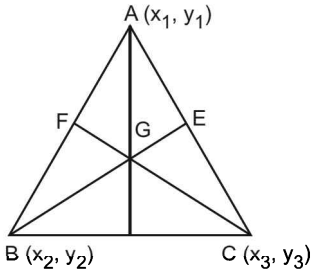
$$= \left[\frac{2\left(\frac{x_1+x_2}{2}\right)+1(x_3)}{2+1}, \frac{2\left(\frac{y_1+y_2}{2}\right)+1(y_3)}{2+1}\right]$$

$$= \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$$

Hence, the required coordinates of point R

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

(iv) Coordinates of centroid of ΔABC



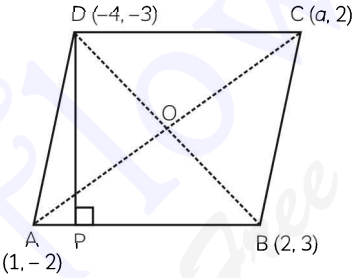
$$= \left(\frac{\text{sum of abscissa of all vertices}}{3}, \frac{\text{sum of ordinate of all vertices}}{3}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

3. If the points A (1, –2), B (2, 3) C (a, 2) and D (–4, –3) form a parallelogram, find the values of a and height of the parallelogram taking AB as base. [CBSE 2012]

Ans.

Given points A (1, –2), B (2, 3), C (a, 2), D (–4, –3) form a parallelogram.



We know that, diagonals of a parallelogram bisect each other.

i.e., midpoint of AC = midpoint of BD.

We know that midpoint of line segment having points (x_1, y_1) and $(x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\Rightarrow$ Midpoint of AC = Midpoint of BD

$$\Rightarrow \left(\frac{1+a}{2}, \frac{-2+2}{2}\right) = \left(\frac{2-4}{2}, \frac{3-3}{2}\right)$$

$$\Rightarrow \left(\frac{1+a}{2}, 0\right) = \left(\frac{-2}{2}, 0\right)$$

$$\Rightarrow \frac{1+a}{2} = \left(\frac{-2}{2}, 0\right)$$

$$\Rightarrow \frac{1+a}{2} = \frac{-2}{2} \Rightarrow 1 + a = -2$$

$$\Rightarrow a = -3$$

Hence, the required value of $a = -3$.

Given that, AB is the base of the parallelogram and a perpendicular is drawn from D to AB which meets AB at P. So, DP is height.

Area of ΔABC

$$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$= \left|\frac{1}{2}[1(3 - 2) + 2(2 + 2) + (-3)(-2 - 3)]\right|$$

$$= \left|\frac{1}{2}[1 + 8 + 15]\right| = \frac{24}{2} = 12 \text{ sq. units.}$$

Also, $\Delta ABC \cong \Delta ADC$ [SAS congruence criterion]

$\Rightarrow$ar (ΔABC) = aR (ΔADC) = 12 sq. units

$\Rightarrow$ Area of parallelogram = ar (ΔABC) + ar (ΔADC)

$\Rightarrow$ AB × DP = 12 + 12 ...(i)

$$AB = \sqrt{(1-2)^2+(-2-3)^2}$$

$$= \sqrt{(-1)^2+(-5)^2} = \sqrt{1+25}$$

$$= \sqrt{26}$$

Putting value of AB in eqn (i), we get

$$\Rightarrow \sqrt{26} \times DP = 24$$

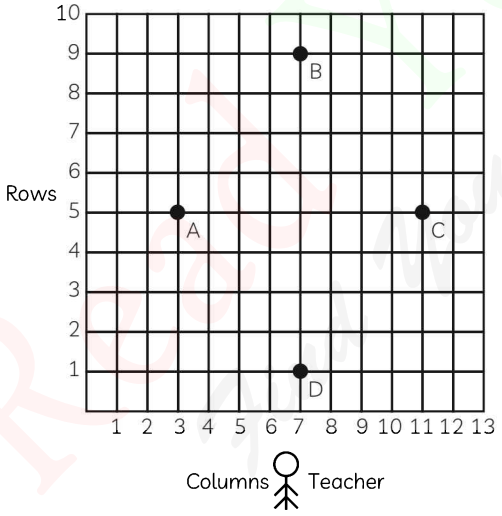
$$\Rightarrow \quad DP = \frac{24}{\sqrt{26}}$$

$$= \frac{24\times\sqrt{26}}{\sqrt{26}\times\sqrt{26}} = \frac{24\sqrt{26}}{26}$$

$$= \frac{12\sqrt{26}}{13} \text{ units}$$

Hence, the required length of the height of the parallelogram is $\frac{12\sqrt{26}}{13}$ units.

4. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in the figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D? If so, what should be his position?



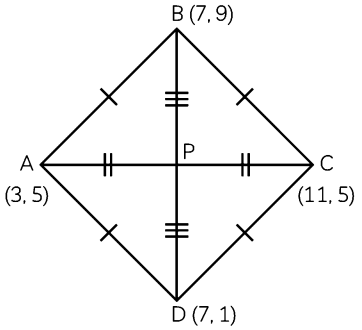
Ans.

Yes , we can place Jaspal in the drill.

By observing the given diagram we get to know the positions of the students A, B, C and D forming a quadrilateral.

So the vertices of this quadrilateral will be as follows A (3, 5), B (7, 9), C (11, 5) and D (7, 1).

To find the type of this quadrilateral , we have to calculate its all sides and also its diagonal.



To find the type of quadrilateral, we will find all its sides

We know that distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$AB = \sqrt{(7-3)^2+(9-5)^2} = \sqrt{4^2+4^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$AB = 4\sqrt{2}$$

$$BC = \sqrt{(11-7)^2+(5-9)^2} = \sqrt{4^2+(-4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

$$CD = \sqrt{(11-7)^2+(5-1)^2} = \sqrt{4^2+4^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

$$AD = \sqrt{(7-3)^2+(1-5)^2} = \sqrt{4^2+(-4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

Now, we will find length of diagonals

$$AC = \sqrt{(11-3)^2+(5-5)^2} = \sqrt{8^2} = 8$$

$$BD = \sqrt{(7-7)^2+(1-9)^2} = \sqrt{(-8)^2} = 8$$

$\Rightarrow$ AB = BC = CD = DA and AC = BD

which represent a square with all its sides equal and diagonals equal.

We know that diagonals of a square bisect each other, so if P should be the position of Jaspal at which he is equidistant from A, B, C and D.

$\therefore$ Coordinates of point P = Midpoint of AC

$$= \left(\frac{3+11}{2}, \frac{5+5}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$$

Hence, the required position of Jaspal is (7, 5).

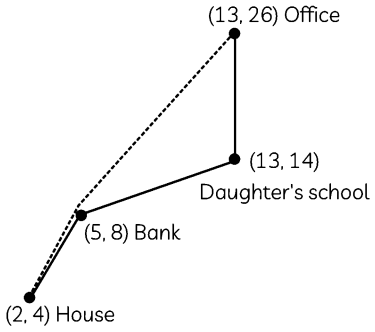
5. Ayush starts walking from his house to his office, Instead of going to the directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is hatisthe extra distance travet

radistance travelled by ed by Ayush in reaching his office (Assume that all uetha-talled distances covered are in straight lines).

The house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at 13 ,26) and coordinates are in km.

Ans.

By given condition, we draw a figure in which every place is indicated with its coordinates.



We know that distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between house and bank

$$= \sqrt{(5-2)^2 + (8-4)^2}$$

$$= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ km}$$

Distance between bank and daughter's school

$$= \sqrt{(13-5)^2 + (14-8)^2} = \sqrt{8^2 + 6^2}$$

$$= \sqrt{64+36} = \sqrt{100} = 10 \text{ km}$$

Distance between daughter's school and office

$$= \sqrt{(13-13)^2 + (26-14)^2} = \sqrt{0+12^2}$$

$$= \sqrt{144} = 12 \text{ km}$$

Total distance travelled = 5 + 10 + 12 = 27 km

Distance between house and office

$$= \sqrt{(13-2)^2 + (26-4)^2} = \sqrt{(11)^2 + (22)^2}$$

$$= \sqrt{121+484}$$

$$= \sqrt{605} = 24.59 \approx 24.6 \text{ km}$$

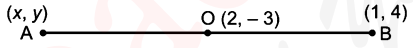
Extra distance travelled = 27 – 24.6 = 2.4 km

Hence, Ayush travels an extra distance of 2.4 km in reaching his office.

1. The coordinate of a point A, where AB is the diameter of a circle whose center is (2, –3) and B (1, 4) are:

Ans.

Let the coordinates of A be (x, y)



Midpoint of AB,

$$\frac{x+1}{2} = 2, \quad \frac{y+4}{2} = -3$$

$$x = 4 - 1 = 3$$

and $$y = -6 - 4 = -10$$

Coordinates of A are (3, –10).

2. Prove that the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear if, $\frac{1}{a} + \frac{1}{b} = 1$.

Ans.

If points are collinear, then area of triangle = 0

i.e., $\frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$$\frac{1}{2}[A(b - 1) + 0 + 1(0 - b)] = 0$$

$$ab - a - b = 0$$

$$a + b = ab$$

Divide both side by ab

$$\frac{a}{ab} + \frac{b}{ab} = \frac{ab}{ab}$$

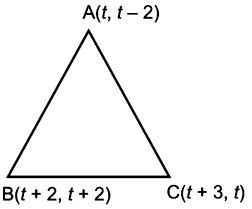
$$\frac{1}{b} + \frac{1}{a} = 1$$

Hence, $$\frac{1}{a} + \frac{1}{b} = 1$$

3. Prove that the area of a triangle with vertices $(t, t - 2)$, $(t + 2, t + 2)$ and $(t + 3, t)$ is independent of t.

Ans.

Let A, B and C are the vertices of a triangle.



$$\text{Area of triangle} = \frac{1}{2}|x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2}|t(t + 2 - t) + (t + 2)(t - t + 2) + (t + 3)(t - 2 - t - 2)|$$

$$= \frac{1}{2}|2t + 2t + 4 - 4t - 12|$$

$$= \frac{1}{2} \times |-8| = 4 \text{ sq units}$$

[area can't be negative]

Hence, area is independent of t.

4. Determine the ratio in which the line $y - x + 2 = 0$ divides the line segment joining the points (3, –1) and (8, 9).

Ans.

Let, line $y - x + 2 = 0$ divides the points (3, –1) and (8, 9) in ratio $k : 1$ at point P.

$\therefore$ x coordinate of the point

$$= \frac{8k+3}{k+1} \qquad \left[\because \frac{mx_2 + nx_1}{m+n}\right]$$

y coordinate of the point

$$= \frac{9k-1}{k+1} \qquad \left[\because \frac{my_2 + ny_1}{m+n}\right]$$

coordinates of the point P are $= \left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$

Also, this point lies on line $y - x + 2 = 0$

$$\therefore \quad \left(\frac{9k-1}{k+1}\right) - \left(\frac{8k+3}{k+1}\right) + 2 = 0$$

$$9k - 1 - 8k - 3 + 2k + 2 = 0$$

$$3k - 2 = 0$$

$$K = \frac{2}{3}$$

Hence, line divides in ratio 2 : 3 internally.

5. The coordinates of houses of Sonu and Labhoo are (7, 3) and (4, 3) respectively. Coordinates of their school is (2, 2). If both leave their house at the same time in the morning and also reach school in same time, then (a) who travel faster.

Ans.

(a) Distance between Sonu's house and school

$$= \sqrt{(2-7)^2 + (2-3)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$= \sqrt{25 + 1} = \sqrt{26}$$

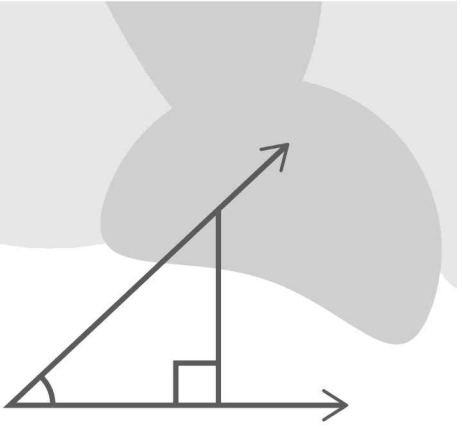
Distance between Labhoo's house and school

$$z = \sqrt{(2-4)^2 + (2-3)^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

So, distance of Sonu's house from school is more. Therefore, Sonu travels faster.

8 Introduction to Trignometry and Its Applications



EXERCISE 8.1

Choose the correct option from the given four options in the following questions:

1. If $\cos A = \frac{4}{5}$, then the value of tan A is:

(A) $\frac{3}{5}$ (B) $\frac{3}{4}$

(C) $\frac{4}{3}$ (D) $\frac{5}{3}$ **[CBSE 2016]**

Ans. (B)

Explanation:

Using the identities $\sin^2 A + \cos^2 A = 1$

and $\tan A = \frac{\sin A}{\cos A}$

Given: $\cos A = \frac{4}{5}$

$\Rightarrow \quad \sin A = \sqrt{1-\cos^2 A}$

$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Now, $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

2. If $\sin A = \frac{1}{2}$, then the value of cot A is:

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2}$ (D) 1 **[CBSE 2016]**

Ans. (A)

Explanation:

Given: $\sin A = \frac{1}{2}$

$\Rightarrow \quad \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Now, $\cot A = \frac{\cos A}{\sin A} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

3. Given that $\sin \theta = \frac{a}{b}$ then $\cos \theta$ is equal to:

(A) $\frac{b}{\sqrt{b^2-a^2}}$ (B) $\frac{b}{a}$

(C) $\frac{\sqrt{b^2-a^2}}{b}$ (D) $\frac{a}{\sqrt{b^2-a^2}}$

[CBSE 2016]

Ans. (C)

Explanation:

Using the identities $\sin^2 \theta + \cos^2 \theta = 1$

Given: $\sin \theta = \frac{a}{b}$

$\cos \theta = \sqrt{1-\sin^2 \theta} = \sqrt{1-\frac{a^2}{b^2}}$

$= \sqrt{\frac{b^2-a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$

4. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to:

(A) $\cos \beta$ (B) $\cos 2\beta$

(C) $\sin \alpha$ (D) $\sin 2\alpha$

Ans. (B)

Explanation:

Given: We know that $\cos 90° = 0$

$\cos(\alpha + \beta) = 0 = \cos 90° \quad [\because \cos 90° = 0]$

$\Rightarrow \quad \cos(\alpha + \beta) = \cos 90°$

$\Rightarrow \quad \alpha + \beta = 90° \Rightarrow \alpha = 90° - \beta \quad ...(i)$

$\sin(\alpha - \beta) = \sin(90° - \beta - \beta)$ [Using eqn (i)]

$= \sin(90° - 2\beta) = \cos 2\beta$

5. If ΔABC is right angled at C, then the value of cos (A + B) is:

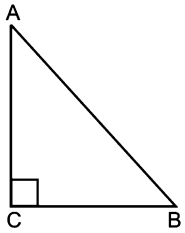
(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ **[CBSE 2017]**

Ans. (A)

Explanation: We know that cos 90° = 0

In ΔABC



sum of 3 angles = 180° [Angle sum property]

$\angle A + \angle B + \angle C = 180$

But $\angle C = 90°$

$\therefore \quad \angle A + \angle B = 90°$

$\Rightarrow \quad \cos(A + B) = \cos 90°$

But $\cos 90° = 0$

$\Rightarrow \quad \cos(A + B) = 0$

6. If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is:

(A) 1 (B) $\frac{1}{2}$

(C) 2 (D) 3

Ans. (A)

Explanation:

We know that $\sin^2\theta + \cos^2\theta = 1$

Given: $\sin A + \sin^2 A = 1$

$\Rightarrow \quad \sin A = 1 - \sin^2 A$

$\Rightarrow \quad \sin A = \cos^2 A \quad [\because \sin^2\theta + \cos^2\theta = 1]$

Squaring both sides

$\Rightarrow \quad \sin^2 A = \cos^4 A$

$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$

$\Rightarrow \cos^2 A + \cos^4 A = 1$

7. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is:

(A) 0° (B) 30°

(C) 60° (D) 90° **[CBSE 2016]**

Ans. (D)

Explanation:

We know that $\sin 30° = \frac{1}{2}$ and $\cos 60° = \frac{1}{2}$

Given: $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$

$\Rightarrow \quad \sin\alpha = \sin 30°$

$\Rightarrow \quad \alpha = 30°$

$\cos\beta = \frac{1}{2}$

$\Rightarrow \quad \cos\beta = \cos 60°$

$\Rightarrow \quad \beta = 60°$

$\therefore \quad \alpha + \beta = 30° + 60° = 90°$

Hence, the value of $(\alpha + \beta) = 90°$

8. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to:

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

[CBSE 2020, 18, 15]

Ans. (C)

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Explanation: $4\tan\theta = 3$

$\Rightarrow \quad \tan\theta = \frac{3}{4}$ ....(i)

$$\Rightarrow \frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} \left[\begin{array}{l}\text{Dividing numerator and}\\ \text{denominator by } \cos\theta\end{array}\right]$$

$$= \frac{4\tan\theta - 1}{4\tan\theta + 1} \quad \left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} \quad \text{[Using eqn. (i)]}$$

$$= \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

9. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is:

(A) 1 (B) $\frac{3}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. (C)

Explanation:

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

Given: $(\sin\theta - \cos\theta) = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1 \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \tan\theta = \tan 45°$ [$\because \tan 45° = 1$]

$\Rightarrow \theta = 45°$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$ $\left[\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Trick Applied

- *Step 1. Calculate the value of θ when tan θ = 1*
- *Step 2. Put the value in the given expression.*

10. **A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is:**

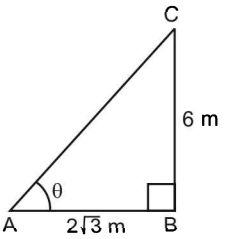
(A) 60° (B) 45°

(C) 30° (D) 90°

[CBSE 2017, 14, 13]

Ans. (A)

Explanation:



Let BC = 6 m be the height of the pole and AB = $2\sqrt{3}$ m be the length of the shadow on the ground.

Let the sun's elevation be θ.

In ΔABC, $\tan\theta = \frac{BC}{AC}$

$\Rightarrow \tan\theta = \frac{6}{2\sqrt{3}} = \sqrt{3}$

Also, $\tan 60° = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60°$

$\Rightarrow \theta = 60°$

Hence, sun's elevation is 60°.

EXERCISE 8.2

Write 'True' or 'False' and justify your answer in each of the following :

1. $\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$

Ans. True.

Explanation: We know that $\sin^2\theta + \cos^2\theta = 1$

and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sqrt{(1-\cos^2\theta)\sec^2\theta} = \sqrt{\sin^2\theta\sec^2\theta}$

[$\because \sin^2\theta + \cos^2\theta = 1$]

$= \sqrt{\sin^2\theta \cdot \frac{1}{\cos^2\theta}} = \sqrt{\tan^2\theta} = \tan\theta$

$\left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

2. **If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.**

Ans. True.

Explanation:

We know that $\sin^2 A + \cos^2 A = 1$

$\cos A + \cos^2 A = 1$ [Given]

$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$

$\Rightarrow \cos A = \sin^2 A$

$\Rightarrow \cos^2 A = \sin^4 A$

$\Rightarrow 1 - \sin^2 A = \sin^4 A$ [$\because \cos^2 A = 1 - \sin^2 A$]

$\Rightarrow \sin^2 A + \sin^4 A = 1$

3. **$(\tan\theta + 2)(2\tan\theta + 1) = 5\tan\theta + \sec^2\theta$.**

Ans. False.

We know that $\sec^2\theta - \tan^2\theta = 1$

Explanation:

$(\tan\theta + 2)(2\tan\theta + 1)$

$= 2\tan^2\theta + \tan\theta + 4\tan\theta + 2$

$= 2(\sec^2\theta - 1) + 5\tan\theta + 2$

[$\because \sec^2\theta - \tan^2\theta = 1$]

$= 2\sec^2\theta - 2 + 5\tan\theta + 2$

$= 2\sec^2\theta + 5\tan\theta$.

4. **If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing.** **[CBSE 2017, 14, 13]**

Ans. False.

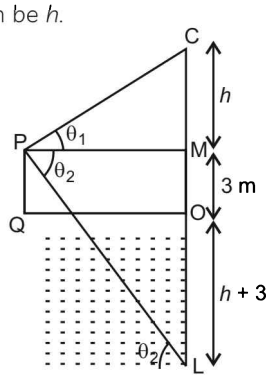
Explanation: We know that if the elevation moves towards the tower, it increases and if its elevation move away from the tower, it decreases. Hence, if the shadow of a tower is increasing, then the angle of the sun is not increasing.

5. **If a man standing on a platform 3 metres above the surface of a lake observes a cloud and its reflection in the lake, then the anlge of elevation of the cloud is equal to the angle of depression of its reflection.** **[CBSE 2010]**

Ans. False.

Explanation: Let a man be standing on a plat-form at point P, 3 m above the surface

of the lake a cloud is observed at point C. Let the height of the cloud from the surface of the platform be h.



In ΔMPC, $\tan\theta_1 = \frac{CM}{PM} = \frac{h}{PM}$

$\Rightarrow \quad PM = \frac{h}{\tan\theta_1}$ ...(i)

In ΔLPM,

$$\tan\theta_2 = \frac{LM}{PM} = \frac{OL+OM}{PM} = \frac{h+6}{PM}$$

$\Rightarrow \quad PM = \frac{h+6}{\tan\theta_2}$ ...(ii)

From eq. (i) and eq. (ii) we get

$$\frac{h}{\tan\theta_1} = \frac{h+6}{\tan\theta_2}$$

$\Rightarrow \quad \tan\theta_2 = \left(\frac{h+6}{h}\right)\tan\theta_1$

Hence $\theta_1 \neq \theta_2$.

6. The value of $2\sin\theta$ can be $a + \frac{1}{a}$, where a is a positive number, and $a \neq 1$.

Ans. False.

Explanation: Let $a = 2$, then

$$a + \frac{1}{a} = 2 + \frac{1}{2} = \frac{5}{2}$$

If $2\sin\theta = a + \frac{1}{a}$, then $2\sin\theta = \frac{5}{2}$

$\Rightarrow \quad \sin\theta = \frac{5}{4} = 1.25$

which is not possible as $\sin\theta \leq 1$.

7. $\cos\theta = \frac{a^2+b^2}{2ab}$, where a and b are two distinct numbers such that $ab > 0$.

Ans. False.

Explanation: We know that

$(a-b)^2 > 0$ [As square of any

$\Rightarrow \quad a^2 + b^2 - 2ab > 0$ number is positive]

$\Rightarrow \quad a^2 + b^2 > 2ab$

$\Rightarrow \quad \frac{a^2+b^2}{2ab} > 1$

But $\cos\theta = \frac{a^2+b^2}{2ab}$

$\Rightarrow \quad \cos\theta > 1$

which is not possible since $-1 \leq \cos\theta \leq 1$.

Hence, $\cos\theta \neq \frac{a^2+b^2}{2ab}$

8. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.

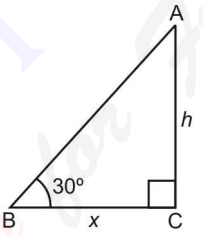
Ans. False.

Explanation: We know that

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

Let AC be the tower with height 'h' and BC = x m

In ΔABC,



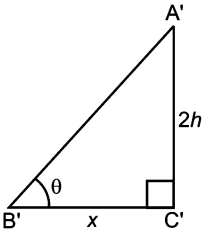
$$\tan 30° = \frac{AC}{BC} = \frac{h}{x}$$

[Here perpendicular, AC = h and base, BC = x]

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$ ...(i)

when height of tower is doubled i.e., A'C' = $2h$

In $\Delta A'B'C'$



$\tan\theta = \frac{A'C'}{B'C'} = \frac{2h}{x}$ [Here, perpendicular A'C' = $2h$ and base B'C' = x]

$\tan\theta = \frac{2}{x} \times h$

$= \frac{2}{x} \times \frac{x}{\sqrt{3}}$ [Using eq^n (i)]

$\tan\theta = \frac{2}{\sqrt{3}}$

But $\tan 60° = \sqrt{3} \neq \frac{2}{\sqrt{3}}$

So $\theta \neq 60°$

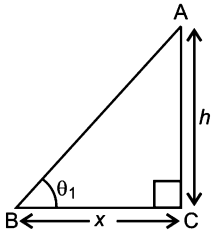
Hence, the required angle is not doubled.

9. If the height of a tower and the distance of the point of observation from its foot, both, are increased by 10%, then the angle of elevation of its top remains unchanged.

Ans. True.

Explanation: Let AC be the tower of height h and the distance of the point of observation from its foot be x.

In ΔABC,



$$\tan\theta_1 = \frac{AC}{BC} = \frac{h}{x} \qquad ...(i)$$

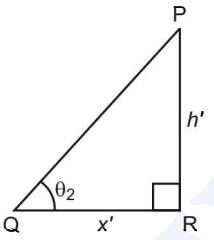
Now, if the height of the tower is increased by 10%, new height $h' = h + 10\%$ of h.

$$= h + \frac{10}{100}h = h + \frac{h}{10} = \frac{11h}{10}$$

Distance of point of observation from its foot is also increased by 10%. New distance x' =

$$x + \frac{10x}{100} = x + \frac{x}{10} = \frac{11x}{10}$$

In ΔPQR,



$$\tan\theta_2 = \frac{PR}{QR} = \frac{h'}{x'} = \frac{\frac{11h}{10}}{\frac{11x}{10}} = \frac{h}{x}$$

$$\Rightarrow \quad \tan\theta_2 = \frac{h}{x} \qquad ...(ii)$$

From eqn (i) and (ii)

$$\tan\theta_1 = \tan\theta_2$$

$$\Rightarrow \quad \theta_1 = \theta_2$$

Hence, the required angle of elevation of its top remains unchanged.

EXERCISE 8.3

Prove the following (from Q.1 to Q.7):

1. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\text{cosec}\,\theta$ **[CBSE 2011]**

Ans. We will use identities $\sin^2\theta + \cos^2\theta = 1$

and $\frac{1}{\sin\theta} = \text{cosec}\,\theta$

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta$$

$$\text{L.H.S.} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta} = \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta \qquad \left[\because \frac{1}{\sin\theta} = \text{cosec}\,\theta\right]$$

= R.H.S. Hence, proved

2. $\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\text{cosec}\,A$

Ans. We will use identities $\sec^2\theta - \tan^2\theta = 1$

and $\sec\theta = \frac{1}{\cos\theta}$

and $\frac{\sin\theta}{\cos\theta} = \tan\theta$

$$\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\text{cosec}\,A$$

$$\text{L.H.S.} = \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A}$$

$$= \tan\left[\frac{1}{1+\sec A} - \frac{1}{1-\sec A}\right]$$

$$= \tan A\left[\frac{1-\sec A - 1 - \sec A}{(1+\sec A)(1-\sec A)}\right] = \frac{-2\tan A.\sec A}{1-\sec^2 A}$$

$$= \frac{2\tan A\sec A}{\sec^2 A - 1} = \frac{2\tan A\sec A}{\tan^2 A} \qquad \left[\begin{array}{l}\because \sec^2 A - 1 \\ = \tan^2 A\end{array}\right]$$

$$= \frac{2\sec A}{\tan A} = 2 \times \frac{1}{\cos A} \times \frac{\cos A}{\sin A} \qquad \left[\begin{array}{l}\because \sec A = \frac{1}{\cos A} \\ \tan A = \frac{\cos A}{\sin A}\end{array}\right]$$

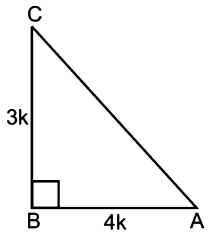
$= \frac{2}{\sin A} = 2\text{cosec } A \qquad \left[\because \text{cosec } A = \frac{1}{\cos A}\right]$

= R.H.S. Hence, proved

3. If $\tan A = \frac{3}{4}$, then $\sin A \cos A = \frac{12}{25}$

Ans.

Given: $\tan A = \frac{3}{4}$



To prove:

$\sin A \cos A = \frac{12}{25}$

Proof: $\tan A = \frac{3}{4}$

We know that, $\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \qquad \tan A = \frac{3}{4} = \frac{P}{B}$

Let P = 3K and B = 4K.

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$= (3K)^2 + (4K)^2$

$= 9K^2 + 16K^2 = 25K^2$

$\Rightarrow \qquad H = 5K$

[As, side cannot be negative]

$\therefore \qquad \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3K}{5K} = \frac{3}{5}$

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4K}{5K} = \frac{4}{5}$

Now, $\sin A \cos A = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$

L.H.S. = R.H.S Hence, proved.

4. $(\sin \alpha + \cos \alpha)(\tan \alpha + \cot \alpha) = \sec \alpha + \text{cosec } \alpha$

Ans.

$(\sin \alpha + \cos \alpha)(\tan \alpha + \cot \alpha) = \sec \alpha + \text{cosec } \alpha$

L.H.S. $(\sin \alpha + \cos \alpha)(\tan \alpha + \cot \alpha)$

$= (\sin\alpha + \cos\alpha)\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$

$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$= (\sin\alpha + \cos\alpha)\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}\right)$

$= (\sin \alpha + \cos \alpha)\left(\frac{1}{\sin\alpha\cos\alpha}\right)$

$[\because \sin^2 \theta + \cos^2 \theta = 1]$

$= \frac{\sin\alpha}{\sin\alpha\cos\alpha} + \frac{\cos\alpha}{\sin\alpha\cos\alpha} = \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha}$

$= \sec \alpha + \text{cosec } \alpha$ = R.H.S. Hence, proved.

5. $(\sqrt{3}+1)(3 - \cot 30°) = \tan^3 60° - 2 \sin 60°$

Ans. $(\sqrt{3}+1)(3 - \cot 30°) = \tan^3 60° - 2\sin 60°$

L.H.S.

$(\sqrt{3}+1)(3 - \cot 30°) = (\sqrt{3}+1)(3 - \sqrt{3})$

$[\because \cot 30° = \sqrt{3}]$

$= (\sqrt{3}+1)\sqrt{3}(\sqrt{3}-1)$

$= \sqrt{3}(\sqrt{3}+1)(\sqrt{3}-1) = \sqrt{3}(3 - 1) = 2\sqrt{3}$

R.H.S.

$\tan^3 60° - 2\sin 60° = (\sqrt{3})^3 - \frac{2 \times \sqrt{3}}{2}$

$= 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$

$\therefore$ L.H.S. = R.H.S. Hence, proved.

6. $1 + \frac{\cot^2 \alpha}{1 + \text{cosec } \alpha} = \text{cosec } \alpha$

Ans.

$1 + \frac{\cot^2 \alpha}{1 + \text{cosec } \alpha} = \text{cosec } \alpha$

L.H.S. $1 + \frac{\cot^2 \alpha}{1 + \text{cosec } \alpha}$

$= \frac{1 + \text{cosec } \alpha + \cot^2 \alpha}{(1 + \text{cosec } \alpha)} = \frac{1 + \cot^2 \alpha + \text{cosec } \alpha}{(1 + \text{cosec } \alpha)}$

$= \frac{\text{cosec}^2 \alpha + \text{cosec } \alpha}{(\text{cosec } \alpha + 1)}$ $[\because 1 + \cot^2\theta = \text{cosec}^2\theta]$

$= \frac{\text{cosec } \alpha (\text{cosec } \alpha + 1)}{(\text{cosec } \alpha + 1)} = \text{cosec } \alpha$ = R.H.S.

Hence, Proved.

7. $\tan \theta + \tan (90° - \theta) = \sec \theta . \sec (90° - \theta)$

Ans. We know that $\tan (90° - \theta) = \cot \theta$ and $\sec (90° - \theta) = \text{cosec } \theta$

$\tan \theta + \tan (90° - \theta) = \sec \theta . \sec (90° - \theta)$

L.H.S. $\tan\theta + \tan(90° - \theta)$

$= \tan\theta + \cot\theta \quad [\because \tan(90° - \theta) = \cot\theta]$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

$= \sec\theta \operatorname{cosec}\theta$

$[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}]$

$= \sec\theta \sec(90° - \theta)$

= R.H.S. Hence, proved.

8. Find the angle of elevation of the Sun when the shadow of a pole *h* metres high is $\sqrt{3}h$ metres long. [CBSE 2017, 15, 12]

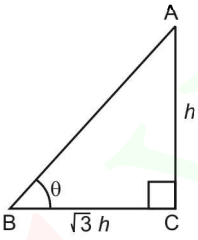
Ans.

Let the angle of elevation of the Sun be θ.

Given: Height of pole = '*h*' m.

Length of shadow = $\sqrt{3}\,h$

In ΔABC,



$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30° \quad \Rightarrow \quad \theta = 30°$$

Hence, the angle of elevation is 30°.

9. If $\sqrt{3}\tan\theta = 1$, find the value of $\sin^2\theta - \cos^2\theta$. [CBSE 2018, 17]

Ans. We know that $\tan 30° = \frac{1}{\sqrt{3}}$, $\sin 30° = \frac{1}{2}$

and $\cos 30° = \frac{\sqrt{3}}{2}$

Given: $\sqrt{3}\tan\theta = 1$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

Now, $\sin^2\theta - \cos^2\theta = \sin^2 30° - \cos^2 30°$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4} = \frac{1-3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

$$\therefore \quad \sin^2\theta - \cos^2\theta = -\frac{1}{2}.$$

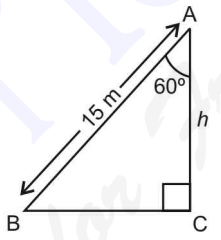
10. A ladder 15 metres long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall. [CBSE 2014, 11]

Ans.

Given: length of ladder, AB = 15 m

Let h be the height of the vertical wall, AC

In ΔPRQ,



The ladder makes an angle of 60° with the wall.

$$\therefore \quad \cos 60° = \frac{AC}{AB} = \frac{h}{15}$$

$$\Rightarrow \frac{1}{2} = \frac{h}{15} \Rightarrow h = \frac{15}{2} = 7.5\text{ m} \left[\because \cos 60° = \frac{1}{2}\right]$$

Hence, the required height of the wall is 7.5 m.

11. Simplify $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$. [CBSE 2013]

Ans. We know that $1 + \tan^2\theta = \sec^2\theta$

$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (1 + \tan^2\theta)(1 - \sin^2\theta) \quad [\because (a-b)(a+b) = a^2 - b^2]$

$= \sec^2\theta \cdot \cos^2\theta \quad [\because 1 + \tan^2\theta = \sec^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta]$

$= \frac{1}{\cos^2\theta}.\cos^2\theta = 1 \quad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

12. If $2\sin^2\theta - \cos^2\theta = 2$, find the value of θ.

Ans.

Given: $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2\sin^2\theta - (1 - \sin^2\theta) = 2$

$[\because \cos^2\theta = 1 - \sin^2\theta]$

$\Rightarrow 2\sin^2\theta - 1 + \sin^2\theta = 2$

$\Rightarrow 3\sin^2\theta = 3 \Rightarrow \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 \Rightarrow \sin\theta = \sin 90°$ [$\because \sin 90° = 1$]

$\Rightarrow \theta = 90°$

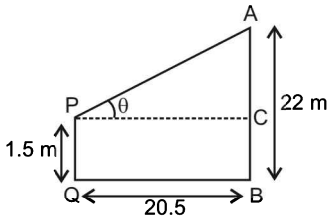
13. An observer 1.5 metres tall is 20.5 metres away from a tower 22 metres high. Determine the angle of elevation of the top of the tower from the eye of the observer. **[CBSE 2016]**

Ans.

Let PQ = 1.5 m is the height of the observer and QB = 20.5 m is the distance of the observer from the tower.

AB = 22 m is the height of the tower. [Given]

Let θ be the angle of elevation of the observer at the top of tower.



AB = 22 m

PQ = CB = 1.5 m

QB = PC = 20.5 m

$\Rightarrow$ AC = AB – BC

= 22 – 1.5

= 20.5 m

In ΔAPM,

$$\tan\theta = \frac{AC}{PC} = \frac{20.5}{20.5} = 1$$

$\Rightarrow \tan\theta = \tan 45°$

$\Rightarrow \theta = 45°$

Hence, the required angle of elevation of the top of the tower from the eye of the observer is 45°.

14. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$. **[CBSE 2015]**

Ans. $\tan^4\theta$ or $\tan^2\theta$ can be converted in $\sec^2\theta$

$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

L.H.S. $\tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

[$\because 1 + \tan^2\theta = \sec^2\theta$]

$= \tan^2\theta \cdot \sec^2\theta$ [$\because \tan^2\theta = \sec^2\theta - 1$]

$= (\sec^2\theta - 1) \cdot \sec^2\theta$

$= \sec^4\theta - \sec^2\theta$ = R.H.S. Hence, proved.

EXERCISE 8.4

1. If $\text{cosec}\,\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$. **[CBSE 2016, 12]**

Ans.

Given: $\text{cosec}\,\theta + \cot\theta = p$

To prove: $\cos\theta = \frac{p^2-1}{p^2+1}$

Proof: $\text{cosec}\,\theta + \cot\theta = p$

$$\Rightarrow \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = p$$

$$\Rightarrow \frac{1+\cos\theta}{\sin\theta} = p$$

Squaring both sides

$$\Rightarrow \left(\frac{1+\cos\theta}{\sin\theta}\right)^2 = p^2$$

$\Rightarrow (1 + \cos\theta)^2 = p^2 \sin^2\theta$

$\Rightarrow (1 + \cos\theta)^2 = p^2(1 - \cos^2\theta)$

$\Rightarrow (1 + \cos\theta)^2 = p^2(1 + \cos\theta)(1 - \cos\theta)$

[$\because a^2 - b^2 = (a - b)(a + b)$]

$\Rightarrow 1 + \cos\theta = p^2 - p^2\cos\theta$

$\Rightarrow \cos\theta + p^2\cos\theta = p^2 - 1$

$\Rightarrow \cos\theta(1 + p^2) = p^2 - 1$

$$\Rightarrow \cos\theta = \frac{(p^2-1)}{(p^2+1)} = \text{R.H.S.}$$

Hence, proved.

Trick Applied

- *Step 1. Reduce given equation into sin θ and cos θ.*
- *Step 2. Simplify the equation.*

2. Prove that $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \tan\theta + \cot\theta$.

Ans. We know that $\sec\theta = \frac{1}{\cos\theta}$

and $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$$\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \tan\theta + \cot\theta$$

L.H.S. $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$

$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$

$$= \sqrt{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}} = \sqrt{\frac{1}{\sin^2\theta\cos^2\theta}} = \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \tan\theta + \cot\theta$ = R.H.S.

3. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 metres towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower. [CBSE 2012, 11]

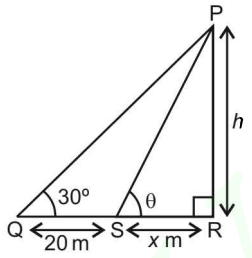
Ans.

Let h be the height of the tower PR.

The observer is standing at Q such that QR = (20 + x) m

$\angle PQR = 30°$

I[st] position of observer makes angle of elevation at tower



In ΔPQR,

$$\tan 30° = \frac{h}{20+x}$$

$$\left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20+x} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad 20 + x = \sqrt{3}h$$

$$\Rightarrow \quad x = \sqrt{3}h - 20 \qquad \ldots(i)$$

Now, the angle of elevation increases by 15° when the observer moves 20 m towards the tower, we have θ = 30° + 15° = 45°.

$$\tan 45° = \frac{h}{x} \Rightarrow h = x \qquad [\because \tan 45° = 1]$$

Substituting $x = h$ in *eqn* (i), we get

$$h = \sqrt{3}h - 20$$

$$\Rightarrow \quad \sqrt{3}h - h = 20$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 20 \Rightarrow h = \frac{20}{\sqrt{3}-1}$$

$$\Rightarrow \quad h = \frac{20}{\sqrt{3}-1}\cdot\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

[By rationalisation]

$$\Rightarrow \quad h = \frac{20(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{20(\sqrt{3}+1)}{3-1} = \frac{20(\sqrt{3}+1)}{2}$$

$$h = 10(\sqrt{3}+1)\ m$$

Hence, required height of tower is $10(\sqrt{3}+1)\ m$.

Trick Applied

- *Step 1. Draw a right angled triangle using given information.*
- *Step 2. Apply trignometric ratio and get the height.*

4. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\frac{1}{2}$.

Ans. To solve and equation in θ, we have to convert into 1 trigonometric ratio

Given: $1 + \sin^2\theta = 3\sin\theta\cos\theta$

To prove: $\tan\theta = 1$ or $\frac{1}{2}$

Proof: $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\sin^2\theta$

$$\Rightarrow \quad \frac{1+\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$

$$\Rightarrow \quad \frac{1}{\sin^2\theta} + 1 = \frac{3\cos\theta}{\sin\theta}$$

$$\left[\because \frac{1}{\sin\theta} = \text{cosec}\,\theta \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$\text{cosec}^2\theta + 1 = 3\cot\theta$

$\Rightarrow \cot^2\theta + 1 + 1 = 3\cot\theta [\because \text{cosec}^2\theta = 1 + \cot^2\theta]$

$\Rightarrow \cot^2\theta - 3\cot\theta + 2 = 0$

$\Rightarrow \cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$

$\Rightarrow \cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$

$\Rightarrow (\cot\theta - 2)(\cot\theta - 1) = 0$

$\Rightarrow \cot\theta = 1$ or 2

$\Rightarrow \tan\theta = 1$ or $\frac{1}{2}$ Hence, proved.

Trick Applied

- *Step 1. Reduce given equation in the form of cot θ.*
- *Step 2. Factorise and get the desired result.*

5. Given that $\sin\theta + 2\cos\theta = 1$, prove that $2\sin\theta - \cos\theta = 2$. [CBSE 2019]

Ans.

Given: $\sin\theta + 2\cos\theta = 1$

To prove: $2\sin\theta - \cos\theta = 2$

Proof: $\sin\theta + 2\cos\theta = 1$

Squaring both sides, we get

$$(\sin\theta + 2\cos\theta)^2 = 1$$

$$\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow 1 - \cos^2\theta + 4 - 4\sin^2\theta + 4\sin\theta\cos\theta = 1$$

$$\Rightarrow -\cos^2\theta - 4\sin^2\theta + 4\sin\theta\cos\theta = -4$$

$$\Rightarrow 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta = 4$$

$$\Rightarrow (2\sin\theta - \cos\theta)^2 = 4$$

$$[\because a^2 + b^2 - 2ab = (a-b)^2]$$

$\Rightarrow 2\sin\theta - \cos\theta = 2$ Hence, proved.

6. The angle of elevation from the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of the tower is $\sqrt{st}$. [CBSE 2017]

Ans.

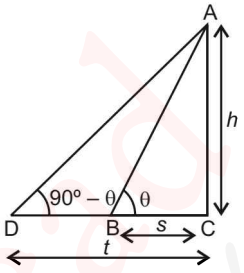
Points of observation D and B are at distance 't' and 's' from tower.

Let AC be the tower of height h

$\Rightarrow$ BC = s, DC = t

and angle of elevation on both positions are complementary.

$$\Rightarrow \quad \angle ADC = 90° - \theta \text{ and } \angle ABC = \theta$$



In ΔABC,

$$\tan\theta = \frac{AC}{BC} = \frac{h}{s} \quad \text{...(i)}$$

and In ΔABC,

$$\tan(90° - \theta) = \frac{AC}{DC}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{t} \quad [\because \tan(90° - \theta) = \cot\theta]$$

$$\Rightarrow \quad \frac{1}{\tan\theta} = \frac{h}{t} \quad \left[\because \cot\theta = \frac{1}{\tan\theta}\right] \quad \text{...(ii)}$$

Multiplying eqn (i) and (ii), we get

$$\tan\theta \cdot \frac{1}{\tan\theta} = \frac{h}{s} \cdot \frac{h}{t}$$

$$\Rightarrow \quad 1 = \frac{h^2}{st} \Rightarrow h^2 = st$$

$$\Rightarrow \quad h = \sqrt{st}$$

Hence, the required height of the tower is $\sqrt{st}$

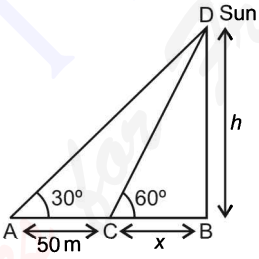
Trick Applied

If two angles are complementary angles, then sum of both angles is equal to 90° first angle + second angle = 90°.

7. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30° than when it is 60°. Find the height of the tower. [CBSE 2012, 11]

Ans. A and C are two position of observation when angle of elevation charges from 30° + 60°.

Let DB be the tower with height h.



Let x be the length of the shadow when angle of elevation is 60°.

i.e., BC = x

Given that AC = 50

$\angle DCB = 60°$ and $\angle DAB = 30°$

In ΔDCB,

$$\tan 60° = \frac{h}{x} \Rightarrow x = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}} \quad \text{...(i)}$$

$$[\because \tan 60° = \sqrt{3}]$$

In ΔABD,

$$\tan 30° = \frac{BD}{AB} = \frac{h}{50 + x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50 + x} \Rightarrow \sqrt{3}h = 50 + x$$

$$\Rightarrow \quad \sqrt{3}h = 50 + \frac{h}{\sqrt{3}} \quad \text{[Using eqn (i)]}$$

$$\Rightarrow \quad \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h = 50 \Rightarrow \left(\frac{3-1}{\sqrt{3}}\right)h = 50$$

$$\Rightarrow \quad h = \frac{50\sqrt{3}}{2} \Rightarrow h = 25\sqrt{3}$$

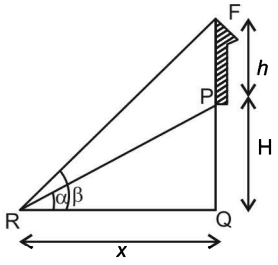
Hence, the required height of the tower is $25\sqrt{3}$.

8. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β, respectively. Prove that the height of the tower is $\left(\frac{h\tan\alpha}{\tan\beta-\tan\alpha}\right)$.

[CBSE 2020, 16, 10]

Ans. The angle of elevation at R from base and to P of flag staff are α, β respectively.

Let PQ be the tower of height H.



Let $QR = x$

Given: height of flag staff, $FP = h$

$\angle PRQ = \alpha$ and $\angle FRQ = \beta$

In ΔPRQ,

$$\tan\alpha = \frac{PQ}{RQ} = \frac{H}{x}$$

$$\Rightarrow \quad x = \frac{H}{\tan\alpha} \qquad ...(i)$$

$$\tan\beta = \frac{FQ}{RQ} = \frac{FP+PQ}{RQ}$$

$$\Rightarrow \quad \tan\beta = \frac{h+H}{x}$$

$$\Rightarrow \quad x = \frac{h+H}{x} \qquad ...(ii)$$

From eqn (i) and (ii), we get

$$\frac{H}{\tan\alpha} = \frac{h+H}{\tan\beta}$$

$$\Rightarrow \quad H\tan\beta = h\tan\alpha + H\tan\alpha$$

$$\Rightarrow H\tan\beta - H\tan\alpha = h\tan\alpha$$

$$\Rightarrow H(\tan\beta - \tan\alpha) = h\tan\alpha$$

$$\Rightarrow \quad H = \frac{h\tan\alpha}{\tan\beta-\tan\alpha}$$

Hence, proved.

9. If $\tan\theta + \sec\theta = l$, prove that $\sec\theta = \frac{l^2+1}{2l}$.

[CBSE 2017]

Ans. We know that $\sec^2\theta - \tan^2\theta = 1$

Here we will change

$(\sec\theta + \tan\theta)$ to $(\sec^2\theta - \tan^2\theta)$

Given: $\tan\theta + \sec\theta = l$

To prove: $\sec\theta = \frac{l^2+1}{2l}$

Proof: $\sec\theta + \tan\theta = l$ ...(i)

Multiplying by $(\sec\theta - \tan\theta)$,

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = l(\sec\theta - \tan\theta)$

$\Rightarrow \sec^2\theta - \tan^2\theta = l(\sec\theta - \tan\theta)$

$\Rightarrow 1 = l(\sec\theta - \tan\theta)$

$$\Rightarrow \quad l = \frac{1}{(\sec\theta-\tan\theta)}$$

$$\Rightarrow \quad \frac{1}{l} = \sec\theta - \tan\theta \qquad ...(ii)$$

Adding eqn (i) and (ii), we get

$$(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta) = l + \frac{1}{l}$$

$$\Rightarrow \quad 2\sec\theta = \frac{l^2+1}{2l}$$

$$\Rightarrow \quad \sec\theta = \frac{l^2+1}{2l}$$ Hence, proved.

Trick Applied

$\sec^2\theta - \tan^2\theta = 1$

$\therefore$ if $\sec\theta - \tan\theta = l$

then $\sec\theta + \tan\theta = \frac{1}{l}$

10. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, then prove that $q(p^2 - 1) = 2p$. [CBSE 2012]

Ans. Here we will change II[nd] expression into $\sin\theta$, $\cos\theta$ and eliminate trigonometric ratio from both.

Given that:

$$\sin\theta + \cos\theta = p$$

$$\sec\theta + \text{cosec}\,\theta = q$$

To prove: $q(p^2 - 1) = 2p$

Proof: $\sin\theta + \cos\theta = p$ ...(i)

$$\sec\theta + \text{cosec}\,\theta = q$$

$$\Rightarrow \quad \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$$

$$\left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$$

$$\Rightarrow \quad \frac{\sin\theta+\cos\theta}{\sin\theta.\cos\theta} = q$$

$$\Rightarrow \quad \frac{p}{\sin\theta.\cos\theta} = q \qquad \text{[Using eqn (i)]}$$

$$\Rightarrow \quad \sin\theta.\cos\theta = \frac{p}{q} \qquad ...(ii)$$

It is given that

$$\sin\theta + \cos\theta = p$$

On squaring both sides, we get

$(\sin\theta + \cos\theta)^2 = p^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\cos\theta = p^2$

$\Rightarrow 1 + 2\sin\theta\cos\theta = p^2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow 1 + \frac{2.p}{q} = p^2$

$\Rightarrow q + 2p = p^2q \Rightarrow p^2q - q = 2p$

$\Rightarrow q(p^2 - 1) = 2p$ Hence, proved.

11. If $a\sin\theta + b\cos\theta = c$, prove that $a\cos\theta - b\sin\theta = \sqrt{a^2+b^2-c^2}$.

Ans. We know that $\sin^2\theta + \cos^2\theta = 1$

Given: $a\sin\theta + b\cos\theta = c$

Squaring both sides

$(a\sin\theta + b\cos\theta)^2 = c^2$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2ab\sin\theta\cos\theta = c^2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2 + b^2 - c^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$

$\Rightarrow a^2 + b^2 - c^2 = (a\cos\theta - b\sin\theta)^2$

$\Rightarrow (a\cos\theta - b\sin\theta) = \sqrt{a^2+b^2-c^2}$

Hence, proved.

12. Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

[CBSE 2017, 15]

Ans. We know that $\sec^2\theta - \tan^2\theta = 1$

To prove: $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

L.H.S. $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$= \frac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(1+\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}$

$= \frac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)+(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}$

$= \frac{(\sec\theta-\tan\theta+1)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)+(\sec^2\theta-\tan^2\theta)}$

$= \frac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta+1)}$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$= \frac{(\sec\theta-\tan\theta+1)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)+(\sec^2\theta-\tan^2\theta)}$

$= \sec\theta - \tan\theta = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{1-\sin\theta}{\cos\theta}$ = R.H.S. Hence, proved.

13. The angle of the elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also height of the other tower.

[CBSE 2020, 15, 14]

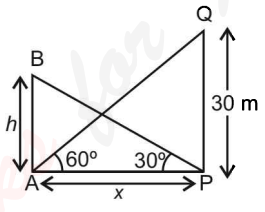
Ans.

Let QP be the tower of height 30 m.

Let BA be the second tower of height h and x be the distance between two towers.

Angle of duration from A to top of 30 m high tower is 60°.

And that to second tower is 30°.



i.e., $AP = x$

$BA = h$

$QP = 30$ m

$\angle QAP = 60°$ and $\angle BPA = 30°$

In ΔQAB,

$\tan 60° = \frac{QP}{AP} = \frac{30}{x}$

$\Rightarrow \sqrt{3} = \frac{30}{x} \quad [\because \tan 60° = \sqrt{3}]$

$x = \frac{30}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$

In ΔBPA,

$\tan 30 = \frac{BA}{AP} = \frac{h}{x} \quad [\because x = 10\sqrt{3}]$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{10\sqrt{3}} \Rightarrow h = 10$

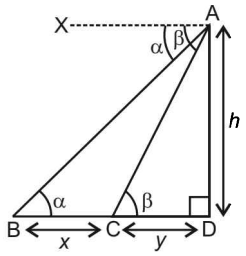
Hence, the required distance and height are $10\sqrt{3}$ and 10 m, respectively.

14. From the top of a tower h m high, the angles of depression of two objects, which are in line with the foot of the tower are α and β ($\beta > \alpha$). Find the distance between the two objects.

[CBSE 2017, 14]

Ans.

Let AD be the tower of height h m.



Let C and B be two objects x metres apart with angles of depression β and α.

Let distance CD = y

$\Rightarrow$ $\angle BAX = \alpha = \angle ABD$ and

$\angle CAX = \beta = \angle ACD$ [Alternate angles]

AD = h, BC = x and CD = y

In right angled ΔACD, we have

$$\tan\beta = \frac{AD}{CD} = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\tan\beta} \qquad ...(i)$$

In right angled ΔABD, we have

$$\tan\alpha = \frac{AD}{BD} = \frac{AD}{BC+CD}$$

$$\Rightarrow \quad \tan\alpha = \frac{h}{x+y} \Rightarrow x+y = \frac{h}{\tan\alpha}$$

$$\Rightarrow \quad y = \frac{h}{\tan\alpha} - x \qquad ...(ii)$$

Equating eqn (i) and (ii), we get

$$\frac{h}{\tan\beta} = \frac{h}{\tan\alpha} - x$$

$$\Rightarrow \quad x = \frac{h}{\tan\alpha} - \frac{h}{\tan\beta}$$

$$\Rightarrow \quad x = h\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right)$$

$$= h(\cot\alpha - \cot\beta) \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

Hence, the required distance between the two objects is h (cot α – cot β).

15. A ladder rests against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p so that its upper end slides a distance q down the wall and then the ladder makes an angle β to the horizontal. Show that $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

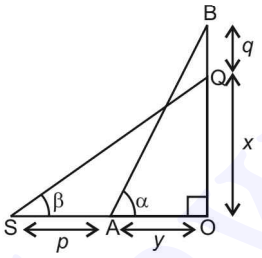
Ans.

Let AB be the ladder at an inclination α to the horizontal and SQ be its position when it makes an angle β to the horizontal.

Given: SA = p, BQ = q

OQ = x and OA = y

$\angle BAO = \alpha$ $\angle QSO = \beta$



In ΔBAO,

$$\cos\alpha = \frac{OA}{AB}$$

$$\Rightarrow \quad \cos\alpha = \frac{y}{AB} \Rightarrow y = AB\cos\alpha = OA \qquad ...(i)$$

and $\sin\alpha = \frac{OB}{AB} \Rightarrow OB = AB\sin\alpha$ ...(ii)

In ΔQSO,

$$\cos\beta = \frac{OS}{SQ}$$

$$\Rightarrow \quad OS = SQ\cos\beta = AB\cos\beta \qquad ...(iii)$$

$$\sin\beta = \frac{OQ}{SQ}$$

$$\Rightarrow \quad OQ = SQ\sin\beta = AB\sin\beta \qquad ...(iv)$$

Now, SA = OS – AO

p = AB cos β – AB cos α

= AB (cos β – cos α) ...(v)

and BQ = BO – QO

= AB sin α – AB sin β

$\Rightarrow$ q = AB (sin α – sin β) ...(vi)

Dividing eq^n (v) by eq^n (vi), we get

$$\frac{p}{q} = \frac{AB(\cos\beta - \cos\alpha)}{AB(\sin\alpha - \sin\beta)}$$

$$\therefore \quad \frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

Trick Applied

- *Step 1. Draw two triangles.*
- *Step 2. Calculate vertical height and horizontal height in both cases.*
- *Step 3. Equate the length of ladder in both the cases.*

16. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first,

its angle of elevation is 45°. Find the height of the tower. **[CBSE 2016]**

Ans.

Let OT be the vertical tower of height H m.

It stands on a horizontal plane,

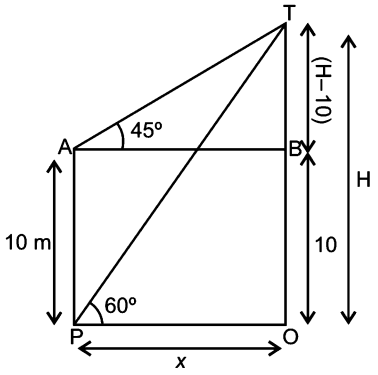
$OP = AB = x$ m.

Given: $AP = 10$ m

Observation point A is 10 m above first observation point P. Angle of elevation from point of observation

$\angle TPO = 60°$

$\angle TAB = 45°$



In right angled triangle ΔTPO, we have

$$\tan 60° = \frac{OT}{OP} = \frac{H}{x}$$

$$\left[\text{As } \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$$\Rightarrow \quad \sqrt{3} = \frac{H}{x} \Rightarrow x = \frac{H}{\sqrt{3}} \qquad ...(i)$$

In right angled ΔTAB, we have

$$\tan 45° = \frac{TB}{AB} = \frac{H-10}{x}$$

$$\Rightarrow \quad 1 = \frac{H-10}{x} \Rightarrow x = H - 10$$

$$\Rightarrow \quad \frac{H}{\sqrt{3}} = H - 10 \qquad \text{[using eqn (i)]}$$

$$\Rightarrow \quad H = \sqrt{3}H - 10\sqrt{3}$$

$$\Rightarrow \quad H(1-\sqrt{3}) = -10\sqrt{3}$$

$$\Rightarrow \quad H(\sqrt{3}-1) = 10\sqrt{3}$$

$$H = \frac{10\sqrt{3}}{(\sqrt{3}-1)}\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{10\sqrt{3}(\sqrt{3}+1)}{3-1} = \frac{10\sqrt{3}(\sqrt{3}+1)}{2}$$

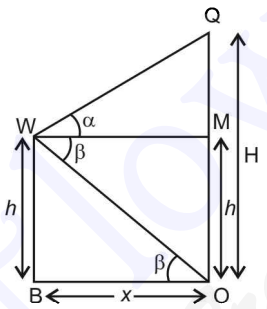
$$H = 5\sqrt{3}(\sqrt{3}+1) = 5(3+\sqrt{3}) \text{ m}$$

Hence, the required height of the tower is $5(3+\sqrt{3})m$.

17. A window of a house is h metres above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β, respectively. Prove that the height of the other house is $h(1 + \tan\alpha \cot\beta)$ metres. **[CBSE 2016, 11, 10]**

Ans.

Let we be the window of the house at a height h metres above the ground. Let OQ be the house located at a distance of x m from the window.



$OQ = H$

$OB = MW = x$

Given: Height of first house $WB = h = MO$

$\angle QWM = \alpha$, $\angle OWM = \beta = \angle WOB$

In ΔQWM,

$$\tan\alpha = \frac{QM}{WM} = \frac{OQ-MO}{WM}$$

$$\Rightarrow \quad \tan\alpha = \frac{H-h}{x}$$

$$\Rightarrow \quad x = \frac{H-h}{\tan\alpha} \qquad ...(i)$$

In ΔWOB,

$$\tan\beta = \frac{WB}{OB} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan\beta} \qquad ...(ii)$$

Equating eqn (i) and eqn (ii), we get

$$\frac{h}{\tan\beta} = \frac{H-h}{\tan\alpha}$$

$$\Rightarrow \quad h\tan\alpha = (H-h)\tan\beta$$

$$\Rightarrow \quad h\tan\alpha = H\tan\beta - h\tan\beta$$

$$\Rightarrow \quad H\tan\beta = h(\tan\alpha + \tan\beta)$$

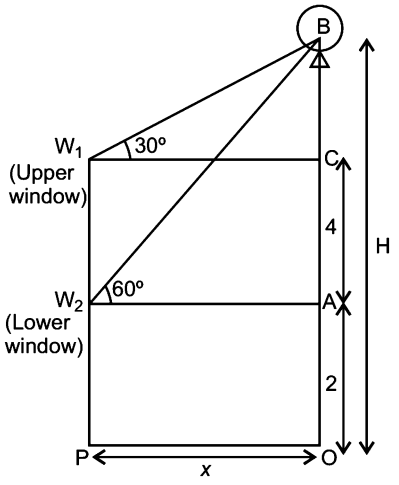
$$\Rightarrow \quad H = \frac{h(\tan\alpha+\tan\beta)}{\tan\beta}$$

$$\Rightarrow \quad H = h\left(1+\frac{\tan\alpha}{\tan\beta}\right) = h\,(1 + \tan\alpha\cot\beta)$$

18. The lower window of a house is at a height of 2 m above the ground and its upper window

is 4 m vertically above the lower window. At certain distance the angles of elevation of a balloon from these window are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground.

Ans. Let B be a balloon at a height OB = H



Let W_2 be the window whose height from the ground, W_2P = 2 m = OR

Let W_1 be the upper window whose Height from lower window = W_1W_2 = 4 m = QR

AC = 4 m

AO = 2 m

$\therefore$ BC = OB − (AC + AO)

= H − (4 + 2)

= H − 6

$\angle BW_1C = 30°$ and $\angle BW_2A = 60°$

In ΔBW_2R,

$$\tan 60° = \frac{BA}{W_2A} = \frac{BC+CA}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{(H-6)+4}{x}$$

$$\Rightarrow \quad x = \frac{H-2}{\sqrt{3}} \qquad ...(i)$$

In ΔBW_1Q,

$$\tan 30° = \frac{BC}{W_1C} = \frac{BC}{OP}$$

$$\Rightarrow \quad \tan 30° = \frac{H-6}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \sqrt{3}(H-6) \qquad ...(ii)$$

From eqn (i) and (ii)

$$\sqrt{3}(H-6) = \frac{(H-2)}{\sqrt{3}}$$

$\Rightarrow$ 3(H − 6) = (H − 2)

$\Rightarrow$ 3H − 18 = H − 2 $\Rightarrow$ 2H = 16 $\Rightarrow$ H = 8

Hence, the height of the balloon is 8 m from the ground.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

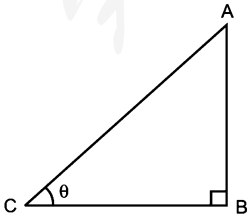
1. If the height and length of the shadow of a man are the same, then the angle of elevation of the sun is:

(A) 45° (B) 60°

(C) 90° (D) 120°

Ans. (A)

Explanation: Let AB be the height of a man and BC be the shadow of a man.



AB = BC [Given]

In right angled ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\frac{AB}{BC} = \tan\theta$$

$$\tan\theta = 1$$

$$\theta = 45°.$$

2. Find A and B, if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + B) = \frac{1}{2}$.

Ans.

Given: $\sin(A + 2B) = \sin 60° \left[\sin 60° = \frac{\sqrt{3}}{2}\right]$

$\therefore$ A + 2B = 60° ...(i)

$$\cos(A + B) = \cos 60° \left[\cos 60° = \frac{1}{2}\right]$$

$\therefore$ A + B = 60° ...(ii)

Subtracting equation (i) and (ii)

B = 0°

Putting the value of B in equation (ii), we get,

$$A = 60° - 0° = 60°$$

So, $A = 60°$ and $B = 0°$.

3. If $x = a\cos^3\theta$, $y = b\sin^3\theta$, prove that

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1.$$

Ans.

$x = a\cos^3\theta$, $y = b\sin^3\theta$

$$\text{LHS} = \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$$

$$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$$

$$= (\cos\theta)^{3\times 2/3} + (\sin\theta)^{3\times 2/3}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

LHS = RHS. Hence, proved.

4. If $a\cos\theta - b\sin\theta = c$, prove that

$a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$.

Ans.

$a\cos\theta - b\sin\theta = c$

On squaring both sides, we get

$(a\cos\theta - b\sin\theta)^2 = c^2$

$(a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta \cdot \sin\theta = c^2$

$a^2(1 - \sin^2\theta) + b^2(1 - \cos^2\theta) - 2ab\cos\theta \cdot \sin\theta = c^2$

$a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\cos\theta \cdot \sin\theta = c^2$

$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta \cdot \sin\theta = a^2 + b^2 - c^2$

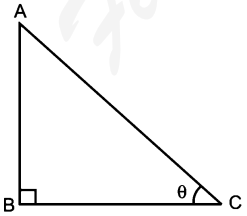
$(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

5. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the sun?

Ans.

Let height of tower be AB and its shadow be BC.



$$\frac{AB}{BC} = \tan\theta$$

But, $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$ [Given]

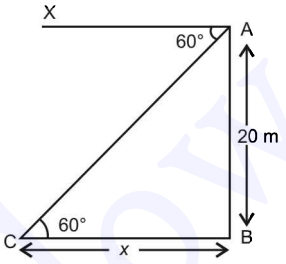
$\therefore \quad \tan\theta = \sqrt{3}$

$\tan\theta = \tan 60°$

$\theta = 60°$.

6. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. Take $\sqrt{3} = 1.732$.

Ans.



In this figure,

Due to alternate angles we obtain,

$$\angle XAC = \angle ACB = 60°$$

In ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$$

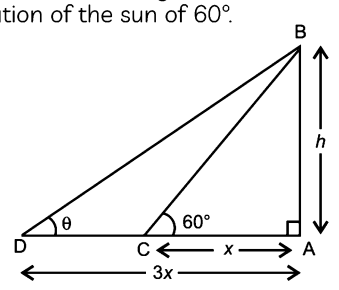
$$= 20\times\frac{1.732}{3} = 11.53$$

Hence, distance between ball and foot of tower is 11.53 m.

7. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°. Find the angle of elevation of the sun of the longer shadow

Ans.

Let AB be tower of height h, AC be the shadow at elevation of the sun of 60°.



In ΔBAC,

$$\frac{AB}{BC} = \tan 60°$$

$$\frac{h}{x} = \sqrt{3}$$

and $h = \sqrt{3}x$ ...(i)

In ΔBAD,

$$\frac{AB}{AD} = \tan\theta$$

$$\frac{h}{3x} = \tan\theta$$

$$\frac{x\sqrt{3}}{3x} = \tan\theta \qquad \text{[using (i)]}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$$

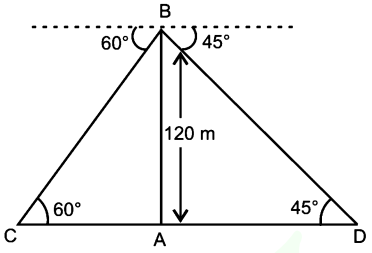
Thus, $\theta = 30°$

The angle of elevation of the sun of the longer shadow is 30°.

8. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of repression as 60° and 45°. Find the distance between two cars.

Ans.

Let AB be the tower of height 120 m. Let C and D be location of car on opposite side of tower.



In ΔBAD, $\frac{AB}{AD} = \tan 45°$

$$\frac{120}{AB} = 1$$

$$AB = 120$$

In ΔBAC, $\frac{AB}{CA} = \tan 60°$

$$\frac{120}{CA} = \sqrt{3}$$

$$CA = \frac{120}{\sqrt{3}} = 40\sqrt{3}$$

$$CD = CA + AD$$

$$= 120 + 40\sqrt{3} = 189.28 \text{ m}$$

Hence, the distance between two men is 189.28 m.

9. Using the formula $\cos 2\theta = 2\cos^2\theta - 1$, find the value of cos 30°, it is being given that $\cos 60° = \frac{1}{2}$.

Ans.

Given, $\cos 2\theta = 2\cos^2\theta - 1$

Let $\theta = 30°$

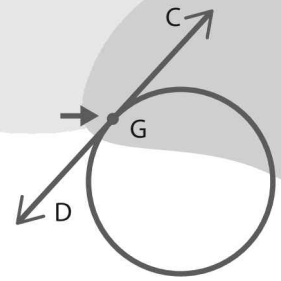
Then, $\cos(2 \times 30°) = 2\cos^2 30° - 1$

$$\cos 60° = 2\cos^2 30° - 1$$

$$\frac{1}{2} + 1 = 2\cos^2 30°$$

$$2\cos^2 30° = \frac{3}{2}$$

$$\cos^2 30° = \frac{3}{4} \text{ and } \cos 30° = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$



9 Circles

EXERCISE 9.1

Choose the correct option from the given four options in the following questions:

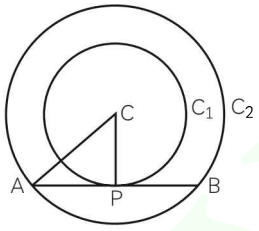
1. If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is the tangent to the other circle is:

(A) 3 cm (B) 6 cm
(C) 9 cm (D) 1 cm

[CBSE 2015, 14]

Ans. (B)

Explanation: AB is tangent to circle C_1 and CP is the radius of circle C_1.



$\therefore \angle CPA = 90°$

[Tangent at any point of circle is perpendicular to radius through the point of contact]

Also, AB is a chord of circle C_2.

So, CP bisects AB

i.e., $AP = BP$

Join AC. In right triangle APC

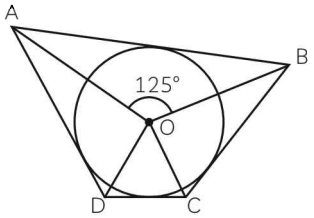
$AC^2 = AP^2 + CP^2$

$\Rightarrow \quad 5^2 = AP^2 + 4^2$

$\Rightarrow \quad AP^2 = 9$ or $AP = 3$

$\Rightarrow \quad AB = 6$ cm

2. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to:



(A) 62.5° (B) 45°
(C) 35° (D) 55°

Ans. (D)

Explanation: As in the given figure ABCD is a quadrilateral circumscribing the circle.

We know that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the cente of the circle.

$\Rightarrow \angle AOB + \angle COD = 180°$

$125° + \angle COD = 180°$

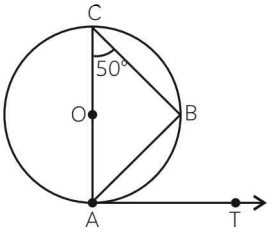
$\angle COD = 180° - 125° = 55°$

Hence, option (D) is verified.

Trick Applied

Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angle of the centre of circle.

3. In the given figure, AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 50°$. If AT is the tangent to the circle at point A, then $\angle BAT$ is equal to:



(A) 65° (B) 60°
(C) 50° (D) 40° **[CBSE 2016]**

Ans. (C)

Explanation: AC is diameter

$\Rightarrow \quad \angle B = 90°$ [$\angle$ in a semi-circle]

$\Rightarrow \quad \angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle BAC = 180° - \angle C - \angle B$

$\Rightarrow \quad \angle BAC = 180° - 50° - 90°$

$= 180° - 140° = 40°$

$\Rightarrow \quad \angle BAC = 40°$ ...(i)

$OA \perp AT$ [As tangent at any point on the circle is perpendicular to the radius through point of contact]

$\Rightarrow$ $\angle OAT = 90°$

$\Rightarrow$ $\angle OAB + \angle BAT = 90°$

$\Rightarrow$ $40° + \angle BAT = 90°$ [Using eqn. (i)]

$\Rightarrow$ $\angle BAT = 90° - 40°$

$\Rightarrow$ $\angle BAT = 50°$

Hence, option (C) is verified.

4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is:

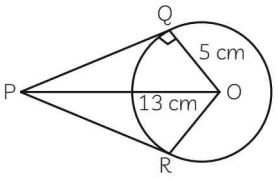
(A) 60 cm^2 **(B) 65 cm^2**

(C) 30 cm^2 **(D) 32.5 cm^2**

Ans. (A)

Explanation: We know that tangent to at any point on the circle is perpendicular to the radius through the point of contact. Hence, we get

$OQ \perp PQ$ and $OR \perp PR$



ΔPOQ and ΔPOR are right-angled triangles.

Using Pythagoras theorem in ΔPQO

$(Base)^2 + (Perpendicular)^2 = (Hypotenuse)^2$

$(PQ)^2 + (OQ)^2 = (OP)^2$

$(PQ)^2 + (5)^2 = (13)^2$

$(PQ)^2 + 25 = 169$

$(PQ)^2 = 144$

$PQ = 12$ cm

We know that tangents through an external point to a circle are equal,

$PQ = PR = 12$ cm

Therefore, area of quadrilateral PQRS,

A = area of ΔPOQ + area of ΔPOR.

Area of right angled triangle

$$= \frac{1}{2} \times \text{base} \times \text{perpendicular}$$

$$A = \left(\frac{1}{2} \times OQ \times PQ\right) + \left(\frac{1}{2} \times OR \times PR\right)$$

$$A = \left(\frac{1}{2} \times 5 \times 12\right) + \left(\frac{1}{2} \times 5 \times 12\right)$$

$A = 30 + 30 = 60\ cm^2$

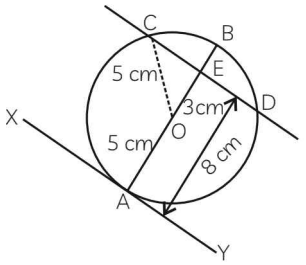
5. At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is:

(A) 4 cm **(B) 5 cm**

(C) 6 cm **(D) 8 cm**

Ans. (D)

Explanation:



According to the question,

Radius of circle, AO = OC = 5cm

AE = 8 cm

AE = AO + OE

$\Rightarrow$ OE = AE − AO

$\Rightarrow$ OE = (8 − 5) = 3 cm

Now, $\angle OAX = \angle BAX = 90°$ [Tangent at any point of a circle is perpendicular to the radius through the point of contact]

Also, CD|| XY meets AB at E, so we have

$90° + \angle AEC = 180°$

$\Rightarrow$ $\angle AEC = 90°$

$\Rightarrow$ $\angle OEC = 90°$

By Pythagoras theorem,

$OC^2 = OE^2 + EC^2$

$\Rightarrow$ $(5)^2 = (3)^2 + (EC)^2$

$\Rightarrow$ $EC^2 = 25 - 9 = 16$

$\Rightarrow$ $EC = 4$

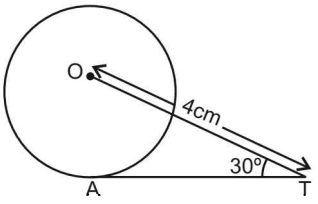
Also, CE = ED [since, perpendicular from center to the chord bisects the chord]

$\Rightarrow$ CD = 2 × EC = 8cm

Trick Applied

Perpendicular from centre to the chord bisects the chord.

6. In the given figure, AT is a tangent to the circle with centre 'O' such that OT = 4 cm and $\angle OTA = 30°$. Then AT is equal to:



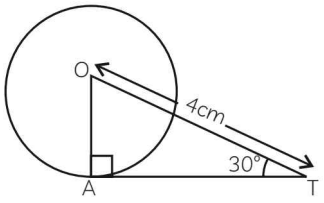
(A) 4 cm **(B) 2 cm**

(C) $2\sqrt{3}$ cm **(D) $4\sqrt{3}$ cm**

Ans. (C)

Explanation: Join OA

OA is radius and AT is tangent at contact point A.



$\Rightarrow$ $\angle OAT = 90°$ [Tangent at any point of a circle is perpendicular to the radius through the point of contact]

$OT = 4$ cm [Given]

In ΔOAT,

Using $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$ we have,

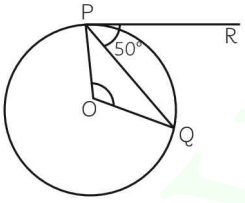
$$\cos 30° = \frac{AT}{OT}$$

$$\frac{\sqrt{3}}{2} = \frac{AT}{4}$$

$\Rightarrow$ $AT = 2\sqrt{3}$ cm

Hence, option (C) is verified.

7. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Then $\angle POQ$ is equal to:



(A) 100° (B) 80°

(C) 90° (D) 75° **[CBSE 2015]**

Ans. (A)

Explanation: OP is radius and PR is tangent at P.

$\Rightarrow$ $\angle OPR = 90°$ [As tangent to at any point on the circle is perpendicular to the radius through point of contact]

$\Rightarrow$ $\angle OPQ + 50° = 90°$

$\Rightarrow$ $\angle OPQ = 90° - 50°$

$\Rightarrow$ $\angle OPQ = 40°$

In ΔOPQ,

$OP = OQ$ [Radii of same circle]

$\Rightarrow$ $\angle OQP = \angle OPQ = 40°$

[Angle opposite to equal sides are equal]

$\angle POQ + \angle OPQ + \angle OQP = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle POQ = 180° - \angle OPQ - \angle OQP$

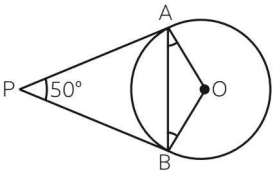
$= 180° - 40° - 40°$

$= 180° - 80° = 100°$

$\Rightarrow$ $\angle POQ = 100°$.

Hence, option (A) is verified.

8. In the given figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to:



(A) 25° (B) 30°

(C) 40° (D) 50° **[CBSE 2015, 11]**

Ans. (A)

Explanation: In ΔOAB, we have

$OA = OB$ [Radii of same circle]

$\Rightarrow$ $\angle OAB = \angle OBA$ [Angle opposite to equal sides are equal]

As OA and PA are radius and tangent respectively at contact point A

$\Rightarrow$ $\angle OAP = 90°$ [As tangent at any point on the circle is perpendicular to the radius through point of contact]

Similarly, $\angle OBP = 90°$

Now, in quadrilateral PAOB,

$\angle OBP + \angle OAP + \angle AOB + \angle APB = 360°$

[By angle sum property of quadrilateral]

$\Rightarrow$ $90° + 90° + \angle AOB + 50° = 360°$

$\Rightarrow$ $\angle AOB = 360° - 90° - 90° - 50°$

$\Rightarrow$ $\angle AOB = 130°$

Again, in ΔOAB,

$\angle AOB + \angle OAB + \angle OBA = 180°$

[By angle sum property]

$\Rightarrow 130° + \angle OAB + \angle OAB = 180°$

[As $\angle OBA = \angle OAB$]

$\Rightarrow$ $2\angle OAB = 180° - 130° = 50°$

$\Rightarrow$ $\angle OAB = 25°$

Hence, $\angle OAB = 25°$ which verifies option (A).

9. If two tangents inclined at angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to:

(A) $\frac{3\sqrt{3}}{2}$ cm (B) 6 cm

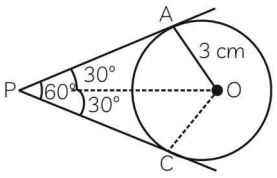
(C) 3 cm (D) $3\sqrt{3}$ cm

[CBSE 2020, 17, 13]

Ans. (D)

Explanation:

Given: A circle with center O and PA and PC are two tangents to the circle at point A and C from an external point P such that $\angle APC = 60°$. [i.e., angle of inclination between two tangents].



To find: AP and PC

In ΔOAP and ΔOCP,

OA = OC [Radii of same circle]

OP = OP [common]

PA = PC [Tangents from an external point to a circle are equal]

∴ ΔOAP ≅ ∠OCP [By SSS Criterion]

⇒ ∠APO = ∠OPC [Corresponding parts of congruent triangles are equal] ...(i)

∠APC = 60° [Given]

⇒ ∠APO + ∠OPC = 60°

⇒ ∠APO + ∠APO = 60° [Using eqn. (i)]

⇒ 2∠APO = 60°

⇒ ∠APO = 30°

OA and PA are the radius and the tangent respectively at contact point A of a circle of radius OA = 3 cm.

⇒ ∠OAP = 90° [As tangent at any point on the circle is perpendicular to the radius through point of contact]

In right angled ΔOAP,

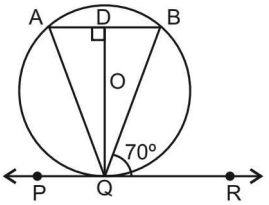
$$\tan 30° = \frac{\text{Perpendicular}}{\text{Base}} = \frac{OA}{AP} = \frac{3}{AP}$$

But $\tan 30° = \frac{1}{\sqrt{3}}$

⇒ $\frac{1}{\sqrt{3}} = \frac{3}{AP}$

⇒ AP = $3\sqrt{3}$ cm which verifies the option (D).

10. In the given figure, if PQR is the tangent to a circle at Q, whose centre is O, AB is a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to:



(A) 20° **(B) 40°**

(C) 35° **(D) 45°**

Ans. (B)

Explanation:

AB || PQR [Given]

∠ABQ = ∠BQR = 70°

[Alternate interior angles] ...(i)

and ∠OQR = ∠ADQ [Alternate interior angles]

As PQR and OQ are tangent and radius at contact point Q

⇒ ∠OQR = 90° [As tangent at any point on the circle is perpendicular to the radius through point of contact]

⇒ ∠OQB + ∠70° = 90°

⇒ ∠OQB = 90° − 70° = 20° ...(ii)

Now, ∠ADQ = 90° and perpendicular from the centre to the chord and bisects the chord

⇒ AD = DB

∠QDA = ∠QDB [Each 90°]

DQ = DQ [Common]

⇒ ΔQMA ≅ ∠QMB

[By SAS criterion of congruence]

⇒ ∠AQD = ∠BQD [By cpct]

∠AQB = ∠AQD + ∠BQD

⇒ ∠AQB = 2∠BQD

[As ∠AQD = ∠BQD]

⇒ ∠AQB = 2(20) = 40° [Using eqn. (ii)]

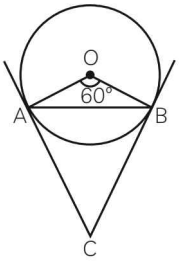
Hence, option (B) is verified.

EXERCISE 9.2

Write True or False and justify your answer in each of the following:

1. If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is also 60°. [CBSE 2011]

Ans. False.



We have a circle with center O and AB as any chord with ∠AOB = 60°.

Now, OA ⊥ AC and OB ⊥ CB [As tangent at any point on the circle is perpendicular to the radius through point of contact]

∠OBC = ∠OAC = 90° ...(i)

In quadrilateral AOBC

∠OBC + ∠OAC + ∠AOB + ∠ACB = 360°

[By angle sum property of quadrilateral]

⇒ 90° + 90° + 60° + ∠ACB = 360°

⇒ ∠ACB = 120° ...(ii)

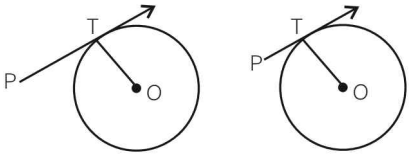
i.e., the angle between two tangents is 120°.

Hence, the given statement is false.

2. The length of tangent from an external point on a circle is always greater than radius of the circle.

Ans. False.

Consider any point P external to a circle away from O.



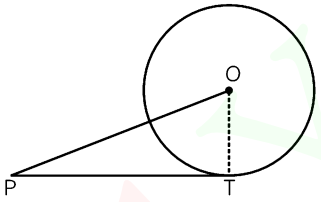
Now, draw tangent PT on the circle. Clearly, the length of the tangent from an external point of a circle may or may not be greater than the radius of the circle. Hence, the given statement is false.

3. The length of the tangent from an external point P on a circle with centre O is always less than OP.

Ans. True.

Consider the circle with centre O.

Let PT be a tangent drawn from an external point P.



Join OT

$\Rightarrow$ $OT \perp PT$ [Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\Rightarrow$ OPT is a right-angled triangle.

We also know that in a right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.

$\Rightarrow$ $OP > PT$

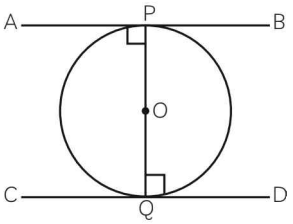
$\Rightarrow$ $PT < OP$

Hence, the length of tangent from an external point P on a circle with center O is always less than OP.

4. The angle between two tangents to a circle may be 0°.

Ans. True.

The angle between two tangents to a circle may be 0° only when both tangent lines coincide or are parallel to each other.



Consider the diameter POQ of a circle with centre O. The tangent at P and Q are drawn.

$\Rightarrow$ $OP \perp AB$ and $OQ \perp CD$ [Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\Rightarrow$ $\angle OPA = 90$ and $\angle OQD = 90$

$\Rightarrow$ $\angle OPA = \angle OQD = 90$

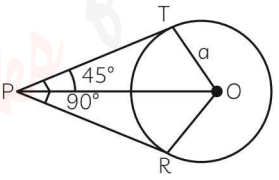
These are alternate interior angles, so the tangent APB || CQD i.e., angle between two tangents to a circle may be zero.

Hence, the given statement is true.

5. If the angle between two tangents drawn from a point P to a circle of radius *a* and centre O is 90°, then OP = $a\sqrt{2}$ [CBSE 2017]

Ans. True.

Let us consider a circle with center O and tangents PT and PR and the angle between them is 90°, and the radius of the circle is a.



In ΔOTP and ΔORP,

$OT = OR$ [Radii of same circle]

$OP = OP$ [Common]

$PT = PR$ [Tangents from an external point to a circle are equal]

$\Delta OTP \cong \Delta ORP$ [By SSS Criterion]

$\Rightarrow$ $\angle TPO = \angle OPR$ [By cpct] ...(i)

$\angle TPR = 90°$ [Given]

$\Rightarrow$ $\angle TPO + \angle OPR = 90°$

$\Rightarrow$ $\angle TPO + \angle TPO = 90°$ [Using eqn. (i)]

$\Rightarrow$ $\angle TPO = 45°$

$OT \perp TP$ [As tangent at any point on the circle is perpendicular to the radius through point of contact]

$\Rightarrow$ $\angle OTP = 90°$

We know that,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

So, ΔPTO is a right-angled triangle

$$\sin(\angle TPO) = \frac{OT}{OP}$$

$$\sin 45° = \frac{a}{OP}$$

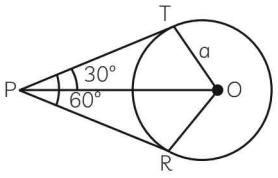
$$OP = \sqrt{2}a \qquad [\text{As } \sin 45° = \frac{1}{\sqrt{2}}]$$

Hence, proved.

6. If the angle between two tangents drawn from a point P to a circle of radius *a* and centre O is 60°, then OP = $a\sqrt{3}$ [CBSE 2017]

Ans. False.

Let us consider a circle with center O and tangents PT and PR, angle between them is 60° and the radius of the circle is *a*



In ΔOTP and ΔORP,

OT = OR [Radii of same circle]

OP = OP [Common]

PT = PR [Tangents from an external point to a circle are equal]

ΔOTP ≅ ΔORP [By SSS Criterion]

⇒ ∠TPO = ∠OPR [By cpct] ...(i)

⇒ ∠TPR = 60° [Given]

⇒ ∠TPO + ∠OPR = 60°

⇒ ∠TPO + ∠TPO = 60° [Using Eqn. (i)]

⇒ ∠TPO = 30°

OT ⊥ TP [As tangent at any point on the circle is perpendicular to the radius through point of contact]

⇒ ∠OTP = 90°

We know that

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

So, ΔPTO is a right-angled triangle

$$\sin(\angle TPO) = \frac{OT}{OP}$$

$$\sin 30° = \frac{a}{OP}$$

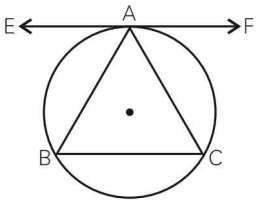
$$OP = 2a \qquad [\text{As } \sin 30° = \frac{1}{2}]$$

Hence, the given statement is false.

7. The tangent to the circum-circle of an isosceles ΔABC at A, in which AB = AC, is parallel to BC. [CBSE 2012]

Ans. True.

Let us consider a circle in which EF is a tangent passing through point A on the circle and ABC is an isosceles triangle in the circle, in which AB = AC



AB = AC [Given]

∠ACB = ∠ABC [Angles opposite to equal sides are equal]....(i)

∠EAB = ∠ACB [Angle between tangent and chord is equal to angle made by chord in alternate segment]

But ∠ABC = ∠ACB

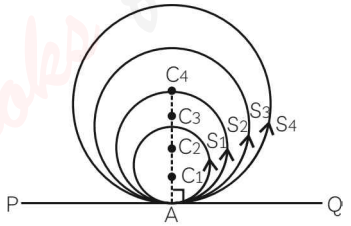
∠EAB = ∠ABC

⇒ EAF || BC [Two lines are parallel if their alternate interior angles are equal]

Hence, the given statement is true.

8. If a number of circles touch a given line segment PQ at a point A, then their centres lie on the perpendicular: bisector of PQ.

Ans. False.



Let the $S_1, S_2, S_3, \ldots, S_n$ be n circles with centers $C_1, C_2, C_3, \ldots, Cn$ respectively.

And PQ is a common tangent to all the circles at point A which is common to all circles.

We know that tangent at any point on the circle is perpendicular to the radius at the point of contact.

⇒ $C_1A \perp PQ$

⇒ $C_2A \perp PQ$

⇒ $C_3A \perp PQ$

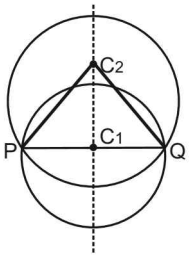
Similarly $C_nA \perp PQ$

So, $C_1\ C_2\ C_3 \ldots\ C_n$ all lie on the perpendicular line to PQ but not on perpendicular bisector as PA may or may not be equal to AQ.

Hence, the given statement is false.

9. If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Ans. True.



Centre of any circle passing through the end points P and Q of a line segment are equidistant from P and Q.

We draw two circles with centre C_1 and C_2 passing through the end points P and Q of a line segment PQ. We know that perpendicular bisectors of a chord of a circle always passes through the centre of circle.

Thus, perpendicular bisector of PQ passes through C_1 and C_2.

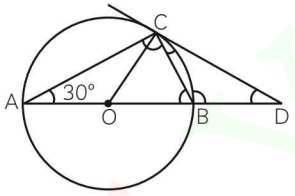
i.e., $C_1P = C_1Q$

$C_2P = C_2Q$

Similarly, all the circles passing through PQ will have their centre on perpendicular bisector of PQ.

10. AB is the diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.

Ans. True.



AB is the diameter of circle with centre O and AC is a chord such that ∠BAC = 30°. Also, tangent at C intersects AB extends at D.

Join OC

OA = OC [Radii of the same circle]

∠OCA = ∠OAC = 30° [Angles opposite to equal sides are equal]

∠ACB = 90° [Angle in a semicircle is a right angle]

∠ABC + ∠ACB + ∠CAB = 180° [Angle sum property]

∠ABC + 90° + 30°= 180° [Angle sum property]

∠ABC = 180 – 120 = 60°

∠OCA + ∠OCB = 90°

30° + ∠OCB = 90°

∠OCB = 60° ...(i)

OC = OB [Radii of same circle]

∠OBC = ∠OCB = 60° [Angles opposite to equal sides are equal]

∠OBC + ∠CBD = 180° [Linear pair]

60° + ∠CBD = 180°

So, ∠CBD = 120° ....(ii)

OC ⊥ CD [Tangent at a point on the circle is perpendicular to the radius through point of contact]

∠OCD = 90°

∠OCB + ∠BCD = 90°

60° + ∠BCD = 90°

∠BCD = 30° ...(iii)

In ΔBCD,

∠CBD + ∠BCD + ∠BDC = 180° [Angle sum property of triangle]

120° + 30° + ∠BDC = 180° [From (ii) and (iii)]

∠BDC = 30° ...(iv)

From eqn. (iii) and eqn. (iv)

∠BCD = ∠BDC = 30°

BC = BD [Sides opposite to equal angles are equal]

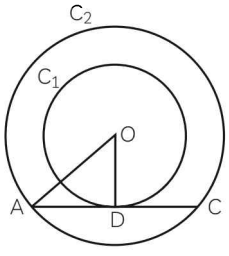
Hence, the given statement is true.

EXERCISE 9.3

1. Out of two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Ans.

Given: Chord AC of circle C_2 is tangent of circle C1 at point D



Join OD.

We know that tangent AC and radius OD at point D are perpendicular.

OD ⊥ AC.

⇒ AD = DC = 4 cm [Perpendicular drawn from centre of circle to any chord, bisects the chord]

In right angled ΔAOD,

$OA^2 = AD^2 + OD^2$

⇒ $OD^2 = OA^2 – AD^2$ [By Pythagoras theorem]

$= 5^2 – 4^2$

$= 25 – 16 = 9$

$\Rightarrow$ OD = 3 cm

Hence, radius of the inner circle is OD = 3 cm.

2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

[CBSE 2011]

Ans.

Given: PR and PQ are two tangents drawn from an external point P to a circle with centre O.

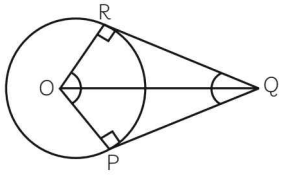
To prove: QORP is a cyclic quadrilateral

Proof: We know that tangent at a point to circle is perpendicular to the radius through the point of contact.

$\Rightarrow$ OR $\perp$ PR and OQ $\perp$ PQ

$\Rightarrow$ $\angle ORP = 90°$ and $\angle OQP = 90°$

$\Rightarrow$ $\angle ORP + \angle OQP = 180°$ ...(i)



Also, in quadrilateral QORP,

$\angle ORP + \angle OQP + \angle ROQ + \angle QPR = 360°$

$\Rightarrow 180° + \angle ROQ + \angle QPR = 360°$

$\Rightarrow$ $\angle ROQ + \angle QPR = 180°$ ...(ii)

From eqn (i) and (ii), we see that the sum of opposite angles in a quadrilateral is 180°.

$\Rightarrow$ Quadrilateral is cyclic.

$\Rightarrow$ QORP is cyclic quadrilateral.

Trick Applied

If sum of opposite angles in quadrilateral is 180°, then quadrilateral is cyclic.

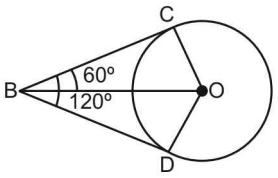
3. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

Ans.

Given: A circle with centre O. BC and BD are two tangents drawn from an external point B to a circle with centre O.

$\angle DBC = 120°$

To prove: BC + BD = BO i.e., BO = 2BC



Construction: Join OC, OD and OB.

Proof: BC and BD are tangents. OC and OD are radius

$\Rightarrow$ OC $\perp$ BC and OD $\perp$ BD [Tangent at a point is perpendicular to the radius through the point of contact]

In ΔOBC and ΔODB,

OC = OD [Radii of same circle]

OB = OB [Common]

BC = BD [Tangents from external point are equal]

$\Rightarrow$ $\Delta OBC \cong \Delta OBD$ [SSS criterion]

$\Rightarrow$ $\angle OBC = \angle OBD$ [By cpct] ...(i)

Also, $\angle DBC = 120°$ [Given]

$\Rightarrow \angle OBC + \angle DBO = 120°$

$\Rightarrow \angle CBO + \angle CBO = 120°$ [using eq^n (i)]

$\Rightarrow \angle CBO = \angle DBO = 60°$

In right angled ΔOBC,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\Rightarrow \cos 60° = \frac{BC}{BO}$$

$$\Rightarrow \frac{1}{2} = \frac{BC}{OB}$$

$\Rightarrow$ OB = 2 BC

Also, BC = BD

$\Rightarrow$ OB = BC + BC $\Rightarrow$ OB = BC + BD

Hence, proved.

Trick Applied

Tangent at any point of circle is perpendicular to radius through point of contact.

4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of angles formed by tangents.

Ans.

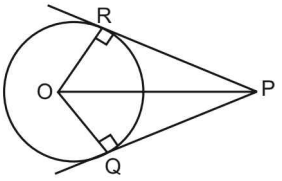
Given: Two intersecting lines PR and PQ intersect at P.

A circle with centre O touches the above lines at R and Q.

To prove: Centre O lies on angle bisector of angle formed by tangents.

Construction: Join OR and OQ.

Proof:



In ΔPRO and ΔPQO,

We know that tangent at any point is perpendicular to the radius through the point.

$\angle PRO = \angle PQO = 90°$ [Tangent at any point is perpendicular to the radius through the point of contact]

OR = OQ [Radii of same circle]

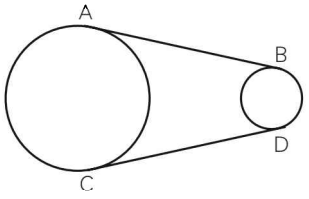
OP = OP [Common]

$\therefore$ $\Delta PRO \cong \Delta PQO$ [R.H.S. criterion]

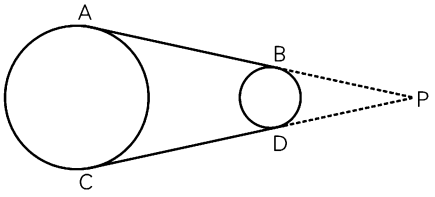
Hence, $\angle RPO = \angle QPO$ [By cpct]

Thus, O lies on angle bisector of $\angle RPQ$.

5. In the given figure AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.



Ans.



Given: AB and CD are common tangents to 2 circles of unequal radii.

To prove: AB = CD.

Construction: Produce AB and CD to intersect at P.

We know that length of tangents drawn from an external point is equal.

Proof:

$\Rightarrow$ For first circle, PA = PC

and For second circle, PB = PD

$\therefore$ PA – PB = PC – PC

$\Rightarrow$ AB = CD

Hence, proved.

6. In Question 5 above, if the radii of the two circles are equal, prove that AB = CD.

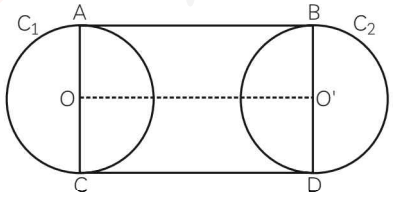
Ans.

Given: AB and CD are two common tangents of two circles of equal radii.

To prove: AB = CD

Construction: Join OA, OC, O'B, O'D, OO'.

Proof:



Since OA = O'B [Given]

Also, $\angle OAB = \angle O'BA = 90°$ [Tangent at any point is perpendicular to radius at the point of contact]

Since perpendicular distance between AB and OO' is the same at two different points:

Hence AB || OO'

Similarly, CD is parallel to OO'

$\Rightarrow$ AB || CD

Also, $\angle AOB = \angle OCD = \angle O'BA$

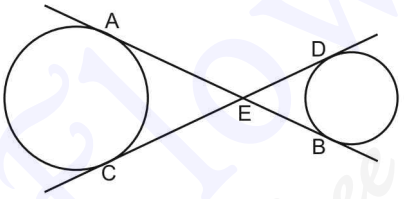
$= \angle O'DC = 90°$

$\Rightarrow$ ABCD is a rectangle.

AB = CD.

Hence, proved.

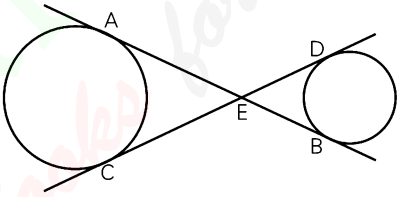
7. In the given figure common tangents AB and CD to two circles intersect at E. Prove that AB = CD. [CBSE 2014]



Ans.

Given: Common tangents AB and CD to two circles intersecting at E.

To prove: AB = CD



Proof: we know that length of tangents drawn from an external point to a circle is equal. Point E is outside of both the circles.

$\therefore$ EB = ED and EA = EC

On adding, we get

EA + EB = EC + ED

$\Rightarrow$ AB = CD

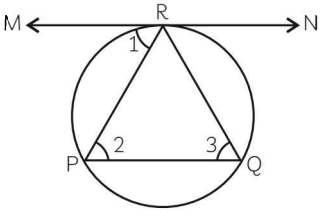
Hence, proved.

8. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Ans.

Given: In a circle, chord PQ is parallel to tangent at R.

To prove: R bisects the arc PRQ.



Proof: Chord RP subtends $\angle 1$ with tangent MN and $\angle 3$ in alternates segment of circle.

$\angle 1 = \angle 2$ [Alternate interior angles]

$\angle 1 = \angle 3$ [Angle between tangent and chord is equal to angle made by chord in alternate segment]

$\Rightarrow$ $\angle 2 = \angle 3$

$\Rightarrow$ PR = QR [Side opposite to equal angles are equal]

$\Rightarrow$ arc PR = arc QR

$\Rightarrow$ Hence, R bisects the arc PQ.

Trick Applied

Angle between tangent and chord is equal to angle made by chord in alternate segment.

9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord. [CBSE 2017]

Ans.

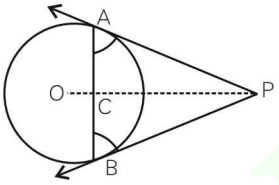
Given: AB be the chord of a circle with centre O. Let AP and BP be tangents at A and B.

Suppose tangents meet at P.

Construction: Join OP.

Suppose OP meets AB at C.

To prove: $\angle PAC = \angle PBC$



In ΔPCA and ΔPCB, we have

PA = PB

[Tangents from an external point are equal]

$\angle APC = \angle BPC$

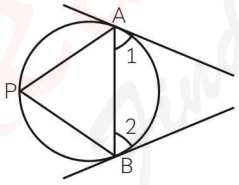
[Since PA and PB are equally inclined to OP]

PC = PC

$\Rightarrow$ $\Delta PAC \cong \Delta PBC$ [By SAS criterion]

$\Rightarrow$ $\angle PAC = \angle PBC$ [By cpct]

Alternate Method:



To prove: $\angle 1 = \angle 2$

Let AB be the chord. Tangents are drawn at A and B.

Let P be another point on the circle.

Join PA and PB.

We know that angles in alternate segment are equal.

$\Rightarrow$ $\angle 2 = \angle P$ and $\angle 1 = \angle P$

$\Rightarrow$ $\angle 1 = \angle 2 = \angle P \Rightarrow \angle 1 = \angle 2$

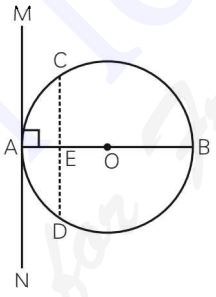
Hence, proved.

10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

Ans.

Given: AB is diameter of circle with centre O.

MAN is tangent to circle at point A.



Construction: Draw a chord CD parallel to tangent MAN.

CD is the chord of the circle and OA is its radius.

$\Rightarrow$ $\angle MAO = 90°$ [Tangent at any point is perpendicular to radius through point of contact]

$\Rightarrow$ $\angle CEO = \angle MAO$ [Corresponding angles]

$\Rightarrow$ $\angle CEO = 90°$

Thus OE bisects CD [Perpendicular from centre to chord bisects the chord]

Similarly, diameter AB bisects all chords which are parallel to the tangent at point A.

EXERCISE 9.4

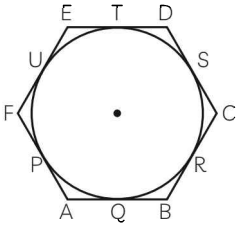
1. If a hexagon ABCDEF circumscribes a circle, prove that AB + CD + EF = BC + DE + FA.

Ans.

Given: Hexagon ABCDEF circumscribes circle.

To prove: AB + CD + EF = BC + DE + FA.

Proof:



Let O be the centre of circle touching sides AB, BC, CD, DE, EF and FA at Q, R, S, T, U, P, respectively.

We know that tangents drawn from an external point to a circle are equal.

$\Rightarrow$ $AQ = AP$ $QB = BR$

$CS = CR$ $DS = DT$

$EU = ET$ $UF = FP$

Now,

$AB + CD + EF = (AQ + QB) + (CS + SD) + (EU + UF)$

$= (AP + BR) + (CR + DT) + (ET + FP)$

$= (AP + FP) + (BR + CR) + (DT + ET)$

$= AF + BC + DE$

$\Rightarrow$ $AB + CD + EF = AF + BC + DE$

Hence, proved.

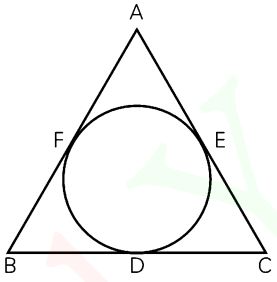
2. Let S denote the semiperimeter of a triangle ABC in which BC = *a*, CA = *b*, AB = *c*. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = *s* – *b*.

Ans.

Given: BC = *a*, CD = *b* and AB = c are sides of ΔABC.

To prove: BD = *s* – *b*

Proof:



We know that tangents drawn from an external point are equal in length. Vertices of ΔABC are in the exterior of circle.

$\Rightarrow$ $AF = AE$, $BD = BF$ and $CD = CE$ ...(i)

We have

$$s = \frac{a+b+c}{2}$$

$$\left[\text{As } s \text{ is semiperimeter } s = \frac{AB+BC+CA}{2}\right]$$

$\Rightarrow$ $a + b + c = 2s$ ...(ii)

Now, $BC + CA + AB = a + b + c$

$(BD + DC) + (CE + EA) + (AF + FB) = a + b + c$

$\Rightarrow (BD + DC) + (CE + EA) + (AF + FB) + AF$

$= a + b + c$

$\Rightarrow 2(BD + CE + AE) = 2s$ [using eqn (i) and (ii)]

$\Rightarrow$ $BD + CE + AE = s$

$\Rightarrow$ $BD = s - (CE + AE)$

Also, $AE + EC = b$

$\Rightarrow$ $BD = (s - b)$

Hence, proved.

3. From an external point P, two tangents, PA and PB, are drawn to a circle with centre O. At one point E on the circle, a tangent is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of the triangle PCD. [CBSE 2014]

Ans.

Given: Two tangents PA and PB to circle with centre O. At point E on the circle, tangent is drawn which intersects PA and PB at C and D respectively.

$PA = 10$ cm

To find: Perimeter of ΔPCD.

We know that tangents from an external point to a circle are equal in length.

$\Rightarrow$ $CE = CA$, $DE = DB$ and $PA = PB$ ...(i)

Perimeter of $\Delta PCD = PC + CD + PD$

$= PC + (CE + ED) + PD$ [As $CD = CE + ED$]

$= PC + CA + DB + PD$ [Using eqn (i)]

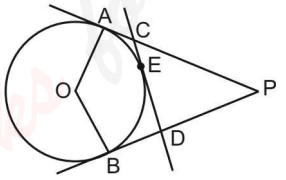
$= PA + PB$

$= 2PA$ [Using eqn (i)]

$= 2 \times 10$ [As PA = 10 cm]

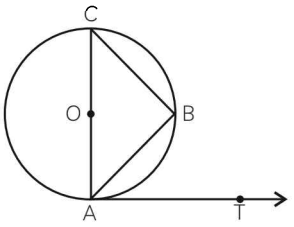
$= 20$ cm

Hence, the perimeter of ΔPCD is 20 cm.



4. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in the figure. Prove that:

$\angle BAT = \angle ACB$ [CBSE 2016]



Ans.

Given: AB is a chord of the circle with centre O.

AOC is the diameter.

AT is tangent at point A.

To prove: $\angle BAT = \angle ACB$

Proof: Radius OA and tangent AT at A are perpendicular.

$\therefore \angle OAT = 90°$ [radius at the point of contact of tangent is perpendicular]

Since AOC is the diameter

$\Rightarrow$ $\angle ABC = 90°$ [Angle in semicircle]

In ΔABC,

$\angle CAB + \angle ABC + \angle BCA = 180°$

[Angle sum property]

$\Rightarrow \angle CAB + \angle BCA = 180° - 90°$

[As $\angle ABC = 90°$]

$\angle CAB + \angle BCA = 90°$ ...(i)

Also, we know that the diameter of a circle is perpendicular to the tangent.

i.e., $CA \perp CT$

$\therefore \angle CAT = 90°$

$\Rightarrow \angle CAB + \angle BAT = 90°$ ...(ii)

From eqn (i) and (ii),

$\angle CAB + \angle BCA = \angle CAB + \angle BAT$

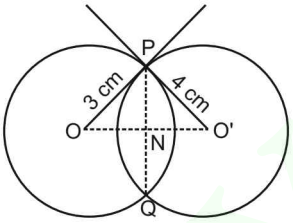
$\Rightarrow \angle ACB = \angle BAT$

$\Rightarrow \angle BAT = \angle ACB$

Hence, proved.

5. Two circles with centres O and O' of radii 3 cm and 4 cm respectively intersect at two points P and Q such that OP and O' P are tangents to the two circles. Find the length of the common chord PQ.

Ans.



Given: Two circles with centres O and O' and radii OP = 3 cm and O'P = 4 cm.

These two circles intersect at P and Q.

To find: PQ = ?

It is given that OP and O'P are two tangents drawn at point P. As radius OP and tangent PO' are at a point of contact P.

$\therefore \angle P = 90°$

$\Rightarrow \angle OPO' = 90°$ [Tangents at any point is perpendicular to radius through point of contact]

Join OO' and PQ such that OO' and PQ intersect at point N.

In $\angle OPO'$:

So, by Pythagoras theorem in right angled $\Delta OPO'$

$(OO')^2 = (OP)^2 + (PO')^2$

$\Rightarrow (OO')^2 = (3)^2 + (4)^2 = 25$

$\Rightarrow OO' = 5$ cm

Also, $PN \perp OO'$

Let $ON = x$, then $NO' = 5 - x$

In right angled $\Delta PNO'$,

$(PO')^2 = (PN)^2 + (NO')^2$

[Pythagoras theorem]

$\Rightarrow (4)^2 = (PN)^2 + (5 - x)^2$

$\Rightarrow (PN)^2 = 16 - (5 - x)^2$ ...(i)

In right angled ΔONP,

$(OP)^2 = (ON)^2 + (NP)^2$

[Pythagoras theorem]

$\Rightarrow (NP)^2 = 3^2 - x^2$

$= 9 - x^2$ ...(ii)

From eqn. (i) and (ii)

$16 - (5 - x)^2 = 9 - x^2$

$\Rightarrow 16 - (25 + x^2 - 10x) = 9 - x^2$

$\Rightarrow 16 - 25 - x^2 + 10x = 9 - x^2$

$\Rightarrow 10x = 9 + 9 \Rightarrow 10x = 18$

$\Rightarrow x = 1.8$

Again, in right angled ΔOPN,

$(OP)^2 = (ON)^2 + (NP)^2$

[Pythagoras theorem]

$\Rightarrow 3^2 = (1.8)^2 + (NP)^2$

$\Rightarrow (NP)^2 = 9 - 3.24 = 5.76$

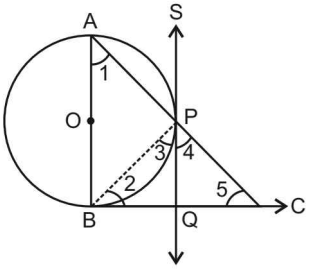
$\Rightarrow NP = 2.4$

$\therefore$ Length of common chord,

$PQ = 2PN = 2 \times 2.4 = 4.8$ cm

6. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

Ans.



Given: ΔABC in which $\angle B = 90°$

Circle with diameter AB intersect the hypotenuse AC at P.

A tangent SPQ at P is drawn to meet BC at Q.

Construction: Join BP.

To prove: BQ = QC

Proof: In ΔABC,

$\angle ABC + \angle BAC + \angle BCA = 180°$

[Angle sum property]

But $\angle ABC = 90°$ [Given]

$\therefore \angle BAC + \angle BCA = 180 - 90 = 90°$

$\Rightarrow \angle 1 + \angle 5 = 90°$

Also, SPQ is tangent and AP is chord at contact point P. Therefore

$\angle 3 = \angle 1$ [Angle between tangent and the chord equals angle made by chord in alternate segment]

$\Rightarrow \angle 3 + \angle 5 = 90°$ ...(i)

Also,

$\angle APB = 90°$ [Angle in semicircle]

$\angle APB + \angle BPC = 180°$ [Linear Pair]

$\Rightarrow 90 + \angle 3 + \angle 4 = 180$

$\Rightarrow \angle 3 + \angle 4 = 90°$ ...(ii)

From eqn (i) and (ii), we get

$\angle 3 + \angle 5 = \angle 3 + \angle 4$

$\Rightarrow \angle 5 = \angle 4$

$\Rightarrow$ PQ = QC [Sides opposite to equal angles are equal]

Also, QP = QB

$\Rightarrow$ QB = QC

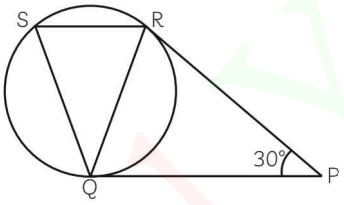
$\Rightarrow$ PQ bisects BC. Q is mid-point of BC.

Hence, proved.

Trick Applied

Tangents drawn from an internal point to a circle are equal.

7. In the given figure tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find the $\angle RQS$. [CBSE 2015]



[Hint: Draw a line through Q and perpendicular to QR]

Ans.

Given: Tangents PQ and PR drawn from an external point P such that $\angle RPQ = 30°$; chord RS || PQ.

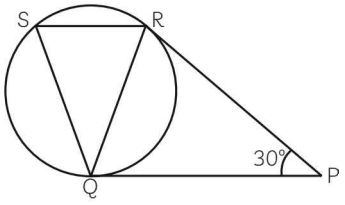
To find: $\angle RQS$.

We know that lengths of tangents drawn from an external point to a circle are equal.

$\Rightarrow$ PQ = PR

$\Rightarrow \angle PQR = \angle QRP$

[Angle opposite to equal sides are equal]



In ΔPQR,

$\angle PQR + \angle QRP + \angle RPQ = 180°$

[Angle sum property]

$\Rightarrow \angle PQR + \angle PQR + 30° = 180°$ [$\angle PQR = \angle QRP$]

$\Rightarrow 2\angle PQR = 180° - 30° = 150°$

$\Rightarrow \angle PQR = 75°$

Also, SR || QP

$\Rightarrow \angle SRQ = \angle RQP = 75°$

[Alternate interior angles]

$\angle PQR = \angle QSR = 75°$ [Angle between tangents and chord is equal to angle made by chord in alternate segment]

In ΔQRS,

$\angle Q + \angle R + \angle S = 180°$

$\angle Q = 180° - (\angle R + \angle S)$

$= 180° - (75° + 75°)$

$= 30°$

Hence, $\angle RQS = 30°$

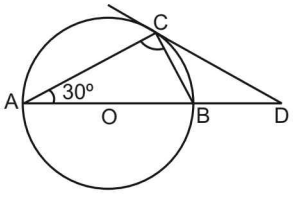
8. AB is a diameter and AC is a chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects extended AB at a point D. Prove that BC = BD. [CBSE 2015]

Ans.

Given: AB is diameter and AC is a chord of a circle with centre O, $\angle BAC = 30°$.

To prove: BC = BD

Construction: Join BC.



Proof: DC is tangent at C and, CB is chord at C, Therefore,

$\angle BCD = \angle CAB$

[Angles in alternate segment]

$\angle CAB = 30°$ [Given]

$\therefore \angle BCD = 30°$

AOB is diameter. [Given]

$\angle ACB = 90°$ [Angle in semicircle]

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ [Angle sum property]

$30° + \angle CBA + 90° = 180°$

$\angle CBA = 180° - 120° = 60°$

Also, $\angle CBA + \angle CBD = 180°$ [Linear pair]

$\Rightarrow \angle CBD = 180° - 60° = 120°$

[$\because \angle CBA = 60°$]

Now, in ΔCBD,

$\angle CBD + \angle BDC + \angle DCB = 180°$

$\Rightarrow 120° + \angle BDC + 30° = 180°$ [Angle sum property]

$\Rightarrow \angle BDC = 30°$ ...(ii)

From eqn (i) and (ii)

$\angle BCD = \angle BDC = 30°$

$\Rightarrow$ BC = BD [Side opposite to equal angles are equal]

Hence, proved.

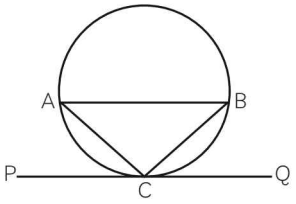
9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. **[CBSE 2015]**

Ans.

Given: arc BAC in which C is mid point of arc BAC. PCQ is tangent at point C.

To prove: AB || PCQ

Construction: Join AB, AC and CB.



Proof: Since C is the midpoint of arc ACB

$\Rightarrow$ arc AC = arc CB

$\Rightarrow$ chord AC = chord CB

In ΔACB,

CA = CB

$\Rightarrow \angle CAB = \angle CBA$ [Equal sides corresponding to equal angles]...(i)

Also, PCQ is tangent

$\Rightarrow \angle ACP = \angle ABC$ [Angles in alternate segments are equal]

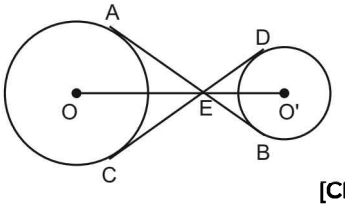
$\Rightarrow \angle ACP = \angle ABC = \angle CAB$ [Using eqn (i)]

But $\angle ACP$ and $\angle CAB$ are alternate angles.

$\Rightarrow$ AB || PCQ

Hence, tangent drawn at midpoint of an arc of a circle is parallel to the chord joining the end point of the arc.

10. In the given figure common tangents, AB and CD to two circles with centres O and O', intersect at E. Prove that the points O, E, O' are collinear.



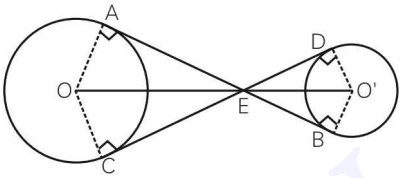
[CBSE 2014]

Ans.

Given: Two circles with centres O and O'. Common tangents, AB and CD to these two circles intersect at E.

To prove: O, E, O' are collinear.

Construction: Join AO, OC, O'D and O'B



Proof: In ΔOAE and OCE,

OA = OC [Radii of same circle]

OE = OE [Common]

EA = EC [Tangents from external point are equal in length]

$\therefore$ ΔEOA ≅ ΔEOC [SSS congruence criterion]

$\Rightarrow \angle OEA = \angle OEC$

i.e., OE is angle bisector of $\angle AEC$.

Similarly, O'E is angle bisector of $\angle DEB$.

$\Rightarrow \angle AEC = 2\angle AEO$ ...(i)

and $\angle DEB = 2\angle O'ED$ ...(ii)

Now as CD is a straight line

$\therefore \angle CEA + \angle AED = 180°$ [Linear pair]

$2\angle AEO = 180° - \angle AED$ [Using eqn (i)]

$$\angle AEO = 90° - \frac{1}{2}\angle AED \quad ...(iii)$$

Now, as AB is a striaght line

$\therefore \angle AED + \angle DEB = 180°$ [Linear pair]

$2\angle O'ED = 180° - \angle DEA$ [Using eqn (ii)]

$$\angle O'ED = 90° - \frac{1}{2}\angle AED \quad ...(iv)$$

Now,

$\angle AEO + \angle AED + \angle O'ED$

$$= 90° - \frac{1}{2}\angle AED + \angle AED + 90° - \frac{1}{2}\angle AED$$

[Using eqn (iii) and (iv)]

$= 90° + 90° = 180°$

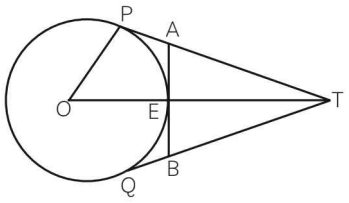
$\Rightarrow \angle AEO + \angle AED + \angle O'ED = 180°$

So O, E and O' lie on the same line

i.e., O, E and O' are collinear.

Hence, proved.

11. In the given figure O is the centre of a circle of radius 5 cm, T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB. **[CBSE 2016]**



Ans.

Given: Circle with centre O having radius 5 cm.

T is a point such that

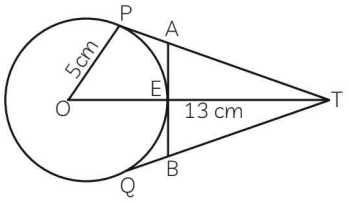
$OT = 13$ cm $OP = 5$ cm

PT, QT and AB are tangents.

To find: AB

We know that tangents at any point is perpendicular to radius through point of contact.

$\Rightarrow$ OP $\perp$ PT, OE $\perp$ AB



by Pythagoras theorem

In ΔOPT,

$$OT^2 = OP^2 + PT^2$$

$$\Rightarrow \quad PT^2 = OT^2 - OP^2 = (13)^2 - (5)^2$$

$$= 169 - 25 = 144$$

$$\Rightarrow \quad PT = 12 \text{ cm}$$

TP and TQ are two tangents from an external point T to a circle.

$$TP = TQ$$

or $QT = 12$ cm

$\therefore$ $QT = 12$ cm

Now $TA = PT - PA$

$\Rightarrow$ $TA = 12 - PA$ ...(i)

and $TB = QT - QB$

$TB = 12 - QB$ ...(ii)

Also, PA = AE and QB = EB [Tangents from external point are equal in length] ...(iii)

$\Rightarrow$ $ET = OT - OE$

But $OT = 13$ cm [Given]

$OE = 5$ cm [Radius]

$\therefore$ $ET = 13 - 5 = 8$ cm

We know that OE $\perp$ AB

$\Rightarrow$ $\angle OEA = 90°$

$\therefore$ $\angle AET = 180° - \angle OEA$ [Linear pair]

$\Rightarrow$ $\angle AET = 180° - 90°$

$\Rightarrow$ $\angle AET = 90°$

In right angled ΔAET,

$$AT^2 = AE^2 + ET^2$$

[By Pythagoras theorem]

Using eq. (i) and (iii)

$$\Rightarrow \quad (12 - PA)^2 = (PA)^2 + (8)^2$$

$$\Rightarrow \quad 144 + (PA)^2 - 24\,PA = (PA)^2 + 64$$

$$\Rightarrow \quad 24PA = 80 \Rightarrow PA = \frac{10}{3} \text{ cm}$$

$$\therefore \quad AE = \frac{10}{3} \text{ cm} \quad \text{[From equation (iii)]}$$

Similarly, $BE = \frac{10}{3}$ cm

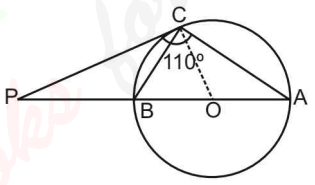
$$AB = AE + EB$$

$$= \frac{10}{3} + \frac{10}{3} = \frac{20}{3} \text{ cm}$$

Hence, the required length of AB is $\frac{20}{3}$ cm.

12. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110°$, find $\angle CBA$ [see figure].

[Hint: Join C with centre O.]



Ans.

Given: AB is diameter of circle with centre O. Tangents at point C and AB when extended intersect at P.

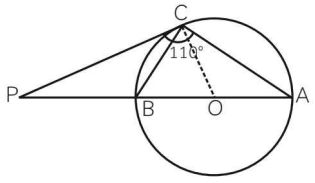
$\angle PCA = 110°$

To find: $\angle CBA$

Construction: Join OC.

OC is radius and CP is tangent.

$\Rightarrow$ OC $\perp$ PC [Tangent at any point is perpendicular to radius through point of contact]



$\angle PCA = 110°$ [Given]

$\Rightarrow$ $\angle PCA + \angle OCA = 110°$

$\Rightarrow$ $\angle OCA = 110° - \angle PCO$

$= 110° - 90°$

$\angle OCA = 20°$

In ΔOAC,

$OA = OC$ [Radii of same circle]

$\therefore \angle OCA = \angle OAC = 20°$ [$\because$ Angles opposite to equal sides are equal]

CP and CB are tangent and chord of a circle at point C and we know that angles in alternate segments are equal

$\therefore \quad \angle BCP = \angle CAB$

As AB is diameter

$\therefore \quad \angle BCA = 90°$ [Angle in a semicircle]

$\angle PCA = \angle PCB + \angle BCA$

$110° = \angle PCB + 90°$

$\Rightarrow \quad \angle PCB = \angle CAB = 20°$

In ΔPAC,

$\angle CPA + \angle PCA + \angle PAC = 180°$

$\angle CPA = 180° - (\angle PCA + \angle PAC)$

$= 180° - (110° + 20°)$

$= 180° - 130° = 50°$

In ΔPBC,

$\angle BPC + \angle PCB + \angle CBP = 180°$

$\Rightarrow 50 + 20° + \angle CBP = 180°$

$\Rightarrow \quad \angle CBP = \angle PBC = 180° - 70°$

$\Rightarrow \quad \angle PBC = 110°$

ABP is a straight line

$\therefore \quad \angle PBC + \angle CBA = 180°$

$\angle CBA = 180° - 110° = 70°$

Hence, $\angle CBA = 70°$

13. If an isosceles triangle ABC, in which AB = AC = 6 cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.

Ans.

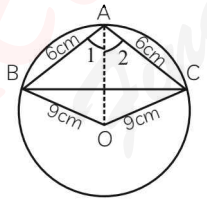
Given: Isosceles triangle ABC with

$AB = AC = 6$ cm

Radius of circle = 9 cm

To find: Area of ΔABC.

Construction: Join OB, OC and OA.



In ΔABO and ΔACO

$AB = AC$ [Given]

$OB = OC$ [Radii of same circle]

$AO = AO$ [Common]

$\Rightarrow \quad \Delta ABO \cong \Delta ACO$

[By SSS congruence criterion]

$\Rightarrow \quad \angle 1 = \angle 2$ [By cpct]

Now in ΔABD and ΔACD,

$AB = AC$ [Given]

$\angle 1 = \angle 2$ [Proved above]

$AD = AD$ [Common]

$\therefore \quad \angle ADB \cong \Delta ADC$

[By SAS congruence criterion]

$\Rightarrow \quad \angle ADB = \angle ADC$ [By cpct]

But $\angle ADB + \angle ADC = 180°$ [Linear pair]

$\Rightarrow \quad \angle ADB = \angle ADC = 90°$

We know that perpendicular from centre of circle bisects the chord.

So, OA is perpendicular bisector of BC.

Let $AD = x$

$OA =$ radius $= 9$ cm

$\therefore \quad OD = OA - AD = (9 - x)$ cm

In right angled triangle ODC,

$OC^2 = OD^2 + DC^2$

[By Pythagoras theorem]

$\Rightarrow \quad DC^2 = OC^2 - OM^2$

$= 9^2 - (9 - x)^2$ ...(i)

In right angled ΔADC,

$AC^2 = AD^2 + DC^2$

[Pythagoras theorem]

$\Rightarrow \quad DC^2 = 6^2 - x^2$ ...(ii)

From eqn (i) and (ii)

$6^2 - x^2 = 9^2 - (9 - x)^2$

$\Rightarrow \quad 36 - x^2 = 81 - (81 + x^2 - 18x)$

$\Rightarrow \quad 36 - x^2 = 81 - 81 - x^2 + 18x$

$\Rightarrow \quad 18x = 36 \Rightarrow x = 2$

$\therefore \quad AD = 2$ cm

From eqn (i)

$DC^2 = 9^2 - (9 - 2)^2$

$= 81 - 49 = 32$

$\Rightarrow \quad DC = 4\sqrt{2}$ cm

$\therefore \quad BC = 2\,MC = 8\sqrt{2}$ cm

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times BC \times AM$$

$$= \frac{1}{2} \times 8\sqrt{2} \times 2$$

$$= 8\sqrt{2}\ \text{cm}^2$$

Hence, the required area of ΔABC is $8\sqrt{2}\ \text{cm}^2$.

14. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on

the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC.

[CBSE 2012]

Ans.

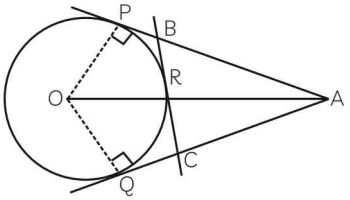
Given: Two tangents AP and AQ, drawn from an external point A to the circle with centre O.

Tangent BC is drawn at point R.

$OA = 13$ cm

Radius of circle = 5 cm

To find: Perimeter of ΔABC.



We know that tangent at any point is perpendicular to radius through point of contact.

$\Rightarrow$ $OP \perp AP$

$\therefore$ $\angle OPA = 90°$

by Pythagoras theorem

In ΔOPA,

$OA^2 = OP^2 + PA^2$

$\Rightarrow$ $13^2 = 5^2 + PA^2$

$\Rightarrow$ $PA^2 = 169 - 25 = 144$

$\Rightarrow$ $PA = 12$ cm

Perimeter of ΔABC = AB + BC + CA [From figure]

= AB + (BR + RC) + CA

= (AB + BR) + (RC + CA)

= (AB + BP) + (CQ + CA)

Points A, B and C are exterior to the circle and tangents drawn from an external point to a circle are equal so PA = QA, BP = BR.

CR = CQ

= AP + AQ

= 2AP

= 2 × 12 = 24 cm.

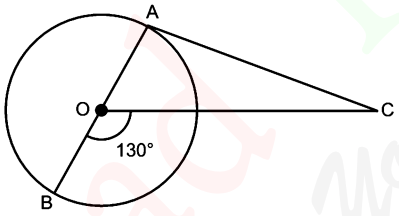
Hence, the perimeter of ΔABC = 24 cm.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. In the given figure, AOB is a diameter of the circle with center O and AC is a tangent to the circle at A. If ∠BOC = 130°, then find ∠ACO.



Ans.

Here,

OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have,

$\angle OAC = 90°$

From exterior angle property,

$\angle BOC = \angle OAC + \angle ACO$

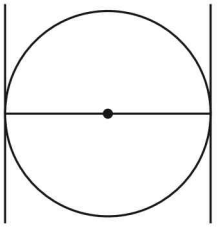
$130° = 90° + \angle ACO$

$\angle ACO = 130° - 90° = 40°$.

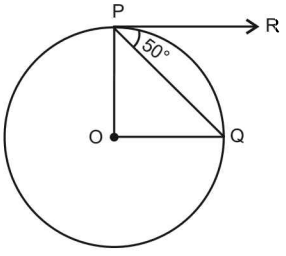
2. What is the maximum number of parallel tangents a circle can have on a diameter?

Ans.

Tangent touches a circle on a distinct point. Thus on the diameter of a circle only two parallel tangents can be drawn. It has been shown in figure given below:



3. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find ∠POQ.



Ans.

In this figure,

$\angle RPQ = 50°$

Since, $\angle OPQ + \angle QPR$ is a right angle.

As tangent makes an angle of 90° with radius:

$\angle OPQ = 90° - 50° = 40°$

Since,

$OP = OQ$ [Radii of a circle]

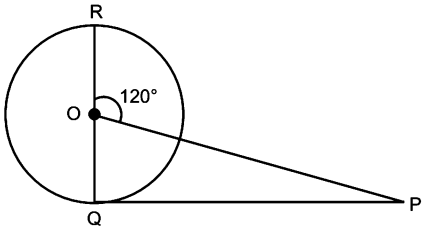
$\angle OPQ = \angle OQP = 40°$

In ΔPOQ,

$\angle POQ = 180° - (40° + 40°)$

$\angle POQ = 100°$.

4. PQ is a tangent drawn from an external point P to a circle with center O, ∠QOR is the diameter of the circle. If ∠POR = 120°. What is the measure of ∠OPQ?



Ans.

Since, PQ is a tangent to the circle, ΔOQP is right angle triangle.

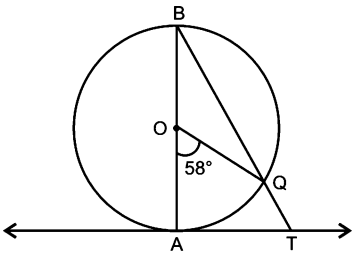
In ΔOQP ,

$\angle POR = \angle OQP + \angle OPQ$ [Exterior angle]

Thus $\angle OPQ = \angle POR - \angle OQP$

$= 120° - 90° = 30°$

5. In the figure, AB is the diameter of a circle with center O and AT is a tangent. If ∠AOQ = 58°, find ∠ATQ.



Ans.

We have $\angle AOQ = 58°$

Since, $\angle ABQ$ and $\angle AOQ$ are the angles formed by the same are on the circumference and centre of the circle respectively.

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2}58°$$

$$= 29°$$

Here, OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus, $\angle BAT = 90°$

$\angle ABQ = \angle ABT$

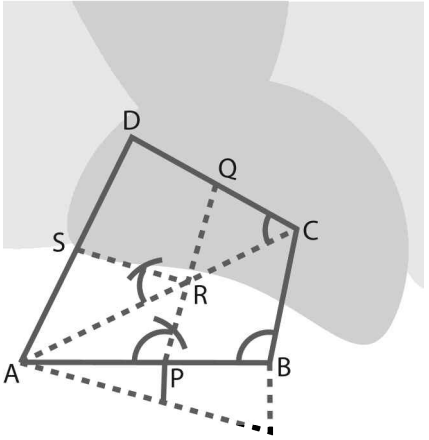
Now in ΔBAT ,

$\angle ATB = 90° - \angle ABT$

$= 90° - 29°$

$= 61°$

Thus, $\angle ATQ = \angle ATB = 61°$.



10 Constructions

EXERCISE 10.1

Choose the correct option from the given four options:

1. To divide a line segment AB in the ratio 5:7, first a ray AX is drawn, so that ∠BAX is an acute angle. Then at equal distances, points are marked on the ray AX such that the minimum number of these points is:

(A) 8 (B) 10

(C) 11 (D) 12

Ans. (D)

Explanation: To divide a line segment AB in the ratio m:n, first a ray AX is drawn which makes an acute angle ∠BAX and then (m + n) points are marked at equal distances on the ray AX.

Here, m = 5 and n = 7.

Therefore, the minimum number of points to be marked on AX = m + n = 5 + 7 = 12.

2. To divide a line segment AB in the ratio 4:7, a ray AX is drawn first such that ∠BAX is an acute angle and then points A_1, A_2, A_3, ... are located at equal distances on the ray AX and the point B is joined to:

(A) A_{12} (B) A_{11}

(C) A_{10} (D) A_9

Ans. (B)

Explanation: To divide a line segment AB in the ratio m:n, first a ray AX is drawn which makes an acute angle ∠BAX and then (m + n) points are marked at equal distances on the ray AX.

Here, m = 4 and n = 7.

Therefore, minimum number of points to be marked on AX = m + n = 4 + 7 = 11.

Since 4 + 7 = 11 points are to be located on AX at equal distances, so B is joined to last point i.e. A_{11}.

3. To divide a line segment AB in the ratio 5:6, draw a ray AX such that ∠BAX is an acute angle. Then draw a ray BY parallel to AX and the points A_1, A_2, A_3,... and B_1, B_2, B_3,... are located at equal distances on ray AX and BY, respectively. Then, the points joined are:

(A) A_5 and B_6 (B) A_6 and B_5

(C) A_4 and B_5 (D) A_5 and B_4

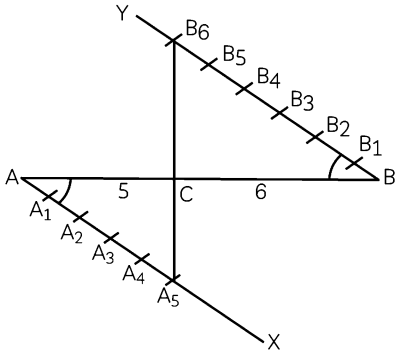
Ans. (A)

Explanation: To divide line segment AB in the ratio 5:6 i.e., m:n = 5:6.

Steps of construction:

1. Draw a ray AX making an acute ∠BAX.
2. Draw a ray BY parallel to AX by taking ∠ABY equal to ∠BAX.
3. Divide AX into five (m = 5) equal parts, AA_1, A_1A_2, A_2A_3, A_3A_4 and A_4A_5
4. Divide BY into six (n = 6) equal parts, BB_1, B_1B_2, B_2B_3, B_3B_4, B_4B_5 and B_5B_6.
5. Join A_5B_6 Let it intersect AB at a point C.

Then, AC:BC = 5:6.



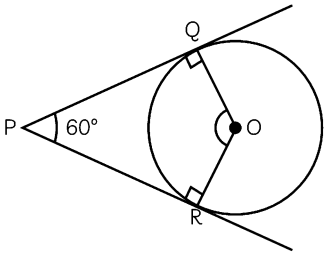
4. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at the end points of those two radii of the circle. The angle between them should be:

(A) 135° (B) 90°

(C) 60° (D) 120°

Ans. (D)

Explanation: Let PQ and PR be the two tangents at points Q and R on a circle with centre O.



Given: ∠RPQ = 60°

We know that the tangent at a point on a circle is perpendicular to the radius through that point.

$\Rightarrow$ $\angle Q = \angle R = 90°$

In quadrilateral PQOR,

$\angle P + \angle Q + \angle O + \angle R = 360°$

$\Rightarrow$ $60° + 90° + 90° + \angle O = 360°$

$\Rightarrow$ $\angle O = 360° - 240° = 120°$

Hence, the required angle between two radii is 120°.

EXERCISE 10.2

Write True or False and give reasons for your answer:

1. By geometrical construction, it is possible to divide a line segment in the ratio $\sqrt{3}:\left(\frac{1}{\sqrt{3}}\right)$.

Ans. True.

According to the question,

$$\text{Ratio} = \sqrt{3}:\left(\frac{1}{\sqrt{3}}\right)$$

On simplifying the given ratio, we get,

$$\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}\times\sqrt{3}}{1} = 3:1$$

Required ratio = 3:1 and 3 and 1 both are positive integers.

Hence, it is possible to divide a line segment in the ratio 3:1 by geometrical construction.

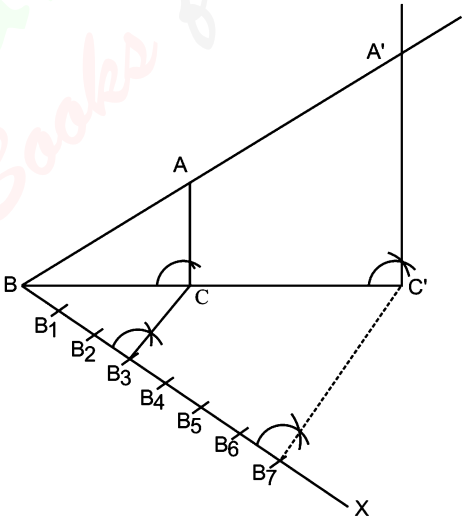
2. To construct a triangle similar to a given ΔABC with its sides $\frac{7}{3}$ of the corresponding sides of ΔABC, draw a ray BX making an acute angle with BC and X lies on the opposite side of A with respect to BC. The points B_1, B_2, ..., B_7 are located at equal distances on BX. B_3 is joined to C and then a line segment is drawn parallel to B_3C where C' lies on BC produced. Finally, line segment A'C is drawn parallel to AC.

Ans. False.

As the sides of new triangle will be equal to $\frac{7}{3}$ of the corresponding sides of ΔABC. Therefore, it will be larger than ΔABC. Hence, B_7C' (larger of $\frac{7}{3}$) instead of B_6C', is drawn parallel to B_3C.

Steps of construction:

1. Draw the given ΔABC.
2. From B, draw any ray BX downwards, making an acute angle CBX.
3. Now, mark seven points B_1, B_2, B_3 ... B_7 on BX, such that $BB_1 = B_1B_2 = B_3B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
4. Join B_3C and draw a line $B_7C' \parallel B_3C$ from B_7 such that it intersects the extended line segment BC at C'.
5. From point C' draw $C'A' \parallel CA$ in such a way that it intersects the extended line segment BA at A'.
6. Then, ΔA'BC' is the required triangle whose sides are $\frac{7}{3}$ of the corresponding sides of ΔABC.



3. A pair of tangents can be constructed to a circle inclined at an angle of 170°.

Ans. True.

It is possible to draw a pair of tangents to a circle, inclined at an angle between 0° and 180°. Hence, we can draw a pair of tangents to a circle inclined at an angle of 170°.

Let us draw two tangents from point P to the circle with centre O at point Q and point R. OQ and OR are joined. We have OQ and OR as radii of the circle.

$\Rightarrow$ $\angle OQP$ and $\angle ORP = 90°$

Since PQ and PR are tangents to the circle at Q and R respectively,

$\Rightarrow$ $\angle OQP + \angle ORP = 180°$ ...(i)

But $\angle QPR + \angle QOR + (\angle OQP + \angle ORP) = 360°$

[Angle sum property of quadrilateral]

$\Rightarrow$ $\angle QPR + \angle QOR + 180° = 360°$

[From eqn. (i)]

$\Rightarrow$ $\angle QPR + \angle QOR = 180°$

Hence, $\angle QPR$ and $\angle QOR < 180°$.

The given angle is 170° which is less than 180°.

Trick Applied

If angle between pair of tangents is greater than 180° then tangents can never be constructed.

EXERCISE 10.3

1. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3:5. [CBSE 2015, 11]

Ans.

Method 1:

Steps of construction:

1. Draw a line segment, AB = 7 cm.
2. Draw a ray AX, making an acute angle with line segment AB i.e., $\angle BAX$.
3. Divide AX, in to 3 + 5 = 8 equal parts and mark the points A_1, A_2, A_3 ... A_8 on AX such that $AA_1 = A_1A_2 = A_2A_3 = ..., A_7A_8$.
4. Join A_8B.
5. From A_3, draw a line parallel to A_8B to meet AB at point P.
6. Thus, P is the required point on AB which divides it in the ratio 3:5.

Hence, AP:PB = 3:5

Justification:

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = ... = A_7A_8 = x$

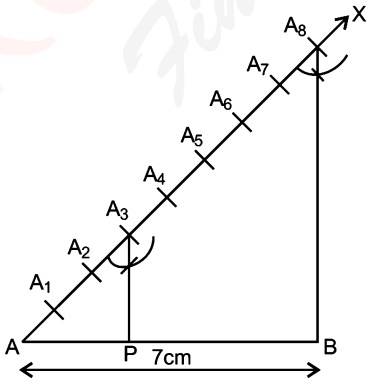
In ΔABA_8, $A_3P \parallel A_8B$

$\Rightarrow$ $\frac{AP}{PB} = \frac{AA_3}{A_3A_8}$

[By basic proportionality theorem]

$\Rightarrow$ $\frac{AA_3}{A_3A_8} = \frac{3x}{5x} = \frac{3}{5}$ [By construction]

Hence, AP:PB = 3:5



Method 2:

Steps of construction:

1. Draw a line segment AB = 7 cm.
2. Draw any ray AX making an acute angle with AB.
3. Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.
4. Locate the points A_1, A_2 and A_3 (m = 3) on AX and B_1, B_2, B_3, B_4 and B_5 (n = 5) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
5. Join A_3B_5.
6. Let it intersect AB at a point P and divides AP:PB = 3:5.

Hence, P is the required point on AB which divides it in 3:5.

Verification (Justification):

In ΔAA_3P and ΔBB_5P,

AX || BY [By construction]

$\angle A = \angle B$ [Alternate angles]

$\angle A_3PA = \angle B_3PB$ [Vertically opp. angles]

$\Delta AA_3P \sim \Delta BB_5P$

[By AA criterion of similarity]

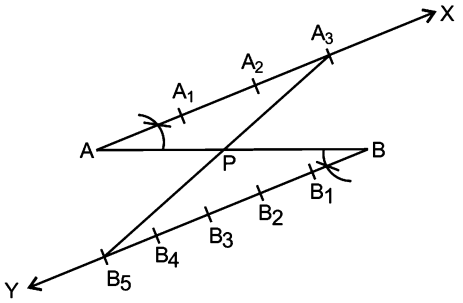
Let each equal part = x cm

$\Rightarrow$ $\frac{AA_3}{AA_5} = \frac{AP}{BP}$

[By basic proportionality theorem]

$\Rightarrow$ $\frac{AP}{BP} = \frac{3x}{5x} = \frac{3}{5}$ [By construction]

Hence, AP:PB = 3:5.



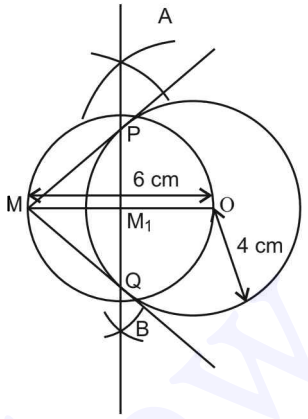
2. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre. [CBSE 2020, 13]

Ans.

This distance of point from which tangents to be drawn should be more than radius so that tangents can be drawn.

Steps of construction:

1. Draw a circle of radius 4 cm. Let O be the centre of the circle.
2. Take a point M at a distance 6 cm away from the radius.
3. Join OM and bisect it.
   (A) With M and O as centres and with radius more than half, draw two arcs on either sides of line OM.
   (B) Let the arc meet at A and B. Join AB such that it meets OM at M_1.
   (C) M_1 will be the mid point of OM.
4. Taking M_1 as centre and M_1O as radius, draw a circle to intersect the circle with radius 4 cm and centre O at two points P and Q.
5. Join PM and QM. PM and QM are the required tangents from M to circle with centre O and radius 4 cm.



Trick Applied

Step (i) Draw perpendicular bisector of distance from centre to external point.
Step (ii) Taking one-half as radius draw circle.
Step (iii) Two circles intersect at two points.
Step (iv) Join these points to an external point and get required tangents.

EXERCISE 10.4

1. Two-line segments AB and AC include an angle of 60°, where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC respectively, such that AP = $\frac{3}{4}$ AB and AQ = $\frac{1}{4}$ AC. Join P and Q and measure the length PQ.

Ans. Justification:

Given that, AB = 5 cm and AC = 7 cm

Also, $AP = \frac{3}{4} AB$ and $AQ = \frac{1}{4} AC$ ...(i)

From eq. (i), we get

$$AP = \frac{3}{4} \times 5 = \frac{15}{4} \text{ cm}$$

Then, PB = AB – AP

$$PB = 5 - \frac{15}{4} = \frac{20-15}{4} = \frac{5}{4} \text{ cm}$$

[As P is any point on AB]

$$AP:PB = \frac{15}{4}:\frac{5}{4}$$

Hence, AP:PB = 3:1 i.e., scale factor of line segment AB is $\frac{3}{1}$.

Again from eq. (i),

$$AQ = \frac{1}{4} AC = \frac{1}{4} \times 7 = \frac{7}{4} \text{ cm}$$

Then, $QC = AC - AQ = \frac{7}{4}:\frac{21}{4} = 1:3$

[As Q is any point on AC]

$$AQ:QC = \frac{7}{4}:\frac{21}{4} = 1:3$$

Hence, AQ:QC = 1:3 i.e., scale factor of line segment AC is $\frac{1}{3}$.

Steps of construction:

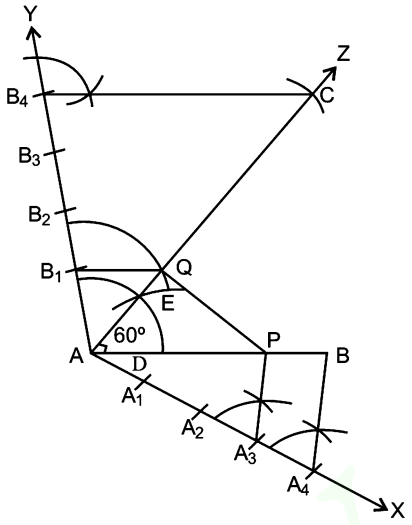
1. Draw a line segment AB = 5 cm.
2. Draw a ray AZ making an acute ∠BAZ = 60°.
   (A) With A as centre and with any radius, draw another arc cutting line AB at D.
   (B) With D as centre and with the same radius, draw an arc cutting the first arc (drawn in step A) at point E.
   (C) Now join ray AZ which forms an angle of 60° with line AB.
3. With centre A and radius 7 cm, draw an arc cutting the line AZ at C.
4. Draw a ray AX, making an acute angle ∠BAX.
5. Divide AX into four equal parts, namely $AA_1 = A_1A_2 = A_2A_3 = A_3A_4$.
6. Join A_4B.
7. From A_3, draw $A_3P \parallel A_4B$ meeting AB at P (by making an angle equal to $\angle AA_4B$)

Then, P is the point on AB which divides it in the ratio 3:1, such that AP:PB = 3:1.

8. Draw a ray AY, making an acute ∠CAY.
9. Divide AY into four parts, namely $AB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
10. Join B_4C.
11. From B_1, draw $B_1Q \parallel B_4C$ meeting AC at Q. (by making an angle equal to $\angle AB_4C$).

    Then, Q is the point on AC which divides it in the ratiol 1:3.

    So, AQ:QC = 1:3
12. Finally, join PQ and its measurement is 3.25 cm.



2. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on the outer circle, construct a pair of tangents to the other. Measure the length of a tangent and verify it by calculation. [CBSE 2016]

Ans.

Steps of construction:

1. Draw two concentric circles C_1, C_2 of radii 3 cm and 5 cm respectively taking 'O' as centre.
2. Taking any point P on the outer circle, join OP.
3. Draw perpendicular bisector MN of OP. Let M' be the mid-point of OP To bisect OP
   (A) With P as centre and any radius more than half of the length of OP, draw two arcs on either side of OP.
   (B) Similarly, with O (centre) and any radius more than half of the length of OP, draw two arcs on either side of OP. It intersect previous arcs at M and N.
   (C) Join MN to meet line OP at M', which is the mid point.
4. Draw a circle Taking M' as the centre and OM' as the radius, which cuts the inner circle at A and B.
5. Join PA and PB. Thus, PA and PB are the required tangents.
6. On measuring, we find that PA = PB = 4 cm.

Verification:

In the right angled ΔOAP,

∠PAO = 90°

According to Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

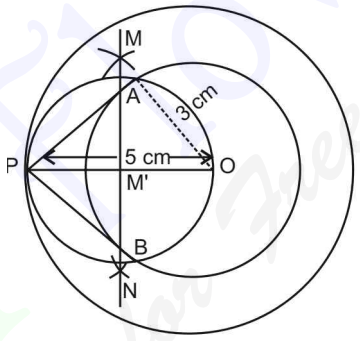
$PA^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

PA = 4 cm

Hence, the length of both the tangents is 4 cm.

Therefore, PA = PB = 4 cm

Hence, verified.



3. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60°. Also justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents. [CBSE 2016, 14, 13, 11]

Ans.

Steps of construction:

1. Draw a circle of radius 4 cm with centre O.
2. Extend the line segment OA to B such that OA = AB = 4 cm.
3. Taking A as the centre, draw a circle of radius 4 cm. This circle intersects the first circle drawn in step 1 at P and Q.
4. Join BP and BQ to get the desired tangents.

Justification: In ΔOAP, we have

OA = OP = 4 cm [Radius]

Also, AP = 4 cm

[Radius of circle with centre A]

∴ ΔOAP is equilateral

So, ∠PAO = 60°

Now, ∠BAP + ∠PAO = 180° [Linear angle]

∠BAP + 60° = 180°

∠BAP = 120°

In ΔBAP, we have BA = AP = 4 cm

[Radii of the circle with centre A]

∠BAP = 120°

As two sides BA and AP are equal, therefore ΔABP is isosceles. So, ∠ABP = ∠APB

Let ∠ABP = ∠APB = α

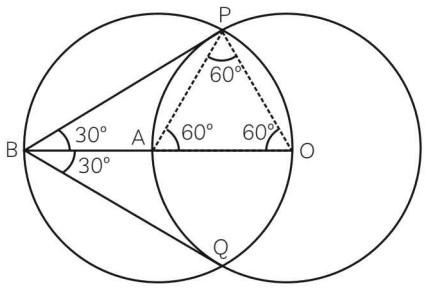
$\angle ABP + \angle APB + \angle BAP = 180°$

$\alpha + \alpha + 120° = 180°$

$2\alpha = 60° \Rightarrow \alpha = 30°$

Therefore, $\angle ABP = \angle APB = 30°$

Hence, $\angle PBQ = 60°$



Alternate method:

Steps of construction:

1. Draw a circle of radius 4 cm with centre O.
2. At O construct radii OA and OB such that $\angle AOB = 120°$ i.e., supplement of the angle between the tangents.

   To draw 120° angle at O:

   (A) With point O as the centre, draw an arc with any radius such that the arc meets ray OA at point C.

   (B) With C as centre and with the same radius as before, draw another arc cutting the previous one at point D.

   (C) Now with D as centre and with the same radius, draw an arc cutting the first arc (drawn in step A) at point E.

   (D) Join OE and produce it to meet the circle at point A.
3. Draw perpendicular to OA and OB at A and B respectively.

   The perpendicular lines to OA ad OB will make an angle of 90° at the point of intersection i.e., at point A and B respectively. (So, we can measure 90° or construct 90° angles by taking A and B as the points of construction like in point 2.)
4. Let us suppose these perpendiculars intersect at P. Then, PA and PB are the required tangents.
5. Now, if we measure the $\angle APB = 60°$ and the distance between PO = 8 cm.

Justification:

From the quadrilateral OAPB, we have

$\angle OAP = \angle OBP = 90°$ [As PA and PB are perpendicular to OA and OB]

$\angle AOB = 120°$ [As per construction]

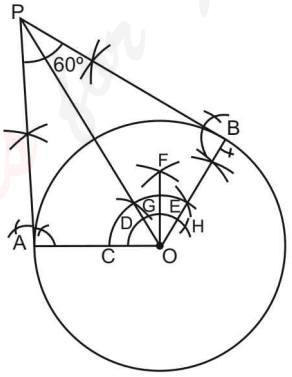
So $\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$

$90° + 90° + 120° + \angle APB = 360°$

$\angle APB = 360° - (90° + 90° + 120°)$

$= 360° - 300° = 60°$

Therefore, $\angle APB = 60°$.



1. To divide a line segment AB in the ratio 2:5, first a ray AX is drawn, so that BAX is an acute angle and then at equal distance points are marked on the ray AX such that the minimum number of these point is:

(A) 2 (B) 5

(C) 4 (D) 7

Ans. (D)

Explanation: We know that,

To divide a line segment AB in the ratio m:n.

First draw a ray AX which makes an acute angle BAX then, marked $m + n$ points at equal distance.

Here, $m = 2$, $n = 5$

Minimum number of these points = 2 + 5 = 7.

2. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as:

(A) scale factor (B) length factor

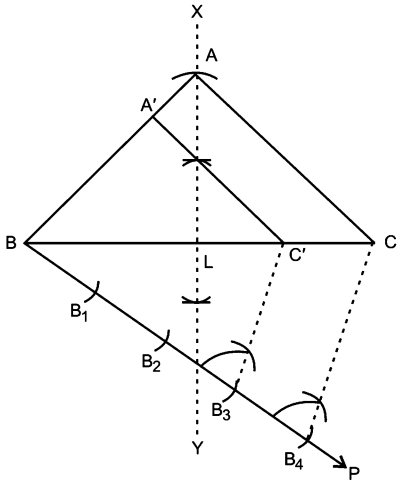
(C) side factor (D) K-factor

Ans. (A)

Explanation: The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as scale factor.

3. Draw an isosceles ΔABC in which BC = 5.5 cm and altitude AL = 3 cm. Then, construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Ans.

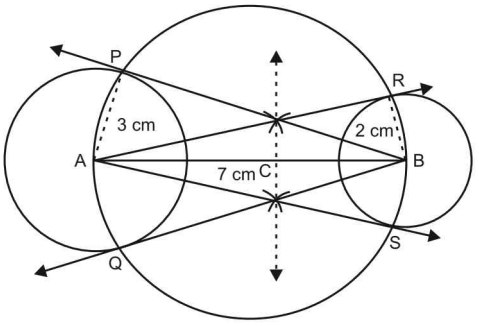


Steps of construction:

1. Draw a line segment BC = 5.5.
2. Perpendicular bisector XY of BC is drawn intersecting BC at L.
3. Point A is marked on XL such that AL = 3 cm
4. AB and AC are joined.
5. An acute angle CBP is drawn below BC.
6. On BP, points B_1, B_2, B_3, and B_4 are marked such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
7. B_4C is joined.
8. B_3C is drawn parallel to B_4C intersecting BC at C.
9. CA is drawn parallel to CA intersecting BA at A.
10. ABC is the required Δ.

4. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Ans.



Required tangents are BP and BQ.

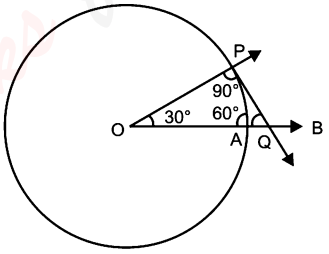
AR and AS .

Steps of construction:

1. Draw AB = 7 cm. Taking A and B as centres, draw two circles of 3 cm and 2 cm radius respectively.
2. Bisect line AB, let mid point of AB be C.
3. Taking C as centre, draw circle of AC radius which will intersect circles at P, Q, R, S.
4. Join BP, BQ, AR, AS. These are the required tangents.

5. Draw a circle of radius 4 cm. Draw a tangent to the circle, making an angle of 60° with a line passing through the centre.

Ans.



Steps of construction:

1. Draw a circle with centre O and radius 4 cm.
2. Make radius OA and produce it to B.
3. Make AOP = 30°.
4. Draw PQ perpendicular to OP, meeting OB at Q.
5. Then PQ is the required tangent.

Justification: In ΔOPQ,

POQ = 30°

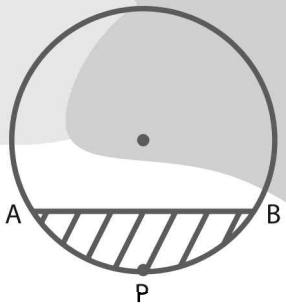
OPQ = 90°

∴ By angle sum property of triangle.

OQP = 180° – (90° + 30°)

= 60°.

11 Area Related To Circles



EXERCISE 11.1

Choose the correct option from the given four options:

1. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then:

(A) $R_1 + R_2 = R$ (B) $R_1^2 + R_2^2 = R^2$

(C) $R_1 + R_2 < R$ (D) $R_1^2 + R_2^2 < R^2$

[CBSE 2011]

Ans. (B)

Explanation: According to the given condition,

Area of circle with radius R = Area of circle with radius R_1 + Area of circle with radius R_2

$\Rightarrow \quad \pi R^2 = \pi {R_1}^2 + \pi {R_2}^2$

$\Rightarrow \quad R^2 = {R_1}^2 + {R_2}^2$

2. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then:

(A) $R_1 + R_2 = R$

(B) $R_1 + R_2 > R$

(C) $R_1 + R_2 < R$

(D) Nothing definite can be said about the relation among R_1 R_2 and R.

Ans. (A)

Explanation: According to the given condition,

Circumference of circle with radius R = Circumference of circle with radius R_1 + Circumference of circle with radius R_2

$\Rightarrow \quad 2\pi R = 2\pi R_1 + 2\pi R_2$

$\Rightarrow \quad R = R_1 + R_2$

3. If the circumference of a circle and the perimeter of a square are equal, then:

(A) Area of the circle = Area of the square

(B) Area of the circle > Area of the square

(C) Area of the circle < Area of the square

(D) Nothing definite can be said about the relation between the areas of the circle and the square.

Ans. (B)

Explanation: It is given that,

Circumference of circle = Perimeter of square

Let r be the radius of circle and a be the side of square

$\Rightarrow \quad 2\pi r = 4a$

$\Rightarrow \quad \frac{22}{7} \times r = 2 \times a$

$\Rightarrow \quad r = \frac{22}{7} \times 2 \times a = \frac{7}{11}a$

$r = \frac{7}{11}a$ ...(i)

Area of square = a^2

Area of circle = πr^2

$= \pi\left(\frac{7}{11}a\right)^2 = \frac{22}{7} \times \frac{7}{11} \times \frac{7}{11}a^2$

$= \frac{14a^2}{11} = \frac{14}{11}$ [Area of square]

$\Rightarrow$ Area of circle = 1.2 area of square

$\Rightarrow$ Area of circle > Area of square.

4. Area of the largest triangle that can be inscribed in a semi-circle of radius r units is:

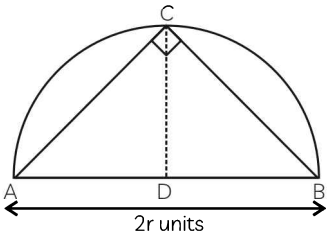
(A) r^2 sq. units (B) $\frac{1}{2}r^2$ sq. units

(C) $2r^2$ sq. units (D) $\sqrt{2}\,r^2$ sq. units

Ans. (A)

Explanation: The largest triangle that can be inscribed in a semicircle of radius r units is the triangle having its base as the diameter of the semicircle and two other sides are taken by considering a point C on the semi circle. Join C with diameter end points A and B.

$\therefore \quad \angle C = 90°$ [Angle in a semicircle]



ΔABC is right angled triangle with base as diameter AB of circle and height CD.

Height of triangle = r

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2r \times r = r^2$ sq. units.

5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is:

(A) 22 : 7 (B) 14 : 11

(C) 7 : 22 (D) 11 : 14

Ans. (B)

Explanation: Let r be the radius of circle and a be the side of square.

It is given that Perimeter of circle = Perimeter of square

$\therefore \quad 2\pi r = 4a$

$\Rightarrow \quad a = \frac{\pi r}{2}$ ...(i)

$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{a^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$$

[Using eqn (i)]

$$= \frac{\pi r^2 \times 4}{\pi^2 r^2} = \frac{4}{\pi} = \frac{4}{22} \times 7 = \frac{28}{22} = \frac{14}{11}$$

$\therefore$ Required ratio = 14 : 11

6. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be:

(A) 10 m (B) 15 m

(C) 20 m (D) 24 m **[CBSE 2012]**

Ans. (A)

Explanation: Let the radius of the new park be R.

$\therefore$ Area of new park = Area of old park I + Area of park II

Also, if r_1 and r_2 are the radius of circle

$r_1 = \frac{d_1}{2} = \frac{16}{2} = 8$

$r_2 = \frac{d_2}{2} = \frac{12}{2} = 6$

Area of the first circular park with diameter 16 m

$$= \pi\left(\frac{16}{2}\right)^2 = \pi(8)^2 = 64\pi \text{ m}^2$$

Area of the second circular park with diameter 12 m

$$= \pi\left(\frac{12}{2}\right)^2 = \pi(6)^2 = 36\pi \text{ m}^2$$

According to the given condition.

$\pi R^2 = 64\pi + 36\pi$

$\Rightarrow \quad \pi R^2 = 100\pi \Rightarrow R^2 = 100$

$\Rightarrow \quad R = 10$ m

7. The area of a circle that can be inscribed in a square of side 6 cm is:

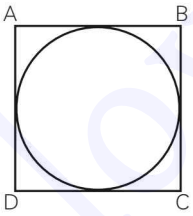
(A) 36 π cm^2 (B) 18 π cm^2

(C) 12 π cm^2 (D) 9 π cm^2

[CBSE 2012]

Ans. (D)

Explanation:



It is given that the side of square = 6 cm

$\Rightarrow$ Diameter of the circle inscribed in a square, d = Side of square = 6 cm

$\therefore$ Radius of circle (r) = $\frac{2r}{2} = \frac{6}{2} = 3$ cm

$\Rightarrow \quad$ Area of circle = $\pi r^2 = \pi(3)^2 = 9\pi$ cm^2

8. The area of a square that can be inscribed in a circle of radius 8 cm is:

(A) 256 cm^2 (B) 128 cm^2

(C) $64\sqrt{2}$ cm^2 (D) 64 cm^2

[CBSE 2015]

Ans. (B)

Explanation: Let the side of square be a cm.

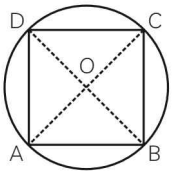
It is given that

Radius of cirlce, r = OC = 8 cm

Diameter of circle = AC = 2 $\times$ OC

$= 2 \times 8 = 16$ cm

Since square is inscribed in circle



$\therefore$ Diagonal of square = Diameter of circle

= 16 cm

$$\text{Area of square} = \frac{(\text{Diagonal})^2}{2} = \frac{(16)^2}{2} = \frac{256}{2}$$

$= 128$ cm^2

Alternate method

Let a be the side of square ABCD

In ΔACB,

$AC^2 = AB^2 + BC^2$ [By pythagoras theorem]

$\Rightarrow (16)^2 = a^2 + a^2$

$\Rightarrow 256 = 2a^2$

$\Rightarrow a^2 = 128$

$\therefore$ Area of square $= 128\ cm^2$

9. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is:

(A) 56 cm (B) 42 cm

(C) 28 cm (D) 16 cm

Ans. (C)

Explanation: According to the given condition, Circumference of circle = Sum of Circumference of two circle

Let r_1 and r_2 be the radius of the two circles then

$$r_1 = \frac{36}{2} = 18 \text{ cm}$$

$$r_2 = \frac{20}{2} = 18 \text{ cm}$$

$\Rightarrow 2\pi r = 2\pi r_1 + 2\pi r_2$

$\Rightarrow 2\pi R = 2\pi r_1 + 2\pi r_2$

$\Rightarrow R = r_1 + r_2 = 18 + 10 = 28$

10. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is:

(A) 31 cm (B) 25 cm

(C) 62 cm (D) 50 cm [CBSE 2011]

Ans. (D)

Explanation: Let radius of 1st circle

$r_1 = 24$ cm

Area of 1st circle $= \pi(r_1)^2 = \pi(24)^2$

$= 576\pi\ cm^2$

Radius of 2nd circle

$r_2 = 7$ cm

Area of 2nd circle $= \pi(r_2)^2 = \pi(7)^2$

$= 49\pi\ cm^2$

Let R be the radius of the new circle.

According to the given condition,

Area of new circle = Area of 1st circle + Area of 2nd circle

$\Rightarrow \pi R^2 = 576\pi + 49\pi$

$\Rightarrow R^2 = 625$

$\Rightarrow R = 25$ cm

$\therefore$ Diameter of new circle $= 2R = 2 \times 25$

$= 50$ cm

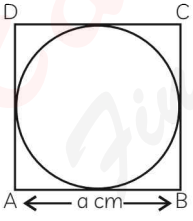
EXERCISE 11.2

1. Is the area of the circle inscribed in a square of side *a* cm, $\pi a^2\ cm^2$? Give reasons for your answer.

Ans. No.

Let ABCD be a square of side *a* cm with a circle inscribed in it. The radius of the circle inscribed in a square of side a cm is *r*.

$\therefore$ Diameter of circle = side of square = *a*



Radius $(r) = \frac{a}{2}$ cm

$\therefore$ Area $= \pi(r)^2 = \pi\left(\frac{a}{2}\right)^2 cm^2$

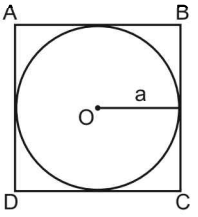
$= \frac{\pi a^2}{4} cm^2$

Hence, area of circle is $\frac{\pi a^2}{4}$

2. Will it be true to say that the perimeter of a square circumscribing *a* circle of radius *a* cm is 8*a* cm? Give reasons for your answer.

Ans. Yes.

Let ABCD be a square of side *a* cm with circle inscribed in it.



The radius of the circle inscribed in a square of side a cm is *r*

Radius of circle, $r = a$ cm

Diameter of circle,

$d = 2 \times r = 2a$ cm

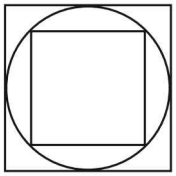
$\Rightarrow$ Side of square ABCD = Diameter of circle, *d*

$= 2a$ cm

$\therefore$ Perimeter of square ABCD = 4 × (side)

$= 4 \times 2a$ cm $= 8a$ cm

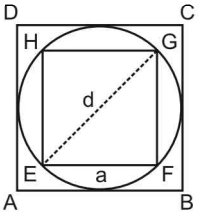
3. In the figure, a square is inscribed in a circle of diameter *d* and another square is circumscribing the circle. Is the area of the outer square four times the area of the inner square? Give reasons for your answer.



Ans. No.

Let ABCD be a square circumscribing a circle of diameter, *d*.

Let EFGH be a square of side a inscribed in the circle.



$\Rightarrow$ Diagonal of inner square

= Diameter of circle = d

In right angled ΔEFG,

$EG^2 = EF^2 + FG^2$

[Using Pythagoras theorem]

$\Rightarrow \quad d^2 = a^2 + a^2$

$\Rightarrow \quad d^2 = 2a^2$

$\Rightarrow \quad a^2 = \frac{d^2}{2}$

Area of inner square EFGH = $a^2 = \frac{d^2}{2}$

Side of outer square ABCD = d

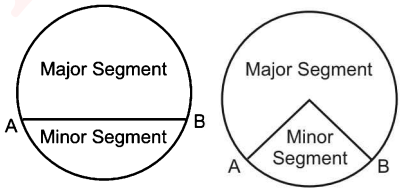
$\Rightarrow$ Area of outer square = $d^2 = 2a^2$

$\Rightarrow$ Area of outer square = 2 times Area of smaller square

So, the given statements is false.

4. Is it true to say that area of a segment of a circle is less than the area of its' corresponding sector? Why?

Ans. No.



It is true only in case of minor segment.

In case of major segment, area of segment is always greater than the area of its corresponding sector.

5. Is it true that the distance travelled by a circular wheel of diameter *d* cm in one revolution is $2\pi d$ cm? Why.

Ans. No

Distance travelled by the wheel in 1 revolution is equal to its circumference

= $2\pi r$ cm

= circumference of wheel

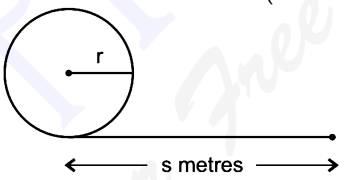
Hence, the given statement is false.

6. In covering a distance s metres, a circular wheel of radius *r* metres makes $\frac{s}{2\pi r}$ revolutions. Is this statement true? Why?

Ans. Yes.

We know that

Distance covered in 1 revolution = $2\pi r$ (circumference)



Let's assure total revolutions needed to cover & no distance in y.

So, $\quad y \times 2\pi r = s$

$\therefore \quad y = \frac{s}{2\pi r}$

7. The numerical value of the area of a circle is greater than the numerical value of its circumference. Is this statement true? Why?

Ans. No.

It will depend on the value of radius.

If $0 < r \leq 2$ then the numerical value of the circumference is greater than the numerical value of its area.

If $r > 2$, its area is greater than its circumference.

8. If the length of an arc of a circle of radius *r* is equal to that of an arc of a circle of radius 2*r*, then the angle of the corresponding sector of the first circle is double the angle of the corresponding sector of the other circle. Is this statement false? Why?

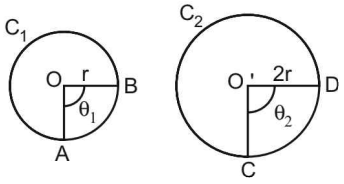
Ans. No.

This statement is true.

Let C_1 and C_2 be the two cirles with radius r and $2r$ respectively.

Arc length of C_1 = Arc length of C_2

i.e., $\overset{\frown}{AB} = \overset{\frown}{CD}$ ...(i)



Let θ_1 be the angle subtended by arc $\overset{\frown}{AB}$ and θ_2 be the angle subtended by arc $\overset{\frown}{CD}$ at the centre.

$$\therefore \quad \overset{\frown}{AB} = \frac{\theta_1}{360} \times 2\pi r \quad \text{...(ii)}$$

and $$\overset{\frown}{CD} = \frac{\theta_2}{360} \times 2\pi(2r) = \frac{\theta_2}{360} \times 4\pi r \quad \text{...(iii)}$$

From eq. (i), (ii) and (iii)

$$\frac{\theta_1}{360} \times 2\pi r = \frac{\theta_2}{360} \times 4\pi r$$

$$\Rightarrow \quad \theta_1 = 2\theta_2$$

Hence, angle of corresponding sector of C_1 is double the angle of the corresponding sector of C_2.

9. The areas of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true? Why?

Ans. No.

The given statement is true for arcs of the same circle. But in differents circles, it is not possible as area of setor

$$A = \frac{lr}{2}$$

where l = corresponding arc length.

r = radius

Hence, area of two setors with equal arc lengths, would be the same in case both the circles have equal radii.

10. The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal? Why?

Ans. No.

The given statement will be true for arcs of the same circle. But in different circles, it is not possible.

Area of 1st sector $= \frac{1}{2}(r_1^2)\theta_1$

where, r_1 is the radius and θ_1 is the angle.

Area of 2nd sector $= \frac{1}{2}(r_2^2)\theta_2$

where, r_2 is the radius and θ_2 is the angle subtended at the centre of the circle by the arc.

It is given that $\frac{1}{2}r_1^2\theta_1 = \frac{1}{2}r_2^2\theta_2$

$$\Rightarrow \quad r_1^2\theta_1 = r_2^2\theta_2$$

Thus it depends both an radius and angle subtended at the centre. But arc lengths depend only on radius.

11. For the largest circle that can be drawn inside a rectangle of length a cm and breadth b cm ($a > b$), is its area πb^2 cm^2? Why?

Ans. No.

Diameter of circle = breadth of rectangle = b

$$\Rightarrow \text{ Radius of circle} = \frac{b}{2}$$

$$\Rightarrow \text{ Area of largest circle} = \pi r^2$$

$$= \pi r^2 = \pi\left(\frac{b}{2}\right)^2 \text{ cm}^2$$

12. Circumferences of two circles are equal. Is it necessary that their areas be equal? Why?

Ans. Yes.

It is given that,

Circumference of circle C_1 = circumference of circle C_2

$$\Rightarrow \quad 2\pi R_1 = 2\pi R_2 \text{ [where, } R_1 \text{ and } R_2 \text{ are radii of two circles]}$$

$$\Rightarrow \quad R_1 = R_2$$

$$\Rightarrow \quad \pi R_1^2 = \pi R_2^2$$

$\Rightarrow$ Areas of two circles are equal.

13. Areas of two circles are equal. Is it necessary that their circumferences are equal? Why?

Ans. Yes.

It is given that,

Area of circle with radius R_1 = Area of circle with radius R_2

$$\Rightarrow \quad \pi(R_1)^2 = \pi(R_2)^2$$

$$\Rightarrow \quad R_1 = R_2$$

$$\Rightarrow \quad 2\pi R_1 = 2\pi R_2$$

Hence, the circumferences are also equal.

14. Is it true to say that the area of a square inscribed in a circle of diameter p cm is p^2 cm^2? Why?

Ans. No.

When the square is inscribed in the circle, the diameter of the circle is equal to the diagonal of the square but not the side of the square.

Let side of square be a.

$\therefore$ Length of diagonal $= \sqrt{a^2 + a^2} = \sqrt{2}a$

$\Rightarrow$ diameter of circle, $p = \sqrt{2}a$

$$\Rightarrow \quad p^2 = 2a^2$$

$\therefore$ Area of circle

$$a^2 = \frac{p^2}{2}$$

EXERCISE 11.3

1. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.

Ans.

Let the radius of the circle be r whose circumference is equal to the sum of the circumferences of two circles with radii

$r_1 = 15$ cm and $r_2 = 18$ cm

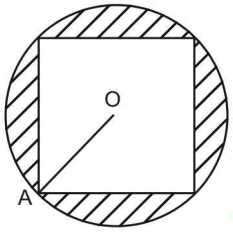
According to the question, Circumference = circumference 1 + circumference 2

$$2\pi r = 2\pi r_1 + 2\pi r_2$$

$$\Rightarrow \quad r = r_1 + r_2 = (15 + 18) = 33 \text{ cm}$$

Hence, $r = 33$ cm is the required radius of the circle.

2. In the given figure, a square of diagonal 8 cm is inscribed in a circle. Find the area of the shaded region.



Ans.

Let the side of the square be a cm and

radius of the circle, r = OA = $\frac{1}{2}$ x diagonal

$\Rightarrow$ r = $\frac{1}{2}$ x 8 = 4 cm

It is given that

length of diagonal of square = 8 cm

$\Rightarrow$ Side of square $a = 4\sqrt{2}$

$\therefore$ Area of circle = $\pi r^2 = \pi \times (4)^2 = 16\pi$ cm^2

Area of square = $a^2 = \left(4\sqrt{2}\right)^2 = 32$ cm^2

So, area of shaded region

= area of circle – area of square

= $\pi r^2 - a^2 = (16\pi - 32)$ cm^2

Hence, the required area of the shaded region is $(16\pi - 32)$ cm^2.

3. Find the area of a sector of a circle of radius 28 cm and central angle 45°.

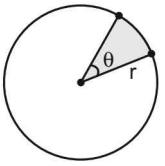
Ans.

Given: Radius of circle,

$r = 28$ cm

Central angle, $\theta = 45°$

We know that,



If θ is measured in degrees then

$$\therefore \quad \text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{45}{360} \times \frac{22}{7} \times 28 \times 28$$

$$= 308 \text{ cm}^2$$

Hence, the required area of the sector of the circle is 308 cm^2.

4. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/h?

Ans.

Given: Radius of wheel, $r = 35$ cm

Speed of wheel, S = 66 km/h

$$S = \frac{66 \times 1000}{60} \text{ m/min} \quad \left[\begin{array}{l}\text{As} \\ 1 \text{ km} = 1000 \text{ m} \\ \text{and } 1 \text{ hr} = 60 \text{ min}\end{array}\right]$$

= 1100 m/min = 1,10,000 cm/min

Circumference of wheel = $2\pi r$

= 2 × 22 × 5 = 220 cm

We know that speed = $\frac{\text{Distance}}{\text{Time}}$

$$S = \frac{2\pi rn}{t} \quad \Rightarrow \quad n = \frac{St}{2\pi r}$$

$\therefore$ Number of revolutions in 1 min, n

$$= \frac{110000}{220} = 500$$

$\Rightarrow n = 500$ revolution per min, n

Hence, the required number of revolutions per minute is 500.

5. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20 m × 16 m. Find the area of the field in which the cow can graze. [CBSE 2010]

Ans.

Field is rectangular. So if cow is tied at its vertex, it will graze the field in the shape of sector. The length of rope is less than length and breath of rectangle. So, the required area is of sector.

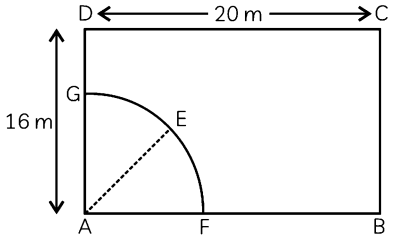
Let ABCD be a rectangular field of dimensions 20 m × 16 m.

Let the cow be tied at point A.

Given : Length of rope AE = 14 m.

∴ Area of field in which cow can graze

= Area of sector AFEG = $\frac{\theta}{360°} \times \pi r^2$



⇒ We know that the angle between two sides of rectangle is 90°

⇒ Area of sector AFEG = $\frac{90}{360} \times \frac{22}{7} \times (14)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ m}^2$

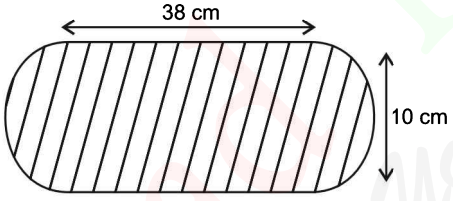
Hence, the required area grazed by cow is 154 m^2.

Trick Applied

Angle between two adjacent sides of rectangle is 90°.

Use it for, area of sector $\frac{\theta}{360°} \times \pi r^2$.

6. Find the area of the flower bed (with semi-circular ends) shown in the given figure



Ans.

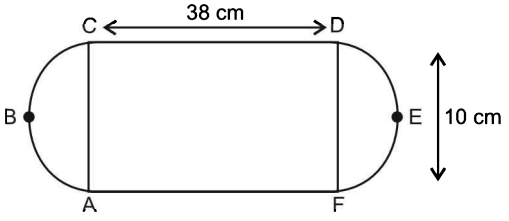
The figure has two semi–circles and one rectangle.

Given: Length of flower bed, l = 38 cm

Breadth of flower bed, b = 10 cm

Both ends of flower bed are semi circle

Radius of semicircle = $\frac{DF}{2} = \frac{10}{2}$ cm = 5 cm



Area of rectangle ACDF

$= l \times b = (38 \times 10) = 380 \text{ cm}^2$

Area of one semicircle = $\frac{\pi r^2}{2} = \frac{\pi}{2}(5)^2 = \frac{25\pi}{2} \text{cm}^2$

∴ Area of two semicircles = $2 \times \frac{25\pi}{2} = 25\pi \text{ cm}^2$

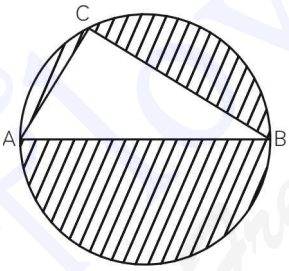
∴ Total area of flower bed

= Area of rectangle + Area of 2 semicircles

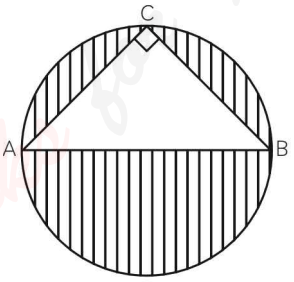
$= l \times b + \pi r^2$

$= (380 + 25\pi) \text{ cm}^2$.

7. In the given figure AB is the diameter of the circle, AC = 6 cm and BC = 8 cm. Find the area of the shaded region (Use π = 3.14).



Ans.



The given figure is a circle, and a right angled triangle (and semicircle, segment also) because AB is diameter and angle in semicircle is 90°.

AB is the diameter of circle,

AC = 6 cm and BC = 8 cm

We know that AB is the diameter

∴ ∠ACB = 90° [Angle in semicircle]

In right angled ΔACB,

$AB^2 = AC^2 + CB^2$

[By Pythagoras theorem]

$= 6^2 + 8^2 = 36 + 64$

$AB^2 = 100$

⇒ $AB = \sqrt{100} = 10$ cm

∴ Area of ΔABC = $\frac{1}{2} \times BC \times AC$

$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$ ...(i)

Diameter of circle, AB = 10 cm

∴ Radius of circle, $r = \frac{10}{2} = 5$ cm

$\Rightarrow$ Area of circle = $\pi r^2 = 3.14 \times (5)^2$

$= 3.14 \times 25 = 78.5 \text{ cm}^2$ ...(ii)

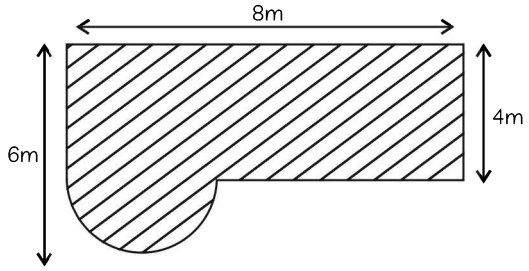
$\therefore$ Area of shaded region

= Area of circle – Area of ΔABC

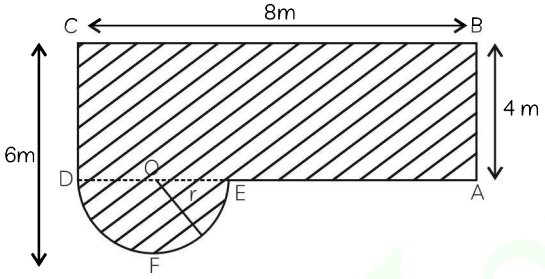
$= (78.5 - 24) \text{ cm}^2$

$= 54.5 \text{ cm}^2$.

8. Find the area of the shaded field shown in the given figure [CBSE 2015]



Ans.



It is clear from the figure that there is one semicircle and one rectangle.

In the figure, join ED.

For Rectangle

length, BC = 8 m and Breadth, AB = 4 m

For Circle radius, $r = 6 - 4 = 2$

Area of rectangle ABCD = BC × AB

$= (8 \times 4) \text{ m}^2 = 32 \text{ m}^2$

Radius of semicircle OD = OE = r

$r = (6 - 4) \text{ m} = 2 \text{ m}$

$\therefore$ Area of semicircle with radius $r = \dfrac{\pi r^2}{2}$

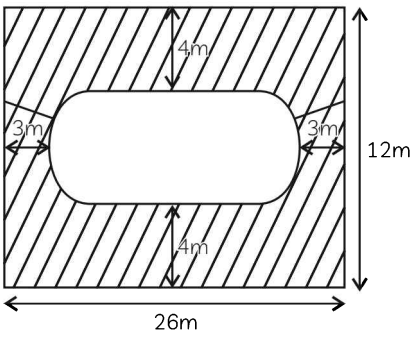
$= \dfrac{\pi}{2} \times (2)^2 = 2\pi \text{ m}^2$

$\therefore$ Area of shaded region = Area of rectangle ABCD + Area of semicircle DFE

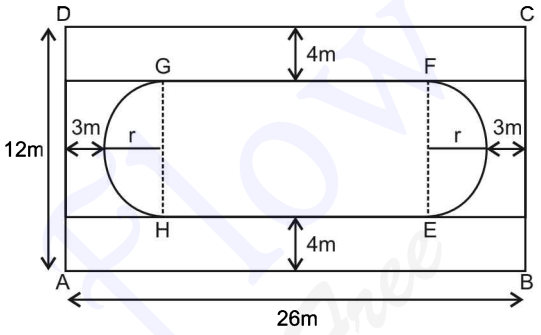
$= (32 + 2\pi) \text{ m}^2$

Hence, the required area of shaded region = $(32 + 2\pi) \text{ m}^2$.

9. Find the area of the shaded field shown in the given figure



Ans.



Join GH and FE such that EFGH is a rectangle.

In rectangle ABCD,

Length of rectangle, AB = 26 m

Breadth of rectangle, AD = 12 m

$\therefore$ Area of rectangle ABCD = 26×12

$= 312 \text{ m}^2$

Inner rectangle EFGH,

Breadth, EF = $[12 - (4 + 4)] = 12 - 8$

= 4 m

Also, EF is equal to the diameter of semicircle

EF = GH = 4 m

$\therefore$ Radius of semicircle EF, (r) = 2 m

Length, EH = $[26 - (5 + 5)] = 26 - 10$

= 16 m

$\therefore$ Area of two semicircles

$= 2\left(\dfrac{\pi r^2}{2}\right) = \dfrac{2 \times \pi \times (2)^2}{2} = 4\pi \text{ m}^2$

Area of inner rectangle, EFGH

$= \text{EH} \times \text{EF} = 16 \times 4 = 64 \text{ m}^2$

$\therefore$ Area of shaded region

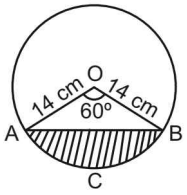
= Area of outer rectangle – (Area of inner rectangle + Area of 2 semicircle)

$= [312 - (64 + 4\pi)] \text{ m}^2 = [248 - 4\pi] \text{ m}^2$

Hence, the area of shaded region = $(248 - 4\pi) \text{ m}^2$.

10. Find the area of the minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is 60°. [CBSE 2019, 15]

Ans.



Shaded region is a minor segment.

Given: radius of circle, r = 14 cm

Angle of sector, $\theta = 60°$

In ΔAOB, $OA = OB = r$

$\Rightarrow \Delta AOB$ is an isosceles triangle.

$\Rightarrow \angle AOB + \angle OAB + \angle OBA = 180°$

[Angle sum property]

$\Rightarrow 60° + \theta + \theta = 180°$

$\Rightarrow 2\theta = 120 \Rightarrow \theta = 60°$

$\Rightarrow \angle AOB = \angle OAB = \angle OBA = 60°$

$\Rightarrow \Delta AOB$ is an eauilateral triangle.

$OA = OB = AB = 14$ cm.

We know that area of equilateral triangle,

Area of $\Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(14)^2 \text{ cm}^2$

$= \frac{\sqrt{3}}{4} \times 196 \text{ cm}^2 = 49\sqrt{3} \text{ cm}^2$

Area of sector $OBCA = \pi r^2 \times \frac{\theta}{360}$

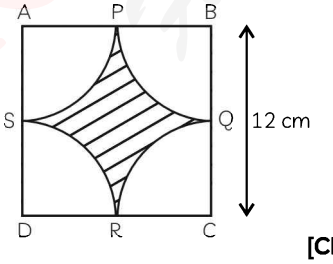
$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360} = \frac{22 \times 2 \times 14}{6} \text{ cm}^2$

$= \frac{308}{3} \text{ cm}^2$

Area of minor segment ACB = Area of sector OBCA – Area of ΔOAB

$= \left(\frac{308}{3} - 49\sqrt{3}\right) \text{ cm}^2$

11. Find the area of the shaded region in the given figure where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA, respectively of a square ABCD (use π = 3.14).



[CBSE 2011]

Ans.

From figure,

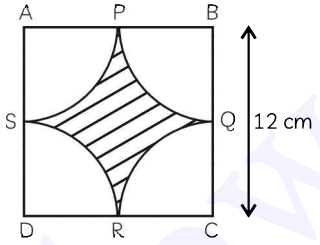
Area of shaded part = Area of square ABCD – Area of 4 sector

A sector is (pizza slice shaped) part of circle. A quadrant is a type of sectorr that happens to be $1/4^{th}$ part of circle. Here APS, PBQ, QCR and RDS are both sectors as well as quadrant.

Given: ABCD is a square with side, BC = 12 cm.

Since, Q is midpoint of BC

$\Rightarrow$ Radius, of sector, PBQ = $\frac{12}{2}$ cm = 6 cm



Now, area of quadrant PBQ = $\frac{\pi r^2}{4}$

$= \frac{3.14 \times (6)^2}{4} = \frac{113.04}{4} \text{ cm}^2$

Area of 4 quadrants = $\frac{4 \times 113.04}{4} = 113.04 \text{ cm}^2$

Area of square ABCD = $(12)^2 = 144 \text{ cm}^2$

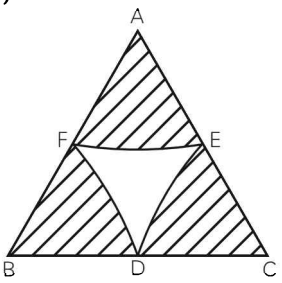
$\therefore$ Area of shaded region

= Area of square ABCD – Area of 4 quadrants

$= 144 - 113.04 = 30.96 \text{ cm}^2$

Hence, area of the shaded region is 30.96 cm^2.

12. In the given figure arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm. to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region (use π = 3.14).



[CBSE 2012]

Ans.

From the given figure, area of the shaded part is equal to the sum of areas of these sectors at points A, B and C.

As ΔABC is equilateral triangle of side 10 cm and radius of the sector is half of the side. All the three sectors are identical.

$\theta = 60°$

ΔABC is an equilateral triangle [Given]

$\Rightarrow \angle A = \angle B = \angle C = 60°$

and AB = BC = AC = 10 cm

E, F and D are the mid-points of sides AC, AB and BC respectively.

$\Rightarrow$ AE = EC = CD = DB = BF = FA = 5 cm

Radius of sector, r = 5 cm

Area of sector CED = $\frac{\theta}{360} \times \pi r^2$

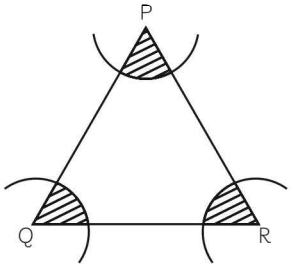
$= \frac{60°}{360°} \times 3.14 \times (5)^2 \text{ cm}^2$

$= \frac{3.14 \times 25}{6} = \frac{78.5}{6} \text{ cm}^2 = 13.0833 \text{ cm}^2$

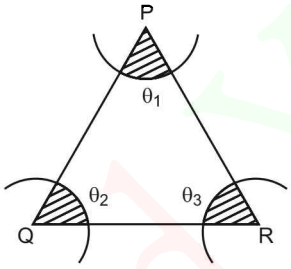
Area of shaded region = 3 (Area of sector CDE)

$= 3 \times 13.0833 \text{ cm}^2 = 39.25 \text{ cm}^2$

13. In the given figure arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



Ans.



The area of the shaded region is equal to the sum of areas of three sectors of same radius but of different angles θ_1, θ_2 and θ_3.

$\theta\angle 1 + \theta\angle 2 + \theta\angle 3 = 180°$

[Angle sum property of Δ]

Given: radii of each arc, r = 14 cm.

Area of sector with central angle P

$= \frac{\angle P}{360} \times \pi r^2 = \frac{\theta_1}{360} \times \pi(14)^2 \text{ cm}^2$

Area of sector with central angle Q

$= \frac{\angle Q}{360} \times \pi r^2 = \frac{\theta_2}{360} \times \pi(14)^2 \text{ cm}^2$

Area of sector with central angle R

$= \frac{\angle R}{360} = \frac{\theta_3}{360} \times \pi(14)^2$

$\Rightarrow$ Sum of areas of 3 sectors

$= \frac{\theta_1}{360} \times \pi(14)^2 + \frac{\theta_2}{360} \times \pi(14)^2 + \frac{\theta_3}{360} \times \pi(14)^2$

$= \frac{\pi}{360} \times (14)^2 \left[\theta_1 + \theta_2 + \theta_3\right]$

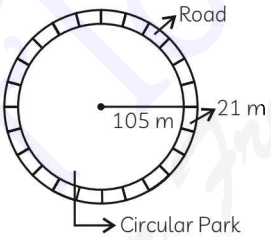
$= \frac{\pi \times 196 \times 180}{360}$ [Since sum of all interior angles in any triangle is 180°]

$= 98\pi = 98 \times \frac{22}{7} = 308 \text{ cm}^2$

Hence, the required area of the shaded region is 308 cm^2.

14. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road. [CBSE 2020]

Ans.



It is given that circular road and park are concentric circles.

Width of road = 21 m

Radius of park, r_2 = 105 m

$\Rightarrow$ Radius of the whole circular portion (park + road)

r_1 = 105 + 21 = 126 m

So, Area of road = Area of park and road – Area of park

$= \pi r_1^2 - \pi r_2^2$

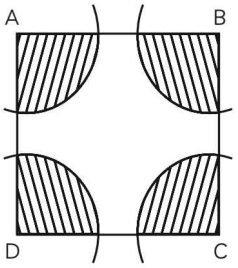
$= \pi\left(r_1^2 - r_2^2\right) = \frac{22}{7}\left[126^2 - 105^2\right] \text{ m}^2$

$= \frac{22}{7} (126 + 105)(126 - 105)$

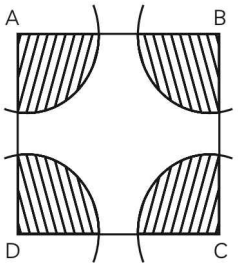
[$\because a^2 - b^2 = (a - b)(a + b)$]

$= \frac{22}{7} \times 231 \times 21 = 15246 \text{ m}^2$

15. In the given figure arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.



Ans.



Specification of quadrilateral are not given, so quadrilateral may be of any shape.

As the radius of each arc, r = 21 cm

Angles made by the arc = ∠A, ∠B, ∠C and ∠D.

So, there are four sectors of ∠A, ∠B, ∠C and ∠D with r = 21 cm.

ABCD is a quadrilateral

radius of each arc, r = 21 cm.

Area of sector with $\angle A = \frac{\angle A}{360} \times \pi r^2$

$= \frac{\angle A}{360} \times \pi (21)^2 \text{ cm}^2$

Similarly,

Area of sector with ∠B

$= \frac{\angle B}{360} \times \pi (21)^2 \text{ cm}^2$

Similarly,

Area of sector with $\angle C = \frac{\angle C}{360} \times \pi (21)^2 \text{ cm}^2$

Similarly,

Area of sector with $\angle D = \frac{\angle D}{360} \times \pi (21)^2 \text{ cm}^2$

We know that sum of all interior angles in quadrilateral = 360°

$\angle A + \angle B + \angle C + \angle D = 360°$ ...(i)

Area of shaded region = sum of areas of four sectors

$= \frac{\angle A}{360} \times \pi(21)^2 + \frac{\angle B}{360} \times \pi(21)^2 + \frac{\angle C}{360} \times \pi(21)^2 + \frac{\angle D}{360} \times \pi(21)^2$

$= \frac{\pi}{360} \times (21)^2 [\angle A + \angle B + \angle C + \angle D]$

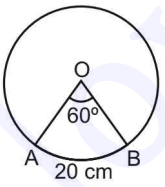
$= \frac{22 \times 21 \times 21}{7 \times 360} \times 360$ [Using equation ...(i)]

$= 22 \times 3 \times 21 = 1386 \text{ cm}^2$

Hence, the required area of the shaded region is 1386 cm^2.

16. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. [CBSE 2017]

Ans.



Arc is a part of circle that makes 60° between radii at end points A and B of wire. So, it form the shape of a sector.

Length of wire = 20 cm

= Length of arc

Sector angle , θ = 60°

We know that

$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$

$\Rightarrow \quad 20 = \frac{60}{360} \times 2\pi r$

$\Rightarrow \quad 2\pi r = 20 \times 6 = 120$

$\Rightarrow \quad r = \frac{120}{2\pi}$

$= \frac{60}{\pi} \text{ cm}$

Hence, radius of the circle is $\frac{60}{\pi}$ cm.

EXERCISE 11.4

1. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹ 50 per metre.

Ans.

Fencing is made on circumference ($2\pi r$) of circular field. So, we require radius for it.

Area of circular playground = 22176 m^2

$\Rightarrow \quad \pi r^2 = 22176$

[Where r is radius of playground]

$\Rightarrow \quad \frac{22}{7} r^2 = 22176$

$r^2 = \frac{22176 \times 7}{22}$

$r^2 = 1008 \times 7 = 7 \times 12 \times 12 \times 7$

$r = 7 \times 12 = 84$ m

For fencing, we need circumference

∴ Length of fencing = cumference of circular playground

$$= 2\pi r = 2 \times \frac{22}{7} \times 84 = 528 \text{ m}$$

Cost of fencing per meter = ₹ 50

$\Rightarrow$ cost of fencing 528 m = ₹ 50 × 528 = ₹ 26400

2. The diameters of the front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that the rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

Ans.

We know that Distance traveled by rear wheel = Distance traveled by front wheel

It is given that

Diameter of front wheel, d_1 = 80 cm

$\therefore$ Radius of front wheel, r_1 = 40 cm

diameter of rear wheel, d_2 = 2 m = 200 cm

$\therefore$ Radius, r_2= 100 cm

Circumference of front wheel = $2\pi r_1$

$$= 2 \times \frac{22}{7} \times 40 = \frac{1760}{7} \text{ cm}$$

$\therefore$ Total distance covered by front wheel in 1400 revolutions = $\frac{1760}{7} \times 1400$

= 1760 × 200 = 352000 cm

Number of revolutions by rear wheel

$$= \frac{\text{Distance covered}}{\text{Circumference of rear wheel}}$$

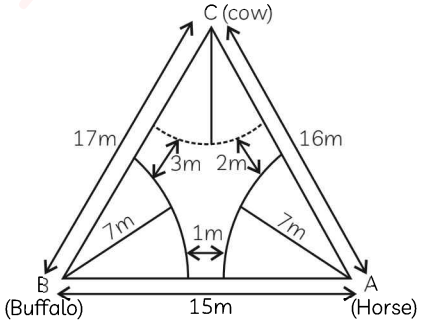
$$= \frac{352000}{2\pi r_2} = \frac{352000}{2 \times \frac{22}{7} \times 100}$$

$$= \frac{352000 \times 7}{2 \times 22 \times 100} = 560$$

Hence, the rear wheel will make 560 revolutions.

3. Sides of a triangular field are 15 m, 16 m and 17 m. In the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.

Ans.



Since with the three corners of the field a cow, a buffalo and a horse and tied separately with ropes of length 7 m each to graze in the field.

Area of field which cannot be grazed by animals = Area of ΔBCA – Area of three sectors

Here, a = 15 m, b = 16 m, c = 17 m

Given: A triangular field ABC with three corners

A → Horse is tied

B → Buffalo is tied

C → Cow is tied

i.e., each animal-grazed field forming a sector in each corner of the field.

Given: Radius of sector = 7 m (length of rope)

Area of sector with $\angle A$

$$= \frac{\angle A}{360} \times \pi r^2 = \frac{\angle A}{360} \times \pi \times (7)^2 = \frac{49\pi \angle A}{360}$$

Similarly,

Area of sector with $\angle B = \frac{49\pi \angle B}{360}$

Area of sector with $\angle C = \frac{49\pi \angle C}{360}$

Sum of areas of three sectors

$$= \frac{49\pi}{360}\angle A + \frac{49\pi}{360}\angle B + \frac{49\pi}{360}\angle C$$

$$= (\angle A + \angle B + \angle C) \times \frac{49\pi}{360}$$

$$= \frac{180 \times 49 \times 22}{360 \times 7}$$ [$\because \angle A + \angle B + \angle C = 180$ Angle sum property of triangle].

= 7 × 11 = 77 m^2

Given sides of triangle

a = 15 m b = 16 m c = 17 m.

Semiperimeter of triangle, $s = \frac{a+b+c}{2}$

$$\Rightarrow s = \frac{15+16+17}{2} = \frac{48}{2} \text{ m} = 24 \text{ m}$$

$\therefore$ Area of triangular field ABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$ [Heron's formula]

$$= \sqrt{24(24-15)(24-16)(24-17)}$$

$$= \sqrt{24 \times 9 \times 8 \times 7} = 27\sqrt{21} \text{ m}^2$$

Area of field which cannot be grazed by the three animals

= Area of ΔCBF – Area of 3 sectors

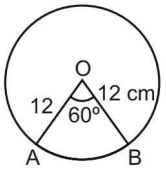
$= \left(24\sqrt{21} - 77\right) m^2$

Hence, the area which cannot be grazed by 3 animals is equal to $\left(24\sqrt{21} - 77\right) m^2$

4. Find the area of the segment of a circle of radius 12 cm whose corresponding sector has a central angle of 60° (use $\pi = 3.14$). **[CBSE 2015]**

Ans.

Area of minor segment = Area of sector – Area of ΔOAB



Given: radius of circle, $r = 12$ cm

Central angle, $\theta = 60°$

To find: Area of segment ABCA

Area of sector $AOBC = \pi r^2 \times \frac{\theta}{360}$

$= \frac{3.14 \times (12)^2 \times 60}{360} = 3.14 \times 2 \times 12$

$= 3.14 \times 24 = 75.36 \text{ cm}^2$

$\because \Delta OAB$ is an isosceles triangle [OA = OB = r]

$\therefore \quad \angle OAB = \angle OBA$

But $\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow \quad \angle OAB + \angle OBA = 180° - 60°$

[As $\angle AOB = \theta = 60°$ (Given)]

$\Rightarrow \quad \angle OAB + \angle OBA = 120°$

or $\angle OAB = \angle OBA = 60°$ [As $\angle OAB = \angle OBA$]

$\Rightarrow \Delta OAB$ is an equilateral triangle

$\therefore$ Area of $\Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(12)^2 = \sqrt{3} \times 3 \times 12 = 36\sqrt{3} \text{ cm}^2$

Hence, area of segment, ABCA

= area of sector OBCA – Area of ΔAOB

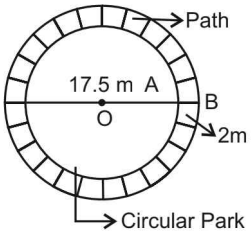
$= \left(75.36 - 36\sqrt{3}\right) \text{cm}^2$

5. A circular pond is is of diameter 17.5 m. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of ₹ 25 per m^2.

Ans.

Given: Diameter of pond, $d = 17.5$ m

$\therefore$ Radius of pond, $r_1 = \frac{d}{2} = \frac{17.5}{2} = 8.75$ m



Width of path around the pond, AB = 2 m

$\therefore$ Inner radius, $r_1 = 8.75$

Outer Radius of the path including pond,

$r_0 = 8.75 + 2 = 10.75$ m

Area of circular path

= Area of outer circle – Area of inner circle

$= \pi r_0^2 - \pi r_1^2$

$= \pi[(10.75)^2 - (8.75)^2]$

$= \pi[(10.75 - 8.75)(10.75 + 8.75)]$

$[\because (a^2 - b^2) = (a - b)(a + b)]$

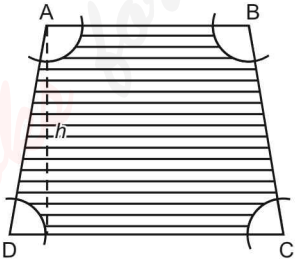
$= \pi \times 2 \times 19.5 = 3.14 \times 2 \times 19.5 = 122.46 \text{ m}^2$

Cost of constructing the path per square metre = ₹ 25

$\therefore$ Cost of constructing the path of 122.46 m^2 area = ₹ (25×122.46) = ₹ 3061.50

Hence, the cost is ₹ 3061.50.

6. In the given figure ABCD is a trapezium with AB II DC, AB = 18 cm, DC = 32 cm and distance between AB and DC =14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure. **[CBSE 2010]**



Ans.

Given: ABCD is trapezium with AB || DC and 4 sectors.

AB = 18 cm, CD = 32 cm

Distance between AB and DC, $h = 14$ cm.

Arc of radii, $r = 7$ cm.

To find: Area of shaded region.

Area of shaded part = Area of trapezium – Area of 4 sectors

Since AB || DC.

$\angle A + \angle D = 180°$

$\angle B + \angle C = 180°$

$\therefore$ Area of sector with $\angle A$ and $\angle D = \frac{\theta}{360} \times \pi r^2$

$= \frac{(\angle A + \angle D)}{360} \times \pi r^2 = \frac{180}{360} \times \pi r^2$

[As $(\angle A + \angle D) = 180°$]

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$

Similarly, area of sector with angle B and C = 77 cm^2.

We know that,

Area of trapezium = $\frac{1}{2}$ (AB + DC) × h

= $\frac{1}{2}(18 + 32) \times 14 = \frac{1}{2} \times 50 \times 14$

= 350 cm^2

Area of shaded region

= (Area of trapezium ABCD – Area of sectors with ∠A, ∠B, ∠C and ∠D)

= [350 – (77 + 77)] = 350 – 154 = 196 cm^2

Hence, area of the shaded region is 196 cm^2.

7. Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles. [CBSE 2011]

Ans.

The three circles are drawn in such a way that each of them touches the other two. By joining the centers of the three circles, we get

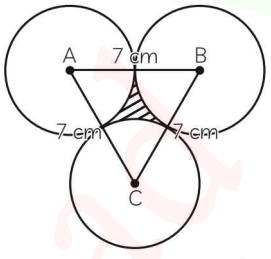
AB = BC = CA = 2(Radius) = 7 cm

Therefore, triangle ABC is an equilateral triangle with each side 7 cm.

This shows that ΔABC is an equilateral triangle with side 7 cm.

⇒ ∠A = ∠B = ∠C = 60°

[Angle of equilateral; Δ]



Area of sector with (∠A = 60°)

= $\frac{\angle A}{360} \times \pi r^2 = \frac{60}{360} \times \pi \times (3.5)^2$

Similarly,

Area of sector with ∠B = Area of sector with ∠C

= $\frac{60}{360} \times \pi \times (3.5)^2$

Area of 3 sectors

= $3 \times \frac{60}{360} \times \frac{22}{7} \times 3.5 \times 3.5$

= $11 \times \frac{5}{10} \times \frac{35}{10}$ $cm^2 = \frac{77}{4}$ cm^2

= 19.25 cm^2

Area of ΔABC = $\frac{\sqrt{3}}{4}$ × (side)^2

= $\frac{\sqrt{3}}{4} \times (7)^2 = \frac{49\sqrt{3}}{4}$ cm^2

Area of shaded region enclosed between these circles

= Area of ΔABC – Area of 3 sectors

= $\left(\frac{49\sqrt{3}}{4} - 19.25\right)$ cm^2

Hence, required area is $\left(\frac{49\sqrt{3}}{4} - 19.25\right)$ cm^2.

8. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Ans.

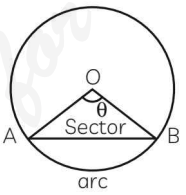
Given: Radius of circle, r = 5 cm

Arc length, l = 3.5 cm

Let angle of sector be θ.

We know that,

Arc length l = $\frac{\theta}{360} \times 2\pi r$



⇒ $3.5 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 5$

⇒ $\theta = \frac{35}{10} \times \frac{7}{22} \times \frac{360}{2 \times 5} = \frac{49 \times 9}{11} = \left(\frac{441}{11}\right)^\circ$

We know that,

Area of sector = $\frac{\theta}{360} \times \pi r^2$

= $\frac{441}{11} \times \frac{1}{360} \times \frac{22}{7} \times 5 \times 5$

= $\frac{5 \times 7}{4} = \frac{35}{4} = 8.75$ cm^2

9. Four circular cardboard pieces of radii 7 cm are placed on a paper in such a way that each piece touches the other two pieces. Find the area of the portion enclosed between these pieces.

Ans.

As we know that point of contact of two circles lies on the line joining their centres.

So, the line segments AB, BC, CD and AD will pass through the corresponding point of contact P, Q, R, S respectively.

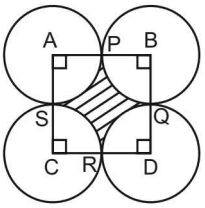
As SD and AS are radius at contact point S.

So, AD will be perpendicular to the tangent through S. It implies that interior angles of quadrilateral are 90° each.

As radius of each circle is equal.

So, quadrilateral ABCD will be square with side $= 2r = 2 \times 7 = 14$ cm.

Thus in the given figure, there is 1 square and 4 sectors.



Since radius of circle, $r = 7$ cm

$\Rightarrow$ AB = 2 × radius = $2r = 2 \times 7$ cm

$= 14$ cm

$\Rightarrow$ AB = BC = CD = DA = 14 cm.

This shows that, quadrilateral ABCD is a square with each of its side length = 14 cm.

$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90°$

$\therefore$ Area of sector with $(\angle A = 90°)$

$$= \frac{\angle A}{360} \times \pi r^2 = \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{1}{4} \times 22 \times 7 = \frac{77}{2} = 38.5 \text{ cm}^2$$

Similarly, area of sector with $\angle B$ = Area of sector with $\angle C$

= Area of sector with $\angle D = 38.5$ cm^2

Area of 4 sectors (with $\angle A$, $\angle B$, $\angle C$ and $\angle D$)

$= 4 \times 38.5 = 154$ cm^2

Area of square ABCD $= (14)^2 = 196$ cm^2

[$\because$ Area of square = (side)^2]

Area of shaded region

= Area of square ABCD – Area of 4 sectors

$= (196 - 154) = 42$ cm^2

10. On a square cardboard sheet of area 784 cm^2, four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates. [CBSE 2016]

Ans.

Let a be the side of Square ABCD.

Area of square = 784 cm^2

$\Rightarrow$ (side)$^2 = 784$ cm^2

$\Rightarrow$ $(a)^2 = (28)^2$

$\Rightarrow$ $a = 28$ cm

Since, all four are congruent circular plates

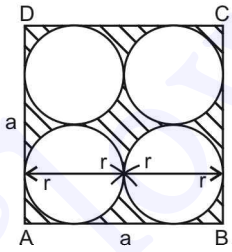
$\therefore$ Diameter of circular plate $= 2r = \frac{a}{2}$

$= \frac{28}{2} = 14$ cm

$\Rightarrow$ Radius of each circular plate $= r = \frac{a}{4}$

$= \frac{14}{2} = 7$ cm

$\therefore$ Area of 1 circular plate $= \frac{22}{7}(7)^2 = 154$ cm^2



$\Rightarrow$ Area of 4 circular plates

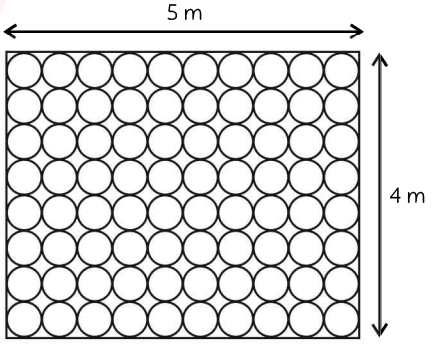
$= 4 \times 154 = 616$ cm^2

Area enclosed between circles and square

= Area of square – Area of 4 circles

$= 784 - 4\pi r^2 = 784 - 616 = 168$ cm^2

11. The floor of a room is of dimensions 5 m × 4 m and it is covered with circular tiles of diameters 50 cm each as shown in the given figure. Find the area of the floor that remains uncovered with tiles. (use $\pi = 3.14$)



Ans.

Given: Dimension of floor 5 m × 4 m

$\Rightarrow$ Length, $l = 5$ m

and breadth, $b = 4$ m

$\Rightarrow$ Area $= l \times b = 5 \times 4 = 20$ m^2

Diameter of circular tile, $d = 50$ cm [Given]

$\Rightarrow$ radius, $r = \frac{50}{2} = 25$ cm

$= \frac{25}{100}$ m $= \frac{1}{4}$ m

$\therefore$ Area of 1 circular tile = πr^2

$$= 3.14 \times \left(\frac{1}{4}\right)^2 = \frac{3.14}{16}\ m^2$$

$\Rightarrow$ No. of circular tiles along the length

$$= 5\ m \div 50\ cm$$

$$= 500\ cm \div 50\ cm = 10$$

No. of circular tiles along the breadth = 4 cm ÷ 50 cm

$$= 400\ cm \div 50\ cm = 8$$

$\therefore$ Total no. of circular tiles = 10 × 8 = 80 tiles

$\therefore$ Area of 80 circular tiles

$$= \left(80 \times \frac{3.14}{6}\right) m^2 = 5 \times 3.14 = 15.7\ m^2$$

$\therefore$ Area of floor that remains uncovered with tiles

= Area of floor – Area of 80 circular tiles

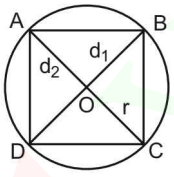
= 20 – 15.7 = 4.3 m^2

12. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if the area of the circle is 1256 cm^2. (use π = 3.14).

[CBSE 2015]

Ans.

It is given that all the vertices of a rhombus lie on a circle so rhombus is a square and its diagonals will be of length 2r cm.



Area of circle = 1256 cm^2

Let r be the radius of circle with centre O and ABCD be the rhombus with vertices positioned on the circle and diagonals d_1 and d_2.

$\Rightarrow$ $\pi r^2 = 1256$

$$r^2 = \frac{1256}{3.14} = 400\ cm^2$$

$\Rightarrow$ $r = 20$ cm

$\Rightarrow$ Diameter of circle, d = 2 × radius

$$= 2 \times 20 = 40\ cm$$

$\Rightarrow$ $d_1 = d_2 = d = 40$ cm

We know that,

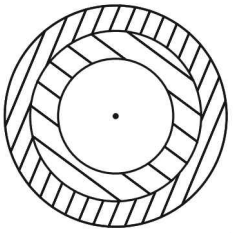
$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 40 \times 40 = 20 \times 40$$

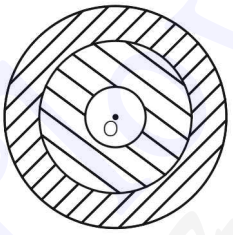
$$= 800\ cm^2$$

Hence, required area of the rhombus is 800 cm^2.

13. An archery target has three regions formed by three concentric circles as shown in the given figure. If the diameters of the concentric circles are in the ratio 1:2:3, then find the ratio of the areas of three regions.



Ans.



Let O be the centre of 3 concentric circles. Since its diameters are in the ratio 1:2:3

$d_1 = x$, $d_2 = 2x$ and $d_3 = 3x$

$\Rightarrow$ Radius of concentric circles

$$r_1 = \frac{x}{2}$$

$$r_2 = \frac{2x}{2} \text{ and } r_3 = \frac{3x}{2}$$

Area of inner circle, $A_1 = \pi\left(\frac{x}{2}\right)^2 = \frac{\pi x^2}{4}$

Area of middle region, $A_2 = \pi x^2 - \frac{\pi x^2}{4} = \frac{3\pi x^2}{4}$

Area of outer region, $A_3 = \pi\left(\frac{3x}{2}\right)^2 - \pi x^2$

$$= \frac{9\pi x^2}{4} - \pi x^2 = \frac{5\pi x^2}{4}$$

$\therefore$ Required ratio = $A_1 : A_2 : A_3$

$$= \frac{\pi x^2}{4} : \frac{3\pi x^2}{4} : \frac{5\pi x^2}{4}$$

= 1:3:5

14. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am and 6:40 a.m.

[CBSE 2012]

Ans.

Length of the minute hand = 5 cm = Radius of the clock

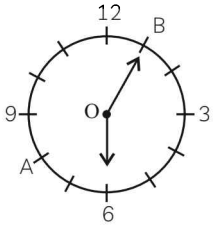
Minutes between the time period 6:05 am to 6:40 am = 35 minutes

In 60 minutes, the minute hand completes one revolution, i.e. 360°.

∴ Angle made by minute hand in 1 minute

= 360°/60° = 60°

Thus angle made by minute hand in 35 minutes = 60° × 35 = 210°



∴ Arc of sector AOBA = $\frac{\theta}{360} \times \pi r^2$

$= \frac{210}{360} \times \frac{22}{7} \times 5 \times 5 = \frac{22 \times 5 \times 5}{12} = \frac{275}{6}$

$= 45\frac{5}{6}$ cm^2

Hence, the required area swept by the minute hand is $45\frac{5}{6}$ cm^2.

15. Area of a sector of central angle 200° of a circle is 770 cm^2. Find the length of the corresponding arc of this sector.

Ans.

Let radius of sector be r'.

It is given that

Given: Central angle of sector, θ = 200°

Area of sector = 770 cm^2.

We know that area of sector = $\frac{\theta}{360} \times \pi r^2$

$\Rightarrow \quad 770 = \frac{200}{360} \times \frac{22}{7} \times r^2$

$\Rightarrow \quad r^2 = \frac{77 \times 18 \times 7}{22} \Rightarrow r = \frac{7 \times 18 \times 7}{2}$

$\Rightarrow \quad r = 7 \times 3 = 21$ cm

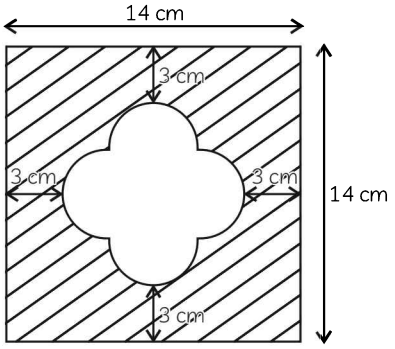
Length of corresponding arc, L = $\frac{\theta}{360} \times 2\pi r$

$= \frac{200}{360} \times 2 \times \frac{22}{7} \times 21$

$= \frac{200 \times 2 \times 22}{120} = \frac{220}{3}$ cm $= 73\frac{1}{3}$ cm

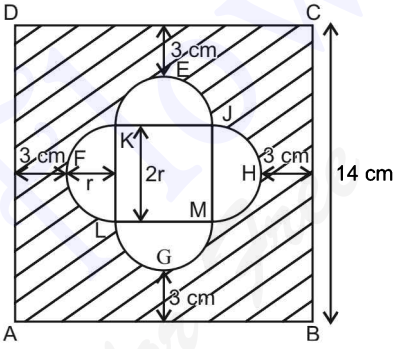
Hence, the required length of the corresponding arc is $73\frac{1}{3}$ cm.

16. Find the area of the shaded region given in the given figure [CBSE 2015]



Ans.

Given: ABCD is a square of side 14 cm. Join JK, KL, LM and MJ.



Identification of shapes of figures:

(i) 4 semi circles of radius *r*

(ii) Square ABCD of side 14 cm

(iii) Square JKLM of side 2r

From the figure,

$AB = 3 + 3 + r + 2r + r$

$\Rightarrow \quad 14 = 6 + 4r$

$\Rightarrow \quad 4r = 14 - 6$

$\Rightarrow \quad r = \frac{8}{4} = 2$

From the figure we have,

ABCD square with AB = 14 cm

4 equal semicircles with diameter JK = 4 cm

JKLM square with side = 4 cm

Area of square ABCD = $(AB)^2 = (14)^2$

$= 196$ cm^2

Area of semicircle with diameter JK

$= \frac{\pi r^2}{2} = \frac{\pi \times (2)^2}{2} = \pi \times 2$ cm^2

Area of 4 semicirlce = $4 \times 2 \times \pi = 8\pi$ cm^2

Area of JKLM square = $(JK)^2 = (4)^2 = 16$ cm^2

Area of shaded region = Area of square ABCD – [Area of 4 semicircles + Area of square JKLM]

$= 196 - [8\pi + 16] = (196 - 8\pi - 16)$ cm^2

$= (180 - 8\pi)$ cm^2

Hence, the required area of the shaded region is $(180 - 8\pi)$ cm^2.

17. Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

Ans.

Given: Area of wheel = 1.54 m^2

Distance covered = 176 m

Let, n be the number of revolutions made by the wheel and r be the radius of the wheel.

Given that, area = 1.54 m^2

$\Rightarrow \quad \pi r^2 = 1.54$

$\Rightarrow \quad \frac{22}{7} r^2 = 1.54 \Rightarrow r^2 = \frac{1.54 \times 7}{22}$

$\Rightarrow \quad r^2 = 0.49$

$\therefore \quad r = 0.7$ m

So, the radius of the wheel is 0.7 m.

We know that,

Distance travelled by wheel in one revolution = Circumference of circular wheel = 4.4 m

Since, distance travelled by a circular wheel = 176 m

Total distance covered by the wheel = No. of revolutions made by wheel × (Distance covered in one revolution)

$\therefore$ No. of revolutions made by wheel = Total Distance / distance in 1 revolution

$= \frac{176}{4.4} = 40$

Hence, the required no. of revolutions is 40.

18. Find the difference of the areas of the two segments of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre.

Ans.

Given: Length of the chord, AB = 5 cm

Let r be the radius of the circle.

Then, OA = OB = r cm

Now, angle subtended at the center of the sector OABO = 90°

Angle subtended at the center of the sector OABO (in radians) = $\theta = \frac{\pi}{2}$

$\therefore$ Triangle AOB is a right-angled triangle.

In ΔAOB,

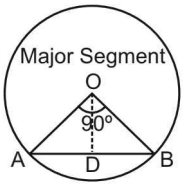
$(AB)^2 = (OA)^2 + (OB)^2$ [Pythagoras theorem]

$\Rightarrow \quad (5)^2 = (OA)^2 + (OB)^2$

$\Rightarrow \quad 25 = r^2 + r^2 \Rightarrow 25 = 2r^2$

$\Rightarrow \quad r = \frac{5}{\sqrt{2}}$ cm

Let OD be the perpendicular drawn on AB.



We know that a perpendicular drawn from the centre to the chord of a circle divides it into two equal parts.

$\Rightarrow \quad AD = DB = \frac{AB}{2} = \frac{5}{2}$ cm

In ΔADO,

$(OA)^2 = OD^2 + AD^2$

$r^2 = (OD)^2 + \left(\frac{5}{2}\right)^2$

$\Rightarrow \quad (OD)^2 = r^2 - \left(\frac{5}{2}\right)^2 = \left(\frac{5}{\sqrt{2}}\right)^2 - \left(\frac{5}{2}\right)^2$

$OD^2 = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$

$OD = \frac{5}{2}$ cm

Area of isosceles ΔAOB = $\frac{1}{2}$ × base × height

Here,

base = AB and height = OD

Area of ΔAOB = $\frac{1}{2} \times 5 \times \frac{5}{2} = \frac{25}{4}$ cm^2

Area of sector AOBA = $\frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times \pi \times \left(\frac{5}{\sqrt{2}}\right)^2 = \frac{\pi}{4} \times \frac{25}{2} = \frac{25\pi}{8}$ cm^2

Area of minor segment

= Area of sector AOBA – Area of ΔAOB

$= \left(\frac{25\pi}{8} - \frac{25}{4}\right)$ cm^2

Area of circle = $\pi r^2 = \pi\left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25\pi}{2}$ cm^2

Area of major segment = Area or circle – Area of minor segment

$= \frac{25\pi}{2} - \left(\frac{25\pi}{8} - \frac{25}{4}\right)$

$= \frac{25\pi}{8} - (4-1) + \frac{25}{4} = \left(\frac{75\pi}{8} + \frac{25}{4}\right)$ cm^2

$\therefore$ Difference of areas of two segments

= Area of major segment – Area of minor segment

$$= \left(\frac{75\pi}{8} + \frac{25}{4}\right) - \left(\frac{25\pi}{8} - \frac{25}{4}\right)$$

$$= \left(\frac{75\pi}{8} - \frac{25\pi}{8}\right) + \left(\frac{25}{4} + \frac{25}{4}\right) = \frac{50\pi}{8} + \frac{50}{4}$$

$$= \left(\frac{25\pi}{4} + \frac{25}{2}\right) \text{cm}^2$$

Hence, the required difference of areas is $\left(\frac{25\pi}{4} + \frac{25}{2}\right) \text{cm}^2$.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. If a circular grass lawn of 35 m in radius has a path 7 m wide running around it on the outside, then the area of the path is

(A) 1450 m^2 (B) 1576 m^2

(C) 1694 m^2 (D) 3368 m^2

Ans. (C)

Explanation: Radius of outer concentric circle

$= (35 + 7)$ m $= 42$ m

Area of path $= \pi(R^2 - r^2)$

$$= \frac{22}{7} \times (1764 - 1225)$$

$$= \frac{22}{7} \times 539 = 1694 \text{ m}^2.$$

2. The diameter of two circles with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

Ans.

Area of circle $= \pi r^2$

Let the radius of circle with centre C = R

According to the question we have,

$$\pi 8^2 + \pi 15^2 = \pi R^2$$

$$64\pi + 225\pi = \pi R^2$$

$$289\pi = \pi R^2$$

$$R^2 = 289$$

$$R = 17 \text{ cm}$$

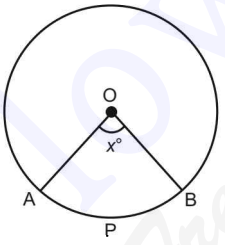
Circumference of circle:

$$2\pi r = 2\pi \times 17$$

$$= 34\pi \text{ cm}$$

So, the circumference of the circle with centre C is 34 π cm.

3. In given figure, O is the centre of a circle. If the area of the sector OAPB is $\frac{5}{36}$ times the area of the circle, then find the value of *x*.



Ans.

Area of sector OAPB is $\frac{5}{36}$ times the area of circle

So, $\pi r^2 \times \frac{x}{360} = \frac{5}{36}\pi r^2$

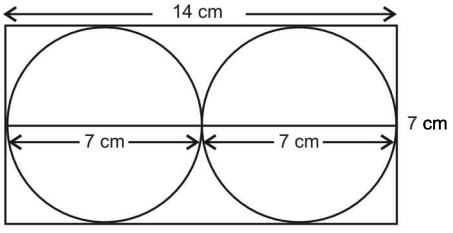
$$\frac{x}{360} = \frac{5}{36}$$

$$x = 50$$

So, the value of *x* is 50°.

4. Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm × 7 cm. Find the area of the remaining cardboard. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Ans.



Area of the remaining cardboard

= Area of rectangular cardboard − 2 × Area of circle

$= l \times b - 2 \times \pi r^2$

$$= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 98 - \frac{44}{7} \times \frac{49}{4}$$

$$= 98 - 77$$

$$= 21$$

Hence, area of reamining cardboard = 21 cm^2.

5. If the perimeter of a semi-circular protractor is 36 cm, find its diameter. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Ans.

$$\text{Perimeter} = \pi r + 2r$$

$$= (\pi + 2)r = 36 \quad \text{[Given]}$$

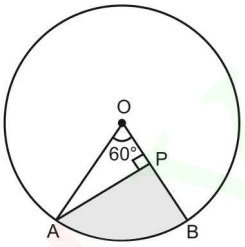
$$\left(\frac{22}{7} + 2\right) r = 36$$

$$\frac{36}{7} \times r = 36$$

$$2r = \frac{7 \times 36 \times 2}{36}$$

So, diameter = $2r$ = 14 cm.

6. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If AP ⊥ OB and AP = 15 cm, find the area of the shaded region.



Ans.

Here,

OA = 17 cm, AP = 15 cm and ΔOPA is right angled triangle.

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2 - 15^2}$$

$$= \sqrt{64}$$

$$= 8 \text{ cm}$$

Area of the shaded region

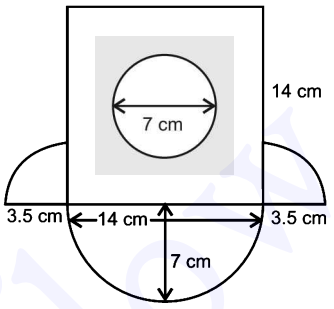
= Area of the sector ΔOAB – Area of ΔOPA

$$= \frac{60}{360} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{60}{360} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$$

= 151.38 – 60 = 91.38 cm^2.

7. In figure, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Ans.

$$\text{Area of square} = a^2$$

$$= (14)^2$$

$$= 196 \text{ cm}^2$$

$$\text{Area of internal circle} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}$$

$$= \frac{77}{2}$$

$$= 38.5 \text{ cm}^2$$

Area of semi circle having diameter

$$= 14 \text{ cm}$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= 77 \text{ cm}^2$$

Area of two quarter circles of radius

$$= \frac{7}{2} \text{ cm}$$

$$= 2 \times \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

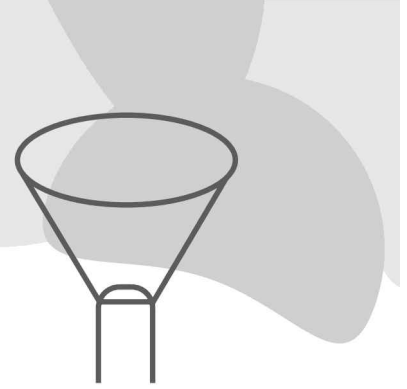
$$= \frac{77}{4}$$

$$= 19.25 \text{ cm}^2$$

Shaded area = 196 – 38.5 + 77 + 19.25

= 292.25 – 38.5

= 253.75 cm^2



12 Surface Areas and Volumes

EXERCISE 12.1

Choose the correct option from the given four options:

1. A cylindrical pencil sharpened at one edge is a combination of:

(A) a cone and a cylinder

(B) frustum of a cone and a cylinder

(C) a hemisphere and a cylinder

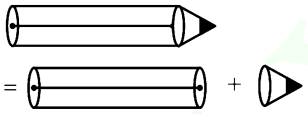
(D) two cylinders.

Ans. (A)

Explanation:

The tip of a sharpened pencil is conical in shape and the rest of the part is cylindricalin shape therefore,

The shape of a sharpened pencil is a cylinder and a cone



= Cylinder + Cone

2. A *surahi* is a combination of:

(A) a sphere and a cylinder

(B) a hemisphere and a cylinder

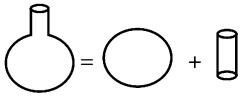
(C) two hemispheres

(D) a cylinder and a cone.

Ans. (A)

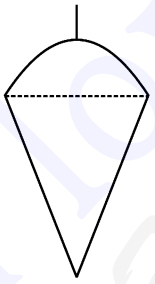
Explanation:

The top part of surahi is cylindrical in shape and bottom part is spherical in shape therefore, surahi is a combination of Sphere and a cylinder.



= Sphere + Cylinder

3. A plumbline (Sahul) is a combination of (see the given figure) :



(A) a cone and a cylinder

(B) a hemisphere and a cone

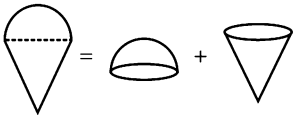
(C) frustum of a cone and a cylinder

(D) sphere and cylinder

Ans. (B)

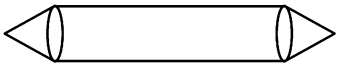
Explanation:

The upper part of plumbline is hemispherical in shape and the bottom part is conical in shape therefore, it is a combination of hemisphere and cone.



= Hemisphere + Cone

4. The shape of a *gilli*, in the *gilli-danda* game (see the given figure), is a combination of:



(A) two cylinders

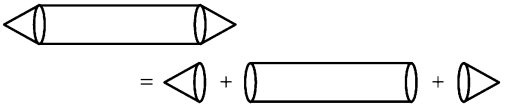
(B) a cone and a cylinder

(C) two cones and a cylinder

(D) two cylinders and a cone

Ans. (C)

Explanation:



As the left and right part of a gilli are conical and the central part is cylindrical

Therefore,

Given figure = Cone + Cylinder + Cone

= Two cones and a cylinder

5. A shuttle cock used for playing badminton has the shape of a combination of:

(A) a cylinder and a sphere

(B) a cylinder and a hemisphere

(C) a sphere and a cone

(D) frustum of a cone and a hemisphere

Ans. (D)

Explanation:

Shuttle cock = Hemisphere Frustum of cone

The upper part of a shuttle cock is hemispherical in shape and the lower part is in the shape of frustum of a cone. Therefore, it is a combination of frustum of a cone and a hemisphere.

6. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accomodate is:

(A) 142296 **(B) 142396**

(C) 142496 **(D) 142596**

Ans. (A)

Explanation:

Given, internal edge of cube = 22 cm

$\therefore$ Volume of cube = $(\text{Side})^3$

$= (22)^3 = 10648 \text{ cm}^3$

Let the spherical marble has radius r.

Diameter of the marble = 0.5 cm

$\therefore$ radius of marble,

$$r = \frac{0.5}{2} = 0.25 \text{ cm}$$

$$\therefore \text{Volume of 1 marble} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.25)^3$$

$$= \frac{1.375}{21} \text{ cm}^3$$

As $\frac{1}{8}$ part of the cube remains unfilled, only $\frac{7}{8}$ part of cube remains filled.

$$\therefore \text{Volume of Filled cube} = \frac{7}{8} \times \text{Volume of cube}$$

$$= \frac{7}{8} \times 10648$$

$$= 7 \times 1331 = 9317 \text{ cm}^3$$

$\therefore$ Required no. of marbles

$$= \frac{\text{Total space filled by marbles}}{\text{Volume of 1 marble}}$$

$$= \frac{9317}{1.375/21} = \frac{9317 \times 21}{1.375}$$

$= 142296$

Hence, cube can accomodate is 142296 number of marbles.

Trick Applied

If we divide the total volume filled by marbles in a cube by volume of a marble, we get required number of marbles.

7. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form of a cone of base diameter 8cm. The height of the cone is:

(A) 12 cm **(B) 14 cm**

(C) 15 cm **(D) 18 cm**

Ans. (B)

Explanation:

We know that during recasting a shape into another its' volume does not change.

Spherical shell:

Internal diameter, $d_1 = 4$ cm

$\therefore$ Internal radius, $r_1 = 2$ cm

External diameter, $d_2 = 8$ cm

$\therefore$ External radius, $r_2 = 4$ cm

$$\text{Volume of spherical shell} = \frac{4}{3}\pi\left[r_2^3 - r_1^3\right]$$

$$= \frac{4}{3}\pi\left[4^3 - 2^3\right] = \frac{4}{3} \times \pi\left[64 - 8\right]$$

$$= \frac{4}{3}\pi \times 56 = \frac{224\pi}{3} \text{ cm}^3$$

Let h be the height of cone.

Diameter of base of cone = 8 cm

$\therefore$ Radius = $\frac{8}{2}$cm = 4 cm

During recasting volume remains same so,

Volume of cone = Volume of spherical shell

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi\left[r_2^3 - r_1^3\right]$$

$$\Rightarrow \quad \frac{1}{3}\times\pi\times(4)^2\times h = \frac{4}{3}\times\pi\times 56$$

$$\Rightarrow \quad h = 14 \text{ cm}$$

Hence, the height of the cone is 14 cm.

Trick Applied

When a solid shape is melted and recast into other solid shape, then volume of both shapes are equal.

8. A solid piece of iron in the form of a cuboid of dimensions 49 cm x 33 cm x 24 cm, is moulded to form a solid sphere. The radius of the sphere is:

(A) 21 cm (B) 23 cm

(C) 25 cm (D) 19 cm

Ans. (A)

Explanation:

Given cuboid with dimensions (49 cm × 33 cm × 24 cm)

$\therefore$ Volume of cuboid = $l \times b \times h$

$= 49 \times 33 \times 24$

$= 38808 \text{ cm}^3$

Let r be the radius of sphere, then

Volume of sphere = $\frac{4}{3}\pi r^3$

During recasting volume remains same so

Volume of sphere = Volume of cuboid

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = l \times b \times h$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 38808$$

$$\Rightarrow \quad r^3 = \frac{38808\times 3\times 7}{4\times 22} = \frac{3528\times 3\times 7}{4\times 2}$$

$$r^3 = 441 \times 21 = 21 \times 21 \times 21$$

$$r = 21$$

Hence, the radius of the sphere is 21 cm.

9. A mason constructs a wall of dimensions 270 cm × 300 crn × 350 cm with bricks, each of size 22.5cm × 11.25cm × 8.75 cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is:

(A) 11100 (B) 11200

(C) 11000 (D) 11300

Ans. (B)

Explanation:

For wall,

Length, l = 270 cm

Breadth, b = 300 cm

Height, h = 350 cm

Volume of wall = l x b x h

$= 270 \times 300 \times 350 \text{ cm}^3$

$= 28350000 \text{ cm}^3$

Fraction of space covered by mortar = $\frac{1}{8}$

Remaining space of wall = $\frac{7}{8}$ × Volume of wall

$= \frac{7}{8} \times 28350000$

$= 7 \times 3543750$

$= 24806250 \text{ cm}^3$

We know that

Volume of cuboid = lbh

where, l = length, b = breadth and h = height

Volume of 1 brick = $l \times b \times h$

$= 22.5 \times 11.25 \times 8.75$

$= 2214.844 \text{ cm}^3$

Required no. of bricks = $\frac{24806250}{2214.844}$

$= 11200$ (approx)

Hence, the no. of bricks required to construct the wall is 11200.

10. 12 solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is:

(A) 4 cm (B) 3 cm

(C) 2 cm (D) 6 cm **[CBSE 2014]**

Ans. (C)

Explanation:

Solid cylinder is recasted into 12 spheres.

So, the volume of 12 spheres will be equal to the volume of the cylinder.

Given, diameter of cylinder = 2 cm

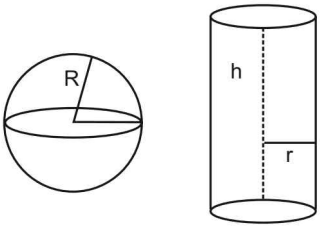
$\therefore$ radius, r = 1 cm

height of cylinder, h = 16 cm

Volume of cylinder = $\pi r^2 h$

$= \pi \times (1)^2 \times 16 = 16\pi \text{ cm}^3$

Let r be the radius of solid sphere.



Volume of 1 sphere = $\frac{4}{3}\pi R^3$

During recasting volume remains same so,

Volume of 12 solid spheres = Volume of cylinder

$\Rightarrow \quad 12 \times \frac{4}{3}\pi R^3 = 16\pi$

$\Rightarrow \quad R^3 = 1 \Rightarrow R = 1$ cm

$\therefore$ Diameter of each sphere, $d = 2r = 2$ cm

Hence, the diameter of each sphere is 2 cm.

11. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. The curved surface area of the bucket is:

(A) **4950 cm^2** (B) **4951 cm^2**

(C) **4952 cm^2** (D) **4953 cm^2**

[CBSE 2017, 10]

Ans. (A)

Explanation:

Clearly, the given bucket is in the form of frustum of a cone.

And we know

Curved surface area of frustum of a cone $= \pi l (R + r)$

Where, l = slant height,

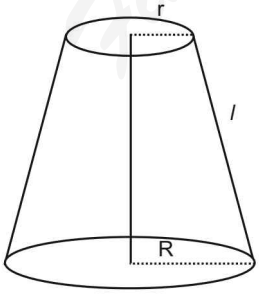
R and r are radii of top and bottom ($R > r$)

$R = 28$ cm

$r = 7$ cm

slant height of bucket, $l = 45$ cm

$\therefore$ Curved surface area of bucket $= \pi l (R + r)$



We know that the curved surface area,

$\Rightarrow$ C.S.A. of bucket $= \pi l(R + r)$

$= \frac{22}{7} \times 45\ (28 + 7) = \frac{22}{7} \times 45 \times 35$

$= 22 \times 45 \times 5 = 4950$ cm^2

12. A medicine -capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. the capacity of the capsule is:

(A) **0.36 cm^3** (B) **0.35 cm^3**

(C) **0.34 cm^3** (D) **0.33 cm^3**

Ans. (A)

Explanation:

Capacity of capsule = Volume of 2 hemispherical part + volume of cylindrical part

Length of capsule = 2 cm

Diameter of capsule = 0.5 cm

$\therefore$ Radius of cylinder, r = radius of hemisphere

Radius of capsule $= \frac{0.5}{2} = 0.25$ cm

Length of entire capsule = radius of 2 hemispherical parts + height of cylindrical part

$\Rightarrow \quad 2 = 2r + h$

$h = 2 - 2r$

Length of cylindrical part of capsule, h

$= 2 - (0.25 + 0.25)$

$= 1.5$ cm

Volume of capsule

= Volume of cylinder part + Volume of 2 hemispheres

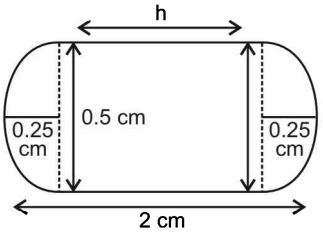
$= \pi r^2 h + 2 \times \frac{2}{3} \times \pi r^3$

$= \frac{22}{7} \times (0.25)^2 \times 1.5 + \frac{2 \times 2}{3} \times \frac{22}{7} \times (0.25)^3$

$= \frac{22}{7} \times (0.25)^2 \left[1.5 + \frac{2 \times 2}{3} \times 0.25\right]$

$= \frac{22 \times 0.0625}{7}\left[1.5 + 0.167 \times 2\right]$

$= 0.36$ cm^3



13. If two solid hemispheres of same base radius r are joined together along their bases, then the curved surface area of this new solid is:

(A) $4\pi r^2$ (B) $6\pi r^2$

(C) $3\pi r^2$ (D) $8\pi r^2$ **[CBSE 2011]**

Ans. (A)

Explanation:

When two hemispheres are joined together along their bases, a sphere of the same base radius is formed.

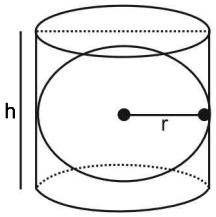
And curved surface area of the sphere = $4\pi r^2$.

14. A right circular cylinder of radius r cm and height h cm ($h > 2r$) just encloses a sphere of diameter:

(A) r cm (B) $2r$ cm

(C) h cm (D) $2h$ cm

Ans. (B)



Explanation:

$\therefore$ As the sphere just encloses in a cylinder, the diameter of sphere will be equal to the diameter of cylinder.

$\Rightarrow$ Diameter of sphere = Diameter of cylinder

$= 2r$ cm.

15. After the conversion of a solid from one shape to another, the volume of the new shape will:

(A) increase (B) decrease

(C) remain unaltered (D) be doubled

Ans. (C)

Explanation:

In a solid, reshaping it con result in a different size (dimensions) like area, length, width or height.

But over all, the volume of that solid remains the same always.

16. In a right circular cone, the cross-section made by a plane parallel to the base is a:

(A) circle (B) frustum of a cone

(C) sphere (D) hemisphere

Ans. (A)

Explanation:

We know that, if a cone is cut by a plane parallel to the base of cone, then all cross sections parallel to the base will be similar to the base i.e., circle.

17. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is

(A) 3:4 (B) 4:3

(C) 9:16 (D) 16:9

Ans. (D)

Explanation:

We know that r_1 and r_2 be the radii of two spheres respectively

Volume of sphere = $\frac{4}{3}\pi r^3$

Given : Ratio of volume is 64:27.

$$\frac{\text{Volume of } 1^{st} \text{ sphere}}{\text{Volume of } 2^{nd} \text{ sphere}} = \frac{64}{27}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi {r_1}^3}{\frac{4}{3}\pi {r_2}^3} = \frac{64}{27}$$

$$\Rightarrow \quad \frac{{r_1}^3}{{r_2}^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

Surface area of sphere = $4\pi r^2$

$$\frac{\text{Surface area of } 1^{st} \text{ sphere}}{\text{Surface area of } 2^{nd} \text{ sphere}}$$

$$= \frac{4\pi {r_1}^2}{4\pi {r_2}^2} = \frac{{r_1}^2}{{r_2}^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the ratio of their surface area is 16:9.

EXERCISE 12.2

Write True or False and give reasons for your answer:

1. Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2$. **[CBSE 2011]**

Ans. False.

When two hemispheres of equal base radius are joined together along their bases, we get a sphere of the same radius.

Curved surface area of hemisphere = $2\pi r^2$

Curved surface area of sphere = $2\pi r^2 + 2\pi r^2$

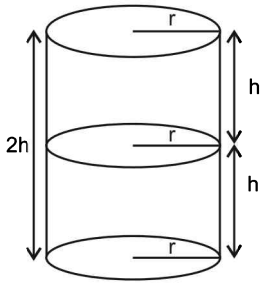
$= 4\pi r^2$

2. A solid cylinder of radius r and height h is placed over an other cylinder of the same height and radius. The total surface area of the shape so formed is $4\pi rh + 4\pi r^2$.

Ans. False.

As one cylinder is placed over another, the base of the first cylinder and the top of the other cylinder will not be covered in total surface area.

When one cylinder is placed over an other cylinder of the same height and radius,



height of new cylinder = $2h$

radius of new cylinder = r

$\therefore$ TSA of new cylinder

$= 2\pi r(2h) + 2\pi r^2$

$= 4\pi rh + 2\pi r^2$

3. A solid cone of radius r and height h is placed over a solid cylinder having the same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r\left[\sqrt{r^2+h^2}+3r+2h\right]$.

Ans. False.

When a solid cone is placed over a solid cylinder of the same radius, the base of the cone and the top of the cylinder will not be covered in the total surface area.

We know that, total surface area of cone of radius r and height h,

TSA = CAS + Area of base = $\pi rl + \pi r^2$,

where $l = \sqrt{l^2+h^2}$

Total surface area of cylinder of radius r and height h

= CSA + Area of both bases

$= 2\pi rh + 2\pi r^2$

When cone is placed over a cylinder, then one base is common for both. So, the total surface area of combined solid,

$= \pi rl + 2\pi rh + \pi r^2$

$= \pi r\left[\sqrt{r^2+h^2}+2h+r\right]$

4. A solid ball is exactly fitted inside a cubical box of side a. The volume of the ball is $\frac{4}{3}\pi a^3$.

Ans. False.

As the ball is exactly fitted into the cubical box of side a,

$\Rightarrow$ Diameter of ball = Edge length of cube

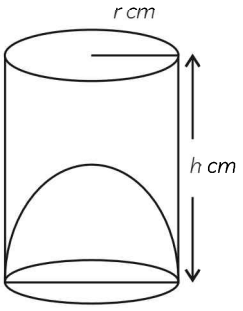
$\Rightarrow \quad 2r = a$

$\Rightarrow \quad r = \frac{a}{2}$

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3}\frac{\pi \times a^3}{8} = \frac{\pi a^3}{6}$

5. The capacity of a cylindrical vessel with a hemispherical portion raised upwards at the bottom as shown in the given figure is $\frac{\pi r^2}{3}[3h-2r]$.



Ans. True.

Capacity of given shape

= Volume of cylinder – Volume of hemispherical portion

Base radius of cylinder = radius of hemisphere

$\Rightarrow$ Capacity of given shape

$= \pi r^2 h - \frac{2}{3}\pi r^3$ [$\because$ Volume of cylinder = $\pi r^2 h$

Volume of heisphere = $\frac{2}{3}\pi r^3$]

$= \frac{\pi r^2}{3}(3h-2r)$

So, the given statement is true.

6. The curved surface area of the frustum of a cone is $\pi l (r_1 + r_2)$, where $l = \sqrt{h^2+(r_1+r_2)^2}$, r_1 and r_2, are the radii of the two ends of the frustum and h is the vertical height.

[CBSE 2010]

Ans. False.

We know that,

Curved surface area of frustum = $\pi l(r_1 + r_2)$

Where, r_1 and r_2 are the radii of two ends ($r_1 > r_2$)

where l = slant height

$$l = \sqrt{h+(r_1-r_2)^2}$$

EXERCISE 12.3

1. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed. **[CBSE 2012]**

Ans.

Given : side of first cube, a_1 = 3cm

side of 2nd cube, a_2 = 4 cm

side of 3rd cue, a_3 = 5 cm

We know that volume of cube = a^3

[Where a is side of cube]

∴ Volume of 1st cube, $V_1 = (3)^3 = 27$ cm^3

Volume of 2nd , $V_2 = (4)^3 = 64$ cm^3

Volume of 3rd cube, $V_3 = (5)^3 = 125$ cm^3

Let edge of resulting cube be x.

Volume of resulting cube = volume of (1st + 2nd + 3rd) cube

$$x^3 = (27 + 64 + 125) \text{ cm}^3$$

$$x^3 = 216 \text{ cm}^3$$

$$\Rightarrow \quad x = 6 \text{ cm}$$

Hence, the edge of cube so formed is 6 cm.

2. How many shots, each having diameter 3 cm, can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm x 12 cm?

Ans.

We know that

Volume of cuboid = lbh

where, l = length, b = breadth and h = height

For cuboidal lead:

Length, l = 9 cm

Breadth, b = 11 cm

Height, h = 12 cm

∴ Volume of cuboid = 9 × 11 × 12 = 1188 cm^3

Diameter of spherical shot = 3 cm

∴ Radius of shot = $\frac{3}{2}$ = 1.5 cm

Volume of 1 shot = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(1.5)^3$

$= \frac{4\times22\times1.5\times1.5\times1.5}{7} = \frac{99}{7} = 14.14 \text{ cm}^3$

Now, required number of shots

$= \frac{\text{Volume of cuboidal lead}}{\text{Volume of 1 shot}} = \frac{1188}{14.14} = 84(\text{approx})$

Hence, 84 shots can be made from the given solid.

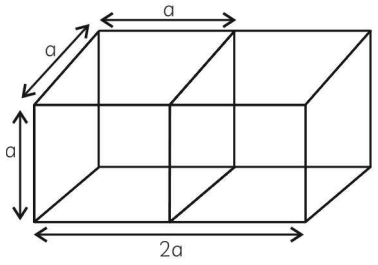
3. Two identical cubes, each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?

[CBSE 2020, 11]

Ans.

Let a be the side of one cube.

As two cubes are joined together, surfaces that are joined together will not be included in the surface area of the resulting cuboid.



Given:

Volume of cube, $a^3 = 64$

$$\Rightarrow \quad a = 4 \text{ cm}$$

On joining 2 cubes, we get cuboid whose dimensions are

length = $2a$ cm

breadth = a cm

height = a cm

Surface area of the resulting cuboid,

= 2 ($lb + bh + hl$)

$= 2\,(2a \cdot a + a \cdot a + a \cdot 2a) = 2\,(2a^2 + a^2 + 2a^2)$

$= 2\,(5a^2) = 10a^2$

$= 10\,(4)^2 = 10 \times 16 = 160\ \text{cm}^2$

Hence, surface area of the resulting cuboid is $160\ \text{cm}^2$.

4. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid. [CBSE 2012]

Ans.

Since the conical cavity is hollowed out from the cube,

Volume of remaining solid = volume of cube - volume of cone

For cube:

side $a = 7$ cm

For cone Height, $h = 7$ cm

radius, $r = 3$ cm

Since conical cavity is hollowed out from cube, volume of remaining solid,

= Volume of cube – Volume of cone.

Volume of cube $= a^3 = (7)^3 = 343$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$$

$$= 66\ \text{cm}^3$$

$\therefore$ Volume of remaining solid $= 343 - 66$

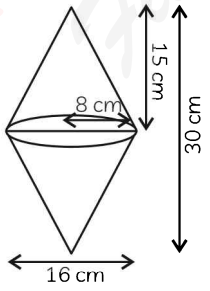
$$= 277\ \text{cm}^3$$

Hence, the required volume of the solid is $277\ \text{cm}^3$.

5. Two cones with the same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Ans.

When two cones with the same base radius and height are joined, the shape so formed will be



Given: radius of cone, $r = 8$ cm

height of cone, $h = 15$ cm.

slant height of cone,

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{64 + 225} = \sqrt{289}$$

$$= 17\ \text{cm}$$

When two identical cones are joined base to base, the total surface area of new solid

becomes equal to the sum of curved surface areas of both the cones.

Surface area of shape formed,

= Curved surface area of 1st cone + curved surface area of 2nd cone

= 2 × surface area of cone

[As both cones are identical]

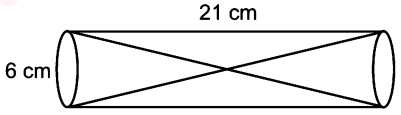
$$= 2\pi rl = 2 \times \frac{22}{7} \times 8 \times 17$$

$$= \frac{5984}{7} = 854.857\ \text{cm}^2$$

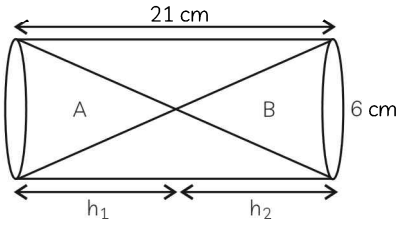
Hence, surface area of the shaped formed is $854.85\ \text{cm}^2$.

$= 855\ \text{cm}^2$ (approx.)

6. Two solid cones A and B are placed in a cylinderical tube as shown in the given figure. The ratio of their capacities are 2:1. Find the heights and capacities of the cones. Also, find the volume of the remaining portion of the cylinder.



Ans.



Diameter of cylinder = 6 cm

Radius of cylinder, $r = 3$ cm

As both cones have equal radius

$\therefore$ Radius of cone A = Radius of cone B = $r = 3$ cm

Let h_1 be height of cone A and h_2 be height of cone B.

$$\text{It is given that } \frac{\text{Volume of cone A}}{\text{Volume of cone B}} = \frac{2}{1}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad \frac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{2}{1} \Rightarrow h_1 = 2h_2$$

Now, total height of cylinder = 21 cm

$\Rightarrow \quad h_1 + h_2 = 21$

$\Rightarrow \quad 2h_2 + h_2 = 21 \Rightarrow 3h_2 = 21$

$\Rightarrow \quad h_2 = 7$ cm

$\Rightarrow \quad h_1 = 2h_2 = 2 \times 7 = 14$ cm

Volume of cone A $= \frac{1}{3}\pi r^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times (3)^2 \times 14 = 132\,\text{cm}^3$

Volume of cone B $= \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3} \times \frac{22}{7} \times (3)^2 \times 7 = 66\ \text{cm}^3$

We know that

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (3)^2 \times 21$

$= 594\ \text{cm}^3$.

Volume of remaining solid

= (Volume of cylinder) – (Volume of cone A + Volume of cone B)

= 594 – (132 + 66) = 594 – 198

= 396 cm^3

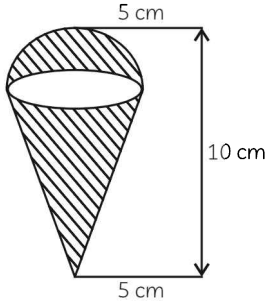
Hence,

Cone A height = 14 cm and volume = 132 cm^3

Cone B height = 7 cm and volume = 66 cm^3

Volume of remaining solid = 396 cm^3

7. An ice cream cone full of ice cream has radius 5 cm and height 10 cm as, shown in the. given figure Calculate the volume of ice cream, provided that its $\frac{1}{6}^{\text{th}}$ part is left unfilled with the ice cream.



Ans.

Ice–cream cone can be considered as a hemisphere surmounted on a cone.

Radius of cone = Radius of hemisphere = 5 cm

Total height = 10 cm

Height of cone = Total height – Radius of hemisphere

= 10 – 5

= 5 cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (5)^3 = \frac{2 \times 22 \times 125}{21}$

$= \frac{5500}{21} = 261.90\,\text{cm}^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 5$

$= \frac{2750}{21} = 130.95\ \text{cm}^3$

Total volume of ice cream cone

= Volume of hemisphere + Volume of cone

= 261.90 + 130.95 = 392.85 cm^3

Since, $\frac{1}{6}^{\text{th}}$ part is left unfilled with ice cream,

this implies $\frac{5}{6}^{\text{th}}$ part is filled with ice cream.

$\therefore$ Required volume of ice cream $= \frac{5}{6} \times 392.85$

$= 327.4\ \text{cm}^3$

Trick Applied

Ice-cream cone is combination of hemisphere and cone.

8. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm. [CBSE 2014]

Ans.

When *n* marbles are dropped into the beaker filled partially with water, the volume of water raised in the beaker, will be equal to the volume of *n* marbles, The shape of water raised in beaker is cylindrical.

Given: For marble

Diameter = 1.4 cm

Radius = 0.7cm

$\Rightarrow$ Volume of one marble = $\frac{4}{3}\pi(0.7)^3$

$= \frac{4}{3}\pi \times 0.343$ [Volume of sphere = $\frac{4}{3}\pi r^3$]

$= \frac{1.372\pi}{3}$ cm^3

For beaker

Diameter = 7 cm

Radius = 3.5 cm

Height of water level raised = 5.6 cm

$\therefore$ Volume of raised water in beaker

$= \pi r^2 h = \pi \times (3.5)^2 \times 5.6$

$= 68.6\ \pi$ cm^3

Volume of n spherical balls = Volume of water raised in cylinder

Required number of marbles, n

$= \frac{\text{Volume of raised water in beaker}}{\text{Volume of one spherical marble}}$

$= \frac{68.6\pi}{1.372\pi} \times 3 = 150$

Hence, 150 marbles are required.

9. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm? [CBSE 2014]

Ans.

Given: For spherical lead shot

Diameter = 4.2 cm

Radius = $\frac{4.2}{2}$ = 2.1 cm

$\therefore$ Volume of spherical lead shot = $\frac{4}{3}\pi r^3$

$\therefore \ = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$

$= \frac{4 \times 22 \times 21 \times 21 \times 21}{21 \times 1000}$ cm^3 ...(i)

For cuboidal lead piece, length l = 66 cm

breadth, b = 42 cm height, h = 21 cm

Volume of cuboidal lead piece = $l \times b \times h$

= 66 × 42 × 21 ...(ii)

It is given that spherical lead shots are made from a solid rectangular lead piece,

$\therefore$ Number of spherical lead shots

$= \frac{\text{Volume of retangular lead piece}}{\text{Volume of spherical lead shot}}$ ...(iii)

Using eqn (i), (ii) & (iii)

Number of spherical lead shots

$= \frac{66 \times 42 \times 21}{4 \times 22 \times 21 \times 21} \times 1000$

$= \frac{3 \times 22 \times 2 \times 21 \times 21}{4 \times 22 \times 21 \times 21} \times 1000$

$= \frac{3 \times 1000 \times 2}{4} = 1500$

Hence, the required number of spherical lead shots is 1500.

10. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm?

Ans.

Given: For spherical lead shot Diameter = 4 cm

$\therefore$ Radius, $r = \frac{4}{2} = 2$ cm

Volume of spherical lead shot = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2)^3 = \frac{4 \times 22 \times 8}{21}$ cm^3

It is given that spherical lead shots are made out of a solid cube of lead,

$\therefore$ Number of spherical shots

$= \frac{\text{Volume of solid cube of lead}}{\text{Volume of spherical lead shot}}$

Also, Edge of solid cube, a = 44 cm

$\therefore$ Volume of solid cube = a^3

$= (44)^3$

$= (44)^3$ cm^3

$\therefore$ Number of spherical lead shots

$= \frac{44 \times 44 \times 44}{4 \times 22 \times 8} \times 21 = 11 \times 11 \times 21$

$= 121 \times 21 = 2541$.

Hence, the required no. of spherical lead shots is 2541.

11. A wall 24 m long, 0.4 m thick and 6 m high is constructed with bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}^{th}$ of the volume of the wall, then find the number of bricks used in constructing the wall.

Ans.

Given: Volume of cuboid = lbh,

where, l = length, b = breadth and h = height

For wall,

Length, l = 24 m

Breadth, $b = 0.4$ m

Height, $h = 6$ m

Volume of wall = $l \times b \times h$

$= 24 \times 0.4 \times 6$

$= \frac{24 \times 4 \times 6}{10}$ m^3

For 1 brick:

length $l_b = 25$ cm $= 0.25$ m

breadth, $b_b = 16$ cm $= 0.16$ m

height, $h_b = 10$ cm $= 0.10$ m

$\therefore$ Volume of 1 brick = $l \times b \times h$

$= 0.25 \times 0.16 \times 0.10$ m^3

It is given that $\left(\frac{1}{10}\right)^{th}$ of the volume of the wall is occupied by mortal, therefore $\left(\frac{9}{10}\right)^{th}$ of the volume is covered by bricks.

$\therefore$ Volume of wall covered by bricks = $\frac{9}{10}$ (Volume of wall)

$= \frac{9}{10}\frac{(24 \times 4 \times 6)}{10}$

No. of bricks

$= \frac{\text{Volume of wall covered by bricks}}{\text{Volume of one brick}}$

$= \frac{\frac{9 \times 24 \times 4 \times 6}{10 \times 10}}{\frac{25 \times 16 \times 10}{1000000}} = \frac{9 \times 24 \times 4 \times 6 \times 1000}{25 \times 16}$

$= 24 \times 6 \times 9 \times 10 = 12960$

Hence, the required no. of bricks in constructing the wall is 12960.

12. Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

[CBSE 2016]

Ans.

Metallic circular disc:

Diameter = 1.5 cm

$\Rightarrow$ Radius = $\frac{1.5}{2} = 0.75$ cm

Height = 0.2 cm

$\therefore$ Volume of circular disc = $\pi r^2 h$

$= \pi \times (0.75)^2 \times 0.2$ cm^3

Right circular cylinder :

Diameter = 4.5 cm

$\Rightarrow$ Radius = $\frac{4.5}{2} = 2.25$ cm

Height = 10 cm

$\therefore$ Volume of right circular cylinder = $\pi r^2 h$

$= \pi \times (2.25)^2 \times 10$ cm^3

No. of metallic circular disc

$= \frac{\text{Volume of right circular cylinder}}{\text{Volume of metallic circular disc}}$

$= \frac{\pi \times 2.25 \times 2.25 \times 10}{\pi \times 0.75 \times 0.75 \times 0.2} = \frac{225 \times 225 \times 10 \times 10}{75 \times 75 \times 2}$

$= 3 \times 3 \times 5 \times 10 = 450$

Hence, the required number of metallic circular discs is 450.

EXERCISE 12.4

1. A solid metallic hemisphere of radius 8 cm is melted and recast into a right circular cone of base radius 6 cm. Determine the height of the cone.

Ans.

For hemisphere, radius, $r = 8$ cm

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times (8)^3 = \frac{1024\pi}{3}$ cm^3

For cone, that is recasted from hemisphere

base radius, = 6 cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2 h$

$= 12\pi h$ cm^3

As volume remains the same, when a body is reformed to another body

$\therefore$ Volume of hemisphere = Volume of cone

$\frac{1024\pi}{3} = 12\,\pi h$

$\Rightarrow$ $h = \frac{1024\pi}{3 \times 12\pi} = \frac{256}{9} = 28.44$ cm

Hence, the height of cone = 28.44 cm.

2. A rectangular water tank of base 11 m × 6 m contains water upto a height of 5 m. If the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

[CBSE 2016]

Ans.

Given:

For cuboidal tank, (rectangular)

length, $l = 11$ m

breadth, $b = 6$ m

height, $h = 5$ m

Volume of tank = $lbh = 11 \times 6 \times 5 = 330\ m^3$

Let cylinderical tank be filled upto a height of 'h' m.

Given radius of cylindrical tank = 3.5 m

Volume of cylinder = $\pi r^2 h$

$\therefore$ Volume of water in cylinderical tank

$= \pi(3.5)^2 \times h = \frac{22}{7} \times 3.5 \times 3.5 \times h$

$= 38.5\ h$

According to the question,

Volume of water is the same in both tanks

$330 = 38.5\ h$

$h = \frac{330}{38.5} = \frac{3300}{385} = 8.57$ or 8.6 m (approx.)

Hence, the height of water level in the cylinderical tank is 8.6 m.

3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm, provided the thickness of the iron is 1.5 cm? If one cubic cm of iron weighs 7.5 g, find the weight of the box.

Ans.

Given: For open box

External length, $L = 36$ cm

External breadth, $b = 25$ cm

External height, $h = 16.5$ cm

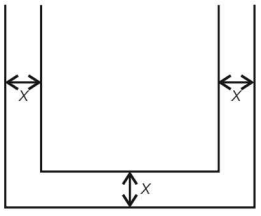
$\therefore$ Volume of external box = lbh

$= 36 \times 25 \times 16.5$

$= 14850\ cm^3$

Given, thickness of iron, $x = 1.5$ cm and box is open from top

For internal box,



length, $l' = l - 2x$

$= 36 - (2 \times 1.5) = 36 - 3 = 33$ cm

breadth, $b' = b - 2x$

$= 25 - (2 \times 1.5) = 25 - 3 = 22$ cm

height, $h' = h - 2x$

$= 16.5 - 1.5 = 15$ cm

$\therefore$ Volume of internal box = $l'b'h'$

$= 33 \times 22 \times 15 = 10890\ cm^3$

Volume of metal used in box

= Volume of external box – Volume of internal box

$= 14850 - 10890 = 3960\ cm^3$

Given: weight of 1 cm^3 of iron = 7.5 gm

$\therefore$ Weight of 3960 cm^3 of iron = 3960×7.5 gm

$= \frac{3960 \times 7.5}{1000} = 29.7$ kg

Hence, the weight of the box is 29.7 km and the volume of the metal = 3960 cm^3.

4. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one fifth of a litre?

Ans.

Given: shape of barrel of fountain pen is cylinder length of barrel i.e., $h = 7$ cm

diameter of barrel i.e., $d = 5$ mm

$= \frac{5}{10}\text{cm} = \frac{1}{2}\text{cm}$

$\therefore$ radius, $r = \frac{1}{2 \times 2} = \frac{1}{4} = 0.25\,\text{cm}$

$\Rightarrow$ Volume of barrel = $\pi r^2 h$

$= \frac{22}{7} \times (0.25)^2 \times 7$

$= 22 \times 0.0625 = 1.375\ cm^3$

According to question,

1.375 cm^3 of ink can write 3300 words $\left(\frac{1}{5}\right)^{\text{th}}$

of a litre $= \frac{1}{5} \times 1000\,cm^3 = 200\ cm^3$.

$\therefore$ 200 cm^3 of ink can write $\frac{3300}{1.375} \times 200$

$= 480000$ words

Hence, $\frac{1}{5}^{\text{th}}$ of a litre of ink can write 480000 words on an average.

5. Water flows at the rate of 10 m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm? [CBSE 2011]

Ans.

When water flows through a pipe of a certain area of cross–section A with velocity v,

then volume of water coming from pipe in time

t = Area of cross–section × Length

Given: speed of water flow = 10 m/min

= 1000 cm / min

For pipe

Diameter, d = 5 mm

$= \frac{5}{10}$ cm

Radius, $r = \frac{5}{10 \times 2}$ cm

= 0.25 cm

$\Rightarrow$ Area of base = πr^2

$= \frac{22}{7} \times (0.25)^2 = 0.1964 \text{ cm}^2$

$\therefore$ Amount of water that flows out of cylindrical pipe in 1 minute = $\pi r^2 h$

$= \pi \times (0.25)^2 \times 10 \times 100$

$= 62.5\pi \text{ cm}^3$

Amount of water required to fill conical vesel

= Volume of conical vesel

$= \frac{1}{3}\pi r^2 h$

Here, radius of conical vessel = $\frac{40}{2}$ = 20 cm

depth i.e., height of conical vessel = 24 cm

$\Rightarrow$ Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (20)^2 \times 24$

$= 3200\pi \text{ cm}^3$

Time required to fill the vessel

$= \frac{\text{Volume of conical vessel}}{\text{Volume of water that flows out in 1 minute}}$

$= \frac{3200\,\pi}{62.5\,\pi} = \frac{32000}{625}$

= 51.2 minutes

= 51 minutes + $\frac{2}{10}$ × 60 seconds

= 51 minutes 12 seconds

Hence, the time required is 51 minutes 12 seconds.

6. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? [CBSE 2018]

Ans.

Given: Heap of rice is in the form of cone with.

Height, h = 3.5 m

Base diameter = 9 m

radius, $r = \frac{9}{2}$m

Volume of rice = Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{9}{2}\right)^2 \times 3.5$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{9 \times 9}{4} \times \frac{35}{10} = \frac{6237}{84}$

$= 74.25 \text{ cm}^2$

Canvas required to just cover heap of rice

= Curved surface area of conical heap.

We know, curved surface area of a cone = πrl

Where r is base radius and l is slant height.

$= \pi r l$

$= \pi r \sqrt{r^2 + h^2}$ [Slant height of cone, $l = \sqrt{r^2 + h^2}$]

$= \frac{22}{7} \times \frac{9}{2} \times \sqrt{\left(\frac{9}{2}\right)^2 + (3.5)^2}$

$= \frac{22}{7} \times \frac{9}{2} \times \sqrt{\frac{81}{4} + 12.25} = \frac{99}{7} \times \sqrt{\frac{130}{4}}$

$= 14.142 \times 5.7 = 80.61 \text{ m}^2$.

Hence, 80.61 m^2 canvas is requred to cover the heap. and volume is 74.25 cm^3.

7. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape, each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹ 0.05 per dm^2.

Ans.

Given: Shape of the pencils is cylindrical.

Let radius of base be r.

length of pencil, h = 25 cm

circumference of base = 1.5 cm

$\Rightarrow$ $2\pi r$ = 1.5 cm

$\Rightarrow$ $r = \frac{1.5}{2\pi}$ cm

Curved surface area of 1 pencil

$= 2\pi rh$

$= 2 \times \pi \times \frac{1.5}{2\pi} \times 25 = 37.5 \text{ cm}^2.$

We know that

$1 \text{ cm} = 0.1 \text{ dm}$

$1 \text{ cm}^2 = 0.01 \text{ dm}^2$

$37.5 \text{ cm}^2 = 0.01 \times 37.5 \text{dm}^2$

$= 0.375 \text{ dm}^2$

$\Rightarrow$ Curved surface area of 120000 pencils

$= 0.375 \times 120000$

$= 45000 \text{ dm}^2.$

Given cost of colouring 1 dm^2 curved surface area of pencil = ₹ 0.05

$\therefore$ Cost of colouring 45000 dm^2 CSA of pencil

= ₹ 0.05 × 45000 = ₹ 2250

8. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm? [CBSE 2011]

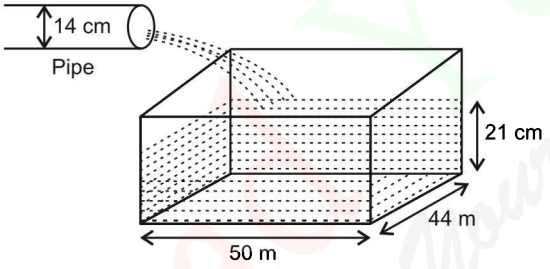
Ans.

Given: Speed of water = 15 km/h

= 15000 m/h

Diameter of pipe, $2r$ = 14 cm

$\Rightarrow$ r = 7 cm



Shape of pipe is cylinder

$\therefore$ Volume of water that flows out of pipe in 1 hour

$= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ m}^3$

$= 231 \text{ m}^3$ ...(i)

It is given that for cuboidal pond

length, l = 50 m

breadth, b = 44 m

Depth required, h = 21 cm = $\frac{21}{100}$m

$\therefore$ Volume of water in coboidal pond

$= lbh$

$= 50 \times 44 \times \frac{21}{100} \text{ m}^2 = 22 \times 21 \text{ m}^3$

$= 462 \text{ m}^3$

From eqn (i)

231 m^3 of water flows out of pipe in 1 hour

$\Rightarrow$ 462 m^3 of water flows out of pipe in

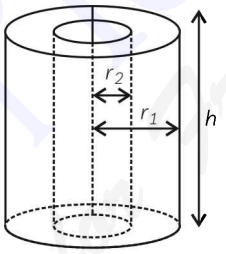
$\frac{1}{231} \times 462 = 2$ hours

Hence, the required time is 2 hours.

9. A solid iron cuboidal block of dimensions 4.4 m × 2.6 m × 1 m is recast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe. [CBSE 2017]

Ans.

For cuboidal block,



length, l = 4.4 m

breadth, b = 2.6 m

height, h = 1 m

Volume = $l \times b \times h = (4.4 \times 2.6 \times 1) \text{ m}^3$

$= 11.44 \text{ m}^3$

As the volume remains the same when a body is recast to another body, we have

$\Rightarrow$ Volume of cylindrical pipe = 11.44 m^3

For cylindrical pipe :

Internal radius, r_2 = 30 cm = 0.3 m

Thickness = 5 cm = 0.05 m

External radius, r_1 = Internal radius + thickness

= 0.3 + 0.05 = 0.35 m

Let h be the length of pipe

$\therefore$ Volume of hollow cylindrical pipe

$= \pi h (r_1^2 - r_2^2)$

$11.44 = \frac{22}{7} \times h \times [(0.35)^2 - (0.3)^2]$

$11.44 = \frac{22}{7} \times h[(0.35 - 0.3)(0.35 + 0.3)]$

$[\because (a^2 - b^2) = (a - b)(a + b)]$

$11.44 = \frac{22}{7} \times h \times 0.05 \times 0.65$

$\Rightarrow h = \frac{11.44 \times 7}{22 \times 0.05 \times 0.65} = 112$ m

Hence, the required length of the pipe is 112 m.

10. **500 people are taking a dip into a cuboidal pond which is 80 m long and 50 rn broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3?**

Ans.

Let h be the rise of the level of water in the pond when 500 people are taking a dip into it.

Given: For cuboidal pond

length, l = 80 m

Breadth, b = 50 m

Height is h.

Volume of water raised in the pond = $l \times b \times h$

$= 80 \times 50 \times h = 4000h\ m^3$ ...(i)

It is given that average displacement by a person = 0.04 m^3

$\therefore$ Average displacement by 500 people = $500 \times 0.04\ m^3 = 20\ m^3$

By given condition

Volume of water raised in pond = Average displacement of water by 500 people

[Using eqn (i)]

$\Rightarrow \quad 4000\ h = 20$

$\Rightarrow \quad h = \frac{20m}{4000} = \frac{20 \times 100}{4000}$ cm

$= 0.5$ cm

Hence, rise of water level in the pond = 0.5 cm.

11. **16 glass spheres each of radius 2 cm are packed into a cuboidal box of internal dimensions 16 cm x 8 cm x 8 cm and then the box is filled with water. Find the volume of water filled in the box.**

Ans.

For cuboidal box,

length, l = 16 cm

breadth, b = 8 cm

height, h = 8 cm

$\therefore$ Volume of cuboidal box = lbh

$= 16 \times 8 \times 8 = 1024\ cm^3$

For glass sphere,

radius = 2 cm

$\therefore$ Volume of 1 glass spheres = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2)^3 = \frac{704}{21} = 33.52\ cm^3$.

Volume of 16 glass sphere = 16×33.52

$= 536.38\ cm^3$

$\therefore$ Required volume of water = Volume of cuboidal box – Volume of 16 glass sphere

$= 1024 - 536.37$

$= 487.62\ cm^3$

Hence, volume of water filled in the box is $487.62\ cm^3$.

12. **A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.** **[CBSE 2019, 14]**

Ans.

For cylindrical bucket,

radius, r = 18 cm

height, h = 32 cm

As we know,

Volume of cylinder = $\pi r^2 h$

Where r is base radius and h is height of cylinder.

Volume of sand in cylindrical bucket

$= \pi r^2 h = \pi(18)^2 \times 32 = 10368\ \pi\ cm^3$.

For conical heap,

height, h = 24 cm

Let r be the radius of conical heap.

Volume of sand in the heap = $\frac{1}{3}\pi r^2 h$.

$= \frac{1}{3} \times \pi \times r^2 \times (24) = 8\pi r^2$

As the volume of sand is constant.

Volume of sand in bucket = Volume of conical heap

$\Rightarrow \quad 10368\pi = 8\pi r^2$

$\Rightarrow \quad 10368 = 8r^2$

$\Rightarrow \quad r^2 = \frac{10368}{8} = 1296$

$\Rightarrow \quad r = 36$ cm

Also, we know that

Slant height of cone, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(36)^2 + (24)^2} = \sqrt{1296 + 576} = \sqrt{1872}$

$\therefore\ l = 43.267$ cm.

Hence, radius of conical heap of sand = 36 cm and slant height of conical heap = 43.267 cm.

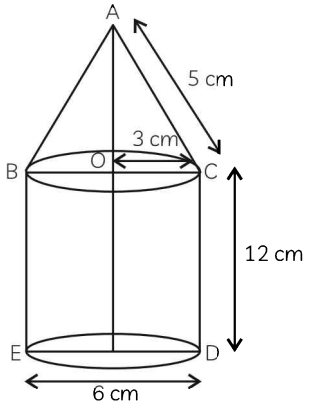
13. **A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm,**

respectively. If the the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket [use π = 3.14].

[CBSE 2011]

Ans.

Given: Rocket is in the form of right circular cylinder at the lower end and a cone at the top.



For upper conical part,

Radius of base, r = 3 cm

slant heigh, l = 5 cm

Let h be height of cone.

In right angled ΔAOC

$l^2 = h^2 + r^2$

$h = \sqrt{l^2 - r^2} = \sqrt{5^2 - 3^2} = \sqrt{25-9}$

$= \sqrt{16} = 4$ cm

⇒ Height of cone, h = 4 cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2 \times 4$

$= 12\ \pi\ cm^3$

Curved surface area of cone = πrl

$= \pi(3)\ (5) = 15\pi\ cm^2$

For cylindrical part

Given, diameter of cylinder = 6 cm

$\therefore$ Radius $= \frac{6}{2} = 3$ cm

Height of cylinder = 12 cm

$\therefore$ Volume of cylinder = $\pi r^2 h$

$= \pi \times (3)^2 \times 12 = 108\ \pi\ cm^3$

Curved surface area of cylinder = $2\pi rh$

$= 2 \times \pi \times 3 \times 12 = 72\pi\ cm^2$

Volume of rocket = Volume of cylinder + Volume of cone

$= 108\ \pi + 12\pi = 120\pi$

$= 120 \times 3.14$

$= 376.8\ cm^3$

Curved surface area of rocket

= Curved surface area of cylinder + Area of base + Curved surface area of cone

$= 72\pi + \pi \times (3)^2 + 15\pi = 72\pi + 9\pi + 15\pi$

$= 96\pi = 301.44\ cm^2$

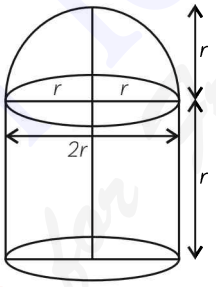
Hence, volume = 376.8 cm^3

Curved surface area = 301.44 cm^2

14. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building.

Ans.

Given: Building in the form of cylinder at the bottom and hemisphere at the top



Volume of building = $41\frac{19}{21}$ m^2

Total height of dome = 2r

Height of hemispherical part = Radius of hemispherical part = r

Height of cylindrical part = r

For cylinder,

Height = $2r - r$ = m

Radius = r

Volume = $\pi r^2 h$

$= \pi \times (r)^2 \times r = \pi r^3\ m^3$

For hemispherical dome

Volume = $\frac{2}{3}\pi r^3 m^3$

Total volume of building

= Volume of cylinder + volume of hemispherical dome

$= \pi r^3 + \frac{2}{3}\pi r^3 = \frac{5}{3}\pi r^3 m^3$

According to the question,

Volume of building = Volume of air

$\Rightarrow \frac{5}{3}\pi r^3 = 41\frac{19}{21}$

$$\Rightarrow \frac{5}{3}\pi r^3 = \frac{880}{21} \Rightarrow r^3 = \frac{880\times 3}{5\times \pi \times 21}$$

$$\Rightarrow \quad r^3 = \frac{880\times 3\times 7}{5\times 22\times 21} = 8$$

$$\Rightarrow \quad r = 2$$

Hence, the height of the building = $2r = 2 \times 2 = 4$ m.

15. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

Ans.

For hemispherical bowl,

Internal radius, $r = 9$ cm

$$\text{Volume} = \frac{2}{3}\pi r^3 = \frac{2}{3}\times \pi \times (9)^3$$

$$= 486\ \pi\ \text{cm}^3$$

For cylindrical shaped bottles,

Radius, $r = 1.5$ cm

Height, $h = 4$ cm

Volume of 1 bottle $= \pi r^2 h$

$$= \pi \times (1.5)^2 \times 4$$

$$1 \text{ bottle} = 9\pi\ \text{cm}^3$$

Let n be the number of bottles needed to empty the bowl.

$\therefore$ Volume of bowl = Volume of n bottles

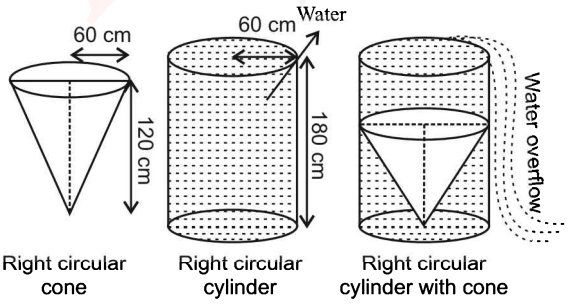
$$486\pi = (9\pi)n$$

$$\Rightarrow \quad n = \frac{486}{9} = 54$$

Hence, 54 bottles are needed to empty the bowl.

16. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

Ans.



When we place a solid right circular cone in a right circular cylinder full of water, then the volume of the right circular cone is equal to the volume of the water that flows out from the cylinder.

$\therefore$ Volume of water left in cylinder

= (Volume of cylinder) – (Volume of cone) ...(i)

For cylinder,

Base radius, $r = 60$ cm

Height, $h = 180$ cm

Volume $= \pi r^2 h = \pi \times (60)^2 \times 180$

$= 648000\ \pi\ \text{cm}^3$

For cone,

Base radius, $r = 60$ cm

Height, $h = 120$ cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\times \pi \times (60)^2 \times 120$$

$$= 144000\pi\ \text{cm}^3$$

Volume of water left in cylinder

$= (648000\pi - 144000\pi)\ \text{cm}^3$ [using eqn (i)]

$= 504000\pi\ \text{cm}^3$

$$= 504000 \times \frac{22}{7} = 1584000\ \text{cm}^3$$

$$= \frac{1584000}{(10)^6}\ \text{m}^3 = 1.584\ \text{m}^3$$

Hence, the required volume of the water left is $1.584\ \text{m}^3$.

17. Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cm/sec in an empty cylindrical tank, the radius of whose base is 40 cm. How much will the water level in the tank rise in half an hour?

[CBSE 2012]

Ans.

Given: For cylindrical pipe Inner radius, $r = 1$ cm

Speed of water = 80 cm / sec

i.e., In 1 sec, water flows 80 cm

In 30 min i.e. 30 × 60 seconds, water flows 80 × 30 × 60 = 144000 cm

Flowing water is filled in cylindrical tank. Hence, the volume of flowing water is equal to volume of water in cylindrical tank.

Volume of water that flows through cylindrical pipe in half an hour $= \pi r^2 h$

$= \pi \times (1)^2 \times 144000 = 144000\pi\ \text{cm}^3$

For cylindrical tank,

Base radius, $r = 40$ cm

Let height of water be raised by h cm

$\therefore$ Volume of cylindrical tank $= \pi(40)^2 \times h$

$= 1600\pi h$

According to the question,

Volume of water in cylindrical tank

= Volume of water flows through cylindrical pipe in half an hour

$\Rightarrow \quad 1600\pi h = 144000\pi$

$\Rightarrow h = \dfrac{144000\pi}{1600\pi} = 90$ cm

Hence, the level of water rises to 90 cm in half an hour.

18. The rain water from a roof of dimensions 22 m x 20 m drains into a cylindrical vessel having the base of diameter 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm. [CBSE 2010]

Ans.

Let the rainfall be 'a' cm = 0.01a m

[because 1 m = 100 cm]

Given: For roof

Length, $l = 22$ m

Breadth, $b = 20$ m

$\therefore$ Volume of water on roof $= l \times b \times h$

$= 22 \times 20 \times \dfrac{a}{100} = \dfrac{22a}{5}$ m^3

For cylindrical vessel,

Diameter of base = 2 m

$\therefore$ radius, $r = 1$ m

height, $h = 3.5$ m

$\therefore$ Volume of water in cylindrical vessel $= \pi r^2 h$.

$= \dfrac{22}{7} \times (1)^2 \times 3.5 = 11$ m^3

According to the question,

Volume of water on the roof

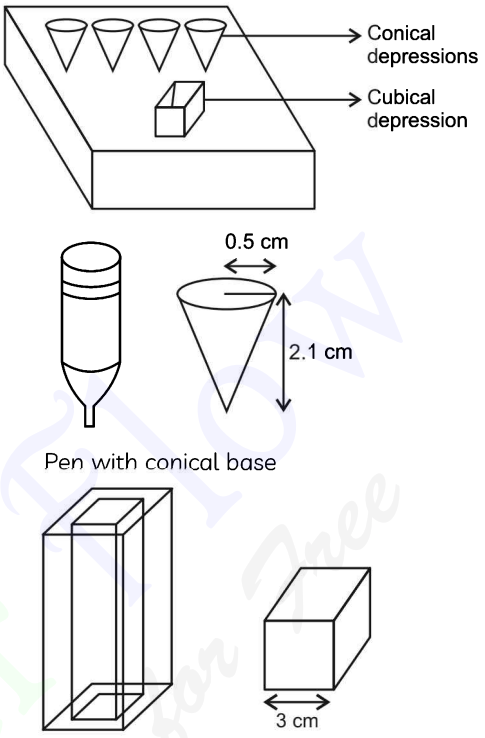
= Volume of water in cylindrical vesel

$\Rightarrow \quad \dfrac{22a}{5} = 11 \Rightarrow a = \dfrac{11 \times 5}{22} = 2.5$ cm

Hence, the rainfall is 2.5 cm.

19. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins, respectively. The dimensions of the cuboid are 10 cm, 5 cm and 4 cm. The radius of each of the conical depressions is 0.5 cm and the depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.

Ans.



For cuboidal stand,

Length, $l = 10$ cm

Breadth, $b = 5$ cm

Height, $h = 4$ cm

Volume of cuboidal pen stand = lbh

$= 10 \times 5 \times 4 = 200$ cm^3

For cuboidal depresssion,

Side, $a = 3$ cm

Volume of cube $= a^3$

$\therefore$ Volume of 1 cuboidal depression

$= (3)^3 = 27$ cm^3.

For conical depression,

Radius, $r = 0.5$ cm

Depth, $h = 2.1$ cm

$\therefore$ Volume of 1 conical depression $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times (0.5)^2 \times 2.1$

$= \dfrac{22}{7} \times 0.5 \times 0.5 \times 0.7$

$= 0.55$ cm^3

$\therefore$ Volume of 4 conical depressions

$= 4 \times 0.55 = 2.2$ cm^3

Volume of wood in entire pen stand

= Volume of cuboidal pen stand

– Volume of 4 conical depression

– Volume of 1 cubical depression

= 200 – 2.2 – 27

= 200 – 29.2 = 170.8 cm^3

Hence, the required volume of wood in the entire stand is 170.8 cm^3.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by:

(A) 200% (B) 500%

(C) 700% (D) 800%

Ans. (C)

Explanation: When the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

2. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is:

(A) 2 : 1 (B) 1 : 2

(C) 1 : 3 (D) 3 : 1

Ans. (A)

Explanation: Since, the readius of cone and cylinder are equal i.e., r.

Then, $\pi rl = 2\pi rh$

$$\frac{l}{h} = \frac{2}{1} = 2:1$$

3. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. Find the ratio of its height to its diameter.

Ans.

Curved surface area of cylinder = $2\pi rh$

= 264 m^2 ...(i)

Volume of cylinder = $\pi r^2 h$

= 924 m^3

According to question,

$$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$$

$$\frac{r}{2} = \frac{7}{2}$$

$r = 7$ m

Putting the value of r in equation (i), we have

$$2 \times \frac{22}{7} \times 7 \times h = 264$$

$h = 6$ m

$$\frac{h}{2r} = \frac{6}{14} = \frac{3}{7}$$

Hence, $h : d = 3 : 7$

4. If the area of three adjacent faces of a cuboid are X Y, and Z respectively, then find the volume of cuboid.

Ans.

Let the length, breadth and height of the cuboid is l, b, and h respectively.

$X = l \times b$

$Y = b \times h$

$Z = l \times h$

$XYZ = l^2 \times b^2 \times h^2$

Volume of cuboid = $l \times b \times h$

$l^2 b^2 h^2 = XYZ$

$lbh = \sqrt{XYZ}$.

5. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?

Ans.

$$\text{Radius of sphere} = \frac{\text{diameter}}{2} = \frac{6}{2} = 3 \text{ cm}$$

$$\text{Radius of cylinder vessel} = \frac{12}{2} = 6 \text{ cm}$$

Let the level of water rise in cylinder be h.

$$\text{Volume of sphere, } V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3 = 36\pi$$

Volume of sphere = Increase volume in cylinder

$36\pi = \pi \times 6 \times 6 \times h$

$h = 1$ cm

Thus level of water rise in vessel is 1 cm.

6. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 cm^2, find the volume of the cylinder. $\left(\pi = \frac{22}{7}\right)$

Ans.

Let `$r$' be the radius and `h' be the height of the cylinder.

$$r + h = 37 \quad \text{...(i)}$$

$$2\pi r(r + h) = 1628 \quad \text{...(ii)}$$

$$2\pi r \times 37 = 1628$$

$$2\pi r = \frac{1628}{37}$$

$$\Rightarrow \quad r = 7\text{cm}$$

putting the value of r in eqn (i), we have

$$h = 37 - 7 = 30 \text{ cm}$$

volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 7 \times 30 = 4620 \text{ cm}^3$$

So, the volume of the cylinder is 4620 cm^3.

7. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of sphere respectively, then find the (R – r) : r.

Ans.

Ratio of the volumes:

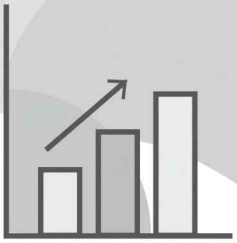
$$\frac{\text{Volume of 1st sphere}}{\text{Volume of 2nd sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$$

$$\frac{r}{R} = \frac{2}{3} \Rightarrow R = \frac{3}{2}r$$

$$(R - r) : r = \left(\frac{3}{2}r - r\right) : r$$

$$= \frac{r}{2} : r = 1 : 2.$$

13 Statistics and Probability



EXERCISE 13.1

Choose the correct option from the given four options:

1. While computing the mean of grouped data, we assume that the frequencies are:

(A) evenly distributed over all the classes.

(B) centred at the classmarks of the classes.

(C) centred at the upper limits of the classes.

(D) centred at the lower limits of the classes.

Ans. (B)

Explanation:

In grouping the data, all the observations between the lower and upper limits of the class intervals are taken as one group; then the mid-value of class mark is taken for further calculation. In computing the mean of a grouped data, the frequencies are centered at the class marks of the classes.

2. If x_i's are the mid-points of the class intervals of the grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $(f_ix_i - \bar{x})$ is equal to:

(A) 0 (B) –1

(C) 1 (D) 2

Ans. (A)

Explanation:

Given that, x_i's are the mid-points of the class intervals, and f_i's are the corresponding frequencies

the mean $\bar{x}$ for the grouped data is

$$\bar{x} = \frac{\Sigma f_i x_i}{n}$$

$$\Rightarrow \quad \bar{x}n = \Sigma f_i x_i \qquad \text{...(i)}$$

Also $\Sigma(f_ix_i - \bar{x}) = \Sigma f_ix_i - \Sigma\bar{x}$

$$\Rightarrow \quad \Sigma(f_ix_i - \bar{x}) = \bar{x}n - \Sigma\bar{x} \qquad \text{[Using eqn. (i)]}$$

$$= \bar{x}n - n\bar{x}$$

$$= 0$$

3. The abscissa of the point of intersection of the less than type cumulative frequency curves of a grouped data gives its:

(A) mean (B) median

(C) mode (D) all the three above

Ans. (B)

Explanation:

The abscissa of the intersection point of less than ogive and more than ogive gives the median.

4. For the following distribution:

Class	Frequency
0 - 5	10
5 - 10	15
10 - 15	12
15 - 20	20
20 - 25	9

the sum of the lower limits of the median class and modal class is:

(A) 8 (B) 10

(C) 11 (D) 12 **[CBSE 2016]**

Ans. (B)

Explanation:

Class	Frequency	Cumulative Frequency
0 - 5	10	10
5 - 10	15	25
10 - 15	12	37
15 - 20	20	57
20 - 25	9	66

Here, $\frac{N}{2} = \frac{66}{2} = 33$, which lies in the interval (10 - 15) as the cumulative frequency just greater than 33 is 37.

Therefore, the median class is 10 - 15 and its lower limit is 10.

The highest frequency is 20, which lies in the interval (15 - 20). Therefore, the modal class is 15 - 20 and its lower limit is 15.

Hence, the required sum is 10 + 15 = 25.

5. **Consider the following frequency distribution:**

Class	Frequency
0 - 5	13
6 - 11	10
12 - 17	15
18 - 23	8
24 - 29	11

The upper limit of the median class is:

(A) 17 (B) 17.5

(C) 18 (D) 18.5 **[CBSE 2015]**

Ans. (B)

Explanation:

Given that the Classes are not continous in the given frequency table, so we make it continous by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class.

Class	Frequency	Cumulative Frequency
0.5 - 5.5	13	13
5.5 - 11.5	10	23
11.5 - 17.5	15	38
17.5 - 23.5	8	46
23.5 - 29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$

Cumulative frequency just greater than 28.5 is 38, which lies in the interval 11.5 - 17.5.

So, the median class is (11.5 - 17.5) and so, the upper limit of the median class is 17.5.

6. **For the following distribution:**

Marks	Number of Students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

the modal class is:

(A) 10 - 20 (B) 20 - 30

(C) 30 - 40 (D) 50 - 60

[CBSE 2017]

Ans. (C)

Explanation:

Marks	Number of Students	Cumulative Frequency
Below 10	3	3
10 - 20	(12 – 3) = 9	12
20 - 30	(27 – 12) = 15	27
30 - 40	(57 – 27) = 30	57
40 - 50	(75 – 57) = 18	75
50 - 60	(80 – 75) = 5	80

Here, we see that the highest frequency is 30, which lies in the interval 30 - 40.

Hence, the modal class is (30 - 40).

7. **Consider the data:**

Class	Frequency
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	7
185 - 205	4

The difference of the upper limit of the median class and the lower limit of the modal class is:

(A) 0 (B) 19

(C) 20 (D) 38

Ans. (C)

Explanation:

Class	Frequency	Cumulative Frequency
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	7	63
185 - 205	4	67

Now, the class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2}$ is called the median class.

Here, $\frac{n}{2} = \frac{67}{2} = 33.5$, which lies in the interval 125 - 145. i.e. median class.

Hence, the upper limit of median class is 145.

Again, the class whose frequency is maximum is called the modal class.

Here, 20 is the highest frequency which lies in the interval 125 -145.

Hence, the lower limit of modal class is 125.

Thus, the required difference = Upper limit of median class – Lower limit of modal class

= 145 – 125 = 20.

8. The times, in seconds, taken by 150 atheletes to run a 110 m hurdle race tabulated below:

Class	Frequency
13.8 - 14	2
14 - 14.2	4
14.2 - 14.4	5
14.4 - 14.6	71
14.6 - 14.8	48
14.8 - 15	20

The number of atheletes who completed the race in less then 14.6 seconds is:

(A) 11 (B) 71

(C) 82 (D) 130

Ans. (C)

Explanation:

The number of athletes who completed the race in less than 14.6 seconds = 2 + 4 + 5 + 71 = 82.

9. Consider the following distribution:

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30 - 40 is:

(A) 3 (B) 4

(C) 48 (D) 51 **[CBSE 2015]**

Ans. (A)

Explanation:

Marks/Class	Number of Students	Frequency
0 - 10	(63 – 58) = 5	5
10 - 20	(58 – 55) = 3	3
20 - 30	(55 – 51) = 4	4
30 - 40	(51 – 48) = 3	3
40 - 50	(48 – 42) = 6	6
More than or equal to 50	(42 – 0) = 42	42

Hence, frequency of the class interval 30 - 40 is 3.

10. If an event cannot occur, then its probability is:

(A) 1 (B) $\frac{3}{4}$

(C) $\frac{1}{2}$ (D) 0

Ans. (D)

Explanation:

The event which cannot occur is called the impossible event with a probability equal to zero.

11. Which of the following cannot be the probability of an event?

(A) $\frac{1}{3}$ (B) 0.1

(C) 3% (D) $\frac{17}{16}$ **[CBSE 2011]**

Ans. (D)

Explanation:

Here, 0.1 = $\frac{1}{10}$, 3% = $\frac{3}{100}$

The probability of an event always lies from 0 to 1.

As $\frac{17}{16} > 1$

∴ Option (D) is correct.

12. An event is very unlikely to happen. Its probability is the closest to:

(A) 0.0001 (B) 0.001

(C) 0.01 (D) 0.1

Ans. (A)

Explanation:

The probability of an event which is very unliked to happen is the closet to zero and from the given options, 0.0001 is the closest to zero.

13. If the probability of an event is p, the probability of its complementary event will be:

(A) $p - 1$ (B) p

(C) $1 - p$ (D) $1 - \frac{1}{p}$

[CBSE 2020, 14]

Ans. (C)

Explanation:

We know that:

probability of an event + probability of its complementary event = 1

$\Rightarrow$ Probability of complementary event

= 1 – probability of an event

= 1 – *p*.

14. The probability expressed as a percentage of a particular occurrence can never be:

(A) less than 100

(B) less than 0

(C) greater than 1

(D) anything but a whole number

Ans. (B)

Explanation:

We know that:

Probability of an event E is $0 \le P(E) \le 1$.

Thus, when probability is expresed in terms of percentage, it always lies from 0 to 100.

Thus it cannot be less than 0.

15. If P(A) denotes the probability of an event A, then:

(A) $P(A) < 0$ **(B)** $P(A) > 1$

(C) $0 \le P(A) \le 1$ **(D)** $-1 \le P(A) \le 0$

[CBSE 2010]

Ans. (C)

Explanation:

If P(A) denotes the probability of an event A, then $0 \le P(A) \le 1$

i.e., probability can be any number that lies from 0 to 1.

16. A card is selected from a deck of 52 cards. The probability of its being a red face card is:

(A) $\frac{3}{26}$ **(B)** $\frac{3}{13}$

(C) $\frac{2}{13}$ **(D)** $\frac{1}{2}$

[CBSE 2012, 10]

Ans. (A)

Explanation:

In a deck of 52 cards:

Total no. of cards = 52

No. of face cards = 12

No. of black face cards = 6

(3 spades and 3 clubs)

No. of red face cards = 6

(3 hearts & 3 diamonds)

Probability of an event E is

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{No. of all possible outcomes of experiment}}$$

$$= \frac{6}{52} = \frac{3}{26}$$

17. The probability that a non leap year selected at random will contain 53 Sundays is:

(A) $\frac{1}{7}$ **(B)** $\frac{2}{7}$

(C) $\frac{3}{7}$ **(D)** $\frac{5}{7}$

[CBSE 2015, 12]

Ans. (A)

Explanation:

A non leap year has 365 days.

365 days = 52 weeks and 1 day.

This 1 day can be any day of the week i.e. Monday or Tuesday or Wednesday or Thursday or Friday or Saturday or Sunday,

Thus, out of the 7 possibilities,

1 favourable event is the event when it is Sunday.

$\therefore$ Required probability = $\frac{1}{7}$

18. When a die is thrown, the probability of getting dice an odd number less than 3 is:

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$

(C) $\frac{1}{2}$ **(D) 0**

Ans. (A)

Explanation:

When a dise is thrown:

Total no. of outcomes = 6 {1, 2, 3, 4, 5, 6}

Odd no. less than 3 = 1 {1}

$\therefore$ No. of favourable outcome = 1

$\therefore$ Required probability

$$= \frac{\text{No. of favourable outcome}}{\text{Total no. of outcomes}}$$

$$= \frac{1}{6}$$

19. A card is drawn from a deck of 52 cards. The event E is that the card is not an ace of hearts. The number of outcomes favourable to E is:

(A) 4 **(B) 13**

(C) 48 **(D) 51**

Ans. (D)

Explanation:

In a deck of 52 cards:

Total no. of outcomes = 52.

No. of heart cards = 13

No. of card with ace of heart = 1.

Hence, no. of favourable outcomes = 52 – 1 = 51.

20. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is:

(A) 7 (B) 14

(C) 21 (D) 58 **[CBSE 2012]**

Ans. (B)

Explanation:

It is given that,

Total no. of eggs = 400

Probability of getting bad egg = 0.035

$\Rightarrow$ Probability of getting bad egg = $\frac{\text{No. of bad eggs}}{\text{Total no. of eggs}} = 0.035$

$\Rightarrow \frac{\text{No. of bad eggs}}{400} = 0.035$

$\Rightarrow$ No. of bad eggs = 0.035 × 400 = 14.

Hence, no. of bad eggs in the lot is 14.

21. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

(A) 40 (B) 240

(C) 480 (D) 750

Ans. (C)

Explanation:

Total no. of tickets sold = 6000.

Probability of winning first prize, P(E) = 0.08.

$\Rightarrow$ P(E) = 0.08

$\Rightarrow \frac{\text{No. of tickets bought by girl}}{\text{Total no. of outcomes}} = 0.08$

$\Rightarrow$ No. of tickets bought by girl = 0.08 × 6000 = 480.

Hence, she bought 480 tickets.

22. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is:

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$

(C) $\frac{4}{5}$ (D) $\frac{1}{3}$

Ans. (A)

Explanation: Total no. of outcomes = 40

Multiples of 5 from 1 to 40 are {5, 10, 15, 20, 25, 30, 35, 40}

$\therefore$ No. of favourable outcomes = 8.

Required probability

$= \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

$= \frac{8}{40} = \frac{1}{5}$

23. Someone is asked to take a number from 1 to 100. The probability that it is a prime is:

(A) $\frac{1}{5}$ (B) $\frac{6}{25}$

(C) $\frac{1}{4}$ (D) $\frac{13}{50}$ **[CBSE 2014]**

Ans. (C)

Explanation: Total no. of outcomes = 100

Prime numbers from 1 to 100 are

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}

$\therefore$ No. of favourable outcomes = 25

Required probability

$= \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

$\therefore$ Required probability = $\frac{25}{100} = \frac{1}{4}$

24. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and the rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is:

(A) $\frac{4}{23}$ (B) $\frac{6}{23}$

(C) $\frac{8}{23}$ (D) $\frac{17}{23}$

Ans. (B)

Explanation: Total no. of outcomes = 23.

No. of students in house A, B & C = 4 + 8 + 5 = 17.

$\therefore$ Remaining students = 23 – 17 = 6.

Required probability

$= \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

Probability that the selected students is not from A, B and C = $\frac{6}{23}$

EXERCISE 13.2

1. The median of an ungrouped data and the median calculated when the same data is grouped are always the same. Do you think that this is a correct statement? Give reason.

Ans. Not always.

For calculating the median of a grouped data, the formula used is based on the assumption that the observations in the classes are uniformly distributed.

2. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

Ans. Not always.

The median, mean and mode can be the same.

They may be equal if the no. of observations are odd and equi spaced i.e., symmetrical distribution.

3. Will the median class and modal class of grouped data always be different? Justify your answer.

Ans. Not always.

Median gives the middle value of data whereas mode gives the maximum occuring frequency.

So it really depends on the data given. The median and modal class may be the same in cases where the distribution of data is perfectly symmetrical.

4. In a family with three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is $\frac{1}{4}$. Is this correct? Justify your answer. [CBSE 2014]

Ans. No (False).

In a family with 3 children, events are (*b*, *b*, *b*), (*g*, *b*, *b*), (*b*, *g*, *b*), (*b*, *b*, *g*), (*g*, *g*, *g*), (*g*, *g*, *b*), (*g*, *b*, *g*), (*g*, *b*, *b*)

$T(E) = 8$

Required probability

$$= \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$$

Total no. of outcomes = 8

Probability of having no girl = $\frac{1}{8}$

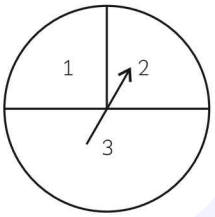
Probability of having 1 girl = $\frac{3}{8}$

Probability of having 2 girls = $\frac{1}{8}$

Probability of having 3 girls = $\frac{3}{8}$

Hence, the probability of each is not $\frac{1}{4}$.

5. A game consists of spinning an arrow which comes to rest pointing at one of the three regions (1, 2 or 3) (see figure). Are the outcomes 1,2 and 3 equally likely to occur? Give reasons.



Ans. No.

The area of region 3 is double than that of 1 or 2. Therefore, the number of outcome for region 3 or its probability is double than the probability of 1 or 2.

Hence, probability of 1 = probability of 2 ≠ probability of 3.

Thus, the outcomes of 1, 2 and 3 are not equally likely to occur.

6. Apoorv throws two dice at once and computes the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has a better chance of getting the number 36. Why?

Ans. Peehu has a better chance.

For Apoorva:

He throws two dice at once

∴ Total no. of outomes T(E) = 36

Number of outcomes for getting product 36 i.e., (6, 6)

Favourable outcome F(E) = 1

∴ P(E) or probability of getting the no. 36 for Apoorv: P(E) for Apoorv = $\frac{F(E)}{T(E)} = \frac{1}{36}$

For Peehu,

she throws one dice.

∴ total no. of outcomes T'(E) = 6.

Number of outcomes for getting square 36 i.e., (6, 6)

Favourable outcome F(E) = 1

∴ Probability of getting square of the no.

is $P(E) = \frac{F'(E)}{T'(E)} = \frac{1}{6}$

Since P'(E) > P(E)

Peehu has a better chance.

7. When we toss a coin, there are two possible outcomes – heads or tails. Therefore, the

probability of each outcome is $\frac{1}{2}$. Justify your answer. [CBSE 2014]

Ans. **Yes (Ture).**

Total no. of outcomes = 2

Probability of heads = Probability of tails = $\frac{1}{2}$

As heads and tails both are equally likely events.

8. A student says that if you throw a die, it will show up 1 or not 1, Therefore, the probability of getting 1 and the probability of getting 'not 1' each is equal to $\frac{1}{2}$. Is this correct? Give reasons.

Ans. **No (False).**

When we throw a dice:

Possible outcomes are S = {1, 2, 3, 4, 5, 6}

$\therefore$ Total no. of outcomes, T(E) = 6

Probability of getting (1) = $\frac{1}{6}$

Probability of not getting 1 = $\frac{5}{6}$

i.e., probability of getting 'not 1'

= 1 – probability of getting 1

= $1 - \frac{1}{6} = \frac{5}{6}$

9. I toss three coins together. The possible out-comes are no heads, 1 head, 2 heads and 3 heads. So, I say that the probability of no heads is $\frac{1}{4}$. What is wrong with this conclusion?

Ans. **No.**

When three coins are tossed together:

Possible outcomes S = {HHH, HTT, THT, TTH, HHT, HTH, THH, and TTT}

Total outcomes, (T) = $2^3 = 8$.

No. of favourable outcomes (getting no heads) F(E) = 1

$\therefore$ Probability of getting no heads = $\frac{F(E)}{T(E)} = \frac{1}{8}$

Outcome 'no heads' means {TTT}

$\therefore$ P(no heads) = $\frac{1}{8}$

Outcome 'one head' means {THT, HTT, TTH}

$\therefore$ P(one head) = $\frac{3}{8}$

Outcome 'two head' means {HHT, HTH, THH}

P(Two head) = $\frac{3}{8}$

Hence, the probability of getting no head is $\frac{1}{8}$ and not $\frac{1}{4}$.

10. If you toss a coin 6 times and it comes down on heads on each occasion. Can you say that the probability of getting a head is 1? Give reasons.

Ans. **No.**

A coin is tossed 6 times, so T(E) = 6

As the outcomes of heads and tails are equally likely outcomes, every time, we get the probability = $\frac{1}{2}$

If we toss a coin 6 times, then the probability will be the same in each case i.e., $\frac{1}{2}$ & not 1.

11. Sushma tosses a coin 3 times and gets tails each time. Do you think that the outcome of the next toss will be a tail? Give reasons.

Ans. **No.**

The coin is tossed 3 times and gets tails each time but it is not necessary that the fourth time will be tails.

The outcome of the next toss may or may not be tails, because on tossing a coin, we get either heads or tails, both are equally likely events.

12. If I toss a coin 3 times and get heads each time, should I expect a tail to have a higher chance in the 4th toss? Give reason in support of your answer.

Ans. **No.**

If we toss a coin, heads and tails, both are equally likely events i.e., the probability of each event is $\frac{1}{2}$. So, a higher chance of getting a tail in the 4^{th} toss cannot be expected.

13. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since this situation has only two possible outcomes, so. the probability of each is $\frac{1}{2}$. Justify.

Ans.

From 1 to 100, there are 50 even and 50 odd numbers.

Total number of outcomes T(E) = 100,

Number of favourable outcomes (even no.), F(E) = 50

So, P(E) i.e., probability of getting an even no.

= $\frac{50}{100} = \frac{1}{2}$

Similarly, probability of getting an odd no. = $\frac{1}{2}$

Hence, probability of getting an odd no

= probabiltiy of getting even no = $\frac{1}{2}$.

EXERCISE 13.3

1. Find the mean of the distribution:

Class	1 - 3	3 - 5	5 - 7	7 - 10
Frequency	9	22	27	17

[CBSE 2013]

Ans.

We first find the class marks (x_i) of each class

Class	**Class marks (x_i)**	**Frequency (f_i)**	**f_ix_i**
1 - 3	2	9	18
3 - 5	4	22	88
5 - 7	6	27	162
7 - 10	8.5	17	144.5
		$\Sigma f_i = 75$	$\Sigma f_ix_i = 412.5$

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{412.5}{75} = 5.5$

Hence, the mean of the given distribution is 35.

2. Calculate the mean of the scores of 20 students in a mathematics test:

Marks	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of Students	2	4	7	6	1

[CBSE 2020, 17, 13]

Ans.

We first find the class marks (x_i) of each class.

Class	**Class marks (x_i)**	**Frequency (f_i)**	**f_ix_i**
10 - 20	15	2	30
20 - 30	25	4	100
30 - 40	35	7	245
40 - 50	45	6	270
50 - 60	55	1	55
		$\Sigma f_i = 20$	$\Sigma f_ix_i = 700$

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{700}{20} = 35$

Hence, the mean of the scores of 20 students is 35.

3. Calculate the mean of the following data:

Class	4 - 7	8 - 11	12 - 15	16 - 19
Frequency	5	4	9	10

Ans.

Given frequency table is in inclusive form, so we need to convert the data into exclusive form or continous by subtracting 0.5 from the lower limit and adding 0.5 in the upper limit of each class.

Also, we find class marks (x_i) of each class and then proceed as follows:

Class	**Class marks (x_i)**	**Frequency (f_i)**	**f_ix_i**
3.5 - 7.5	5.5	5	27.5
7.5 - 11.5	9.5	4	38
11.5 - 15.5	13.5	9	121.5
15.5 - 19.5	17.5	10	175
		$\Sigma f_i = 28$	$\Sigma f_ix_i = 362$

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{362}{28} = 12.93$.

[Alternate Method]

Class	Class marks (x_i)	$d_i = (x_i - a)$	Frequency (f_i)	$f_i d_i$
3.5 - 7.5	5.5	–4	5	–20
7.5 - 11.5	$\boxed{9.5 = a}$	0	4	0
11.5 - 15.5	13.5	+4	9	36
15.5 - 19.5	17.5	+8	10	80
			$\Sigma f_i = 28$	$\Sigma f_i d_i = 96$

Here, a = assumed mean = 9.5,

d_i = deviation from mean

$\bar{x} = a + \frac{\Sigma f_i x_i}{\Sigma f_i} = 9.5 + \frac{96}{28}$

$= 9.5 + 3.43$

$\therefore \quad \bar{x} = 12.93$

Hence, the mean of the given data is 12.93.

4. The following table gives the number of pages written by Sarika for completing her own book for 30 days:

Number of pages written per day	**16 - 18**	**19 - 21**	**22 - 24**	**25 - 27**	**28 - 30**
Number of Days	**1**	**3**	**4**	**9**	**13**

[CBSE 2013]

Ans.

We need to convert the data into continuous classes by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class.

Class	Class marks (x_i)	Number of days (f_i)	$f_i x_i$
15.5 - 18.5	17	1	17
18.5 - 21.5	20	3	60
21.5 - 24.5	23	4	92
24.5 - 27.5	26	9	234
27.5 - 30.5	29	13	377
		$\Sigma f_i = 30$	$\Sigma f_i x_i = 780$

$\therefore$ Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{780}{30} = 26$.

[Alternate Method]

Class	Class marks (x_i)	$d_i = (x_i - a)$	Frequency i.e., No. of days (f_i)	$f_i d_i$
15.5 - 18.5	17	–6	1	$a - 6$
18.5 - 21.5	20	–3	3	–9
21.5 - 24.5	$\boxed{23 = a}$	0	4	0
24.5 - 27.5	26	3	9	27
27.5 - 30.5	29	6	13	78
			$\Sigma f_i = 30$	$\Sigma f_i d_i = 90$

a = assumed mean = 23

Mean, $\bar{x} = a + \frac{\Sigma f_i x_i}{\Sigma f_i} = 23 + \frac{90}{30} = 23 + 3$

$\therefore \quad \bar{x} = 26$

Hence, the mean the number of the pages written per day is 26.

5. The daily income of a sample of 50 employees are tabulated as follows :

Income (in ₹)	1 - 200	201 - 400	401 - 600	601 - 800
Number of employees	14	15	17	7

Find the mean daily income of the employees. **[CBSE 2014]**

Ans.

The given data is not continues, so we need to convert the data into continues by subtracting 0.5 from the lower limit and adding 0.5 in the upper limit of each class:

Class Income (₹)	**Class marks (x_i)**	**No. of employees (f_i)**	$u_i = \frac{x_i - a}{h} = \frac{x_i - 300.5}{200}$	f_iu_i
0.5 - 200.5	100.5	14	–1	–14
200.5 - 400.5	$\boxed{300.5 = a}$	15	0	0
400.5 - 600.5	500.5	14	+1	14
600.5 - 800.5	700.5	7	+2	14
		$\Sigma f_i = 50$		$\Sigma f_iu_i = 14$

Assumed mean, $a = 300.5$

Class width, $h = 200$

$N = \Sigma f_i = 50$

By step deviation method:

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 300.5 + \frac{14}{50} \times 200 = 300.5 + 56$$

Mean = ₹ 356.50

Thus, the mean daily income of the employees is ₹ 356.50.

[Alternate Method]

Class Income	**Class marks (x_i)**	$d_i = (x_i - a)$	f_i	f_id_i
0.5 - 200.5	100.5	–200	14	–2800
200.5 - 400.5	$\boxed{300.5 = a}$	0	15	0
400.5 - 600.5	500.5	+200	14	2800
600.5 - 800.5	700.5	+400	7	2800
			$\Sigma f_i = 50$	$\Sigma f_id_i = 2800$

a = assumed mean = 300.5

$d_i = x_i - a$.

Mean $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 300.5 + \frac{2800}{50}$

$= 300.5 + 56$

$\bar{x} = 356.5$

Thus, the mean daily income of the employees is ₹ 356.50.

6. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table:

Numbe of seats	100 - 104	104 - 108	108 - 112	112 - 116	116 - 120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied during the flights.

Ans.

Class Interval (No. of seats)	Class marks (x_i)	Deviation $d_i = (x_i - a)$	Frequency (f_i)	$f_i d_i$
100 - 104	102	–8	15	–120
104 - 108	106	–4	20	–80
108 - 112	$a = 110$	0	32	0
112 - 116	114	+4	18	72
116 - 120	118	+8	15	120
			$\Sigma f_i = 100$	$\Sigma f_i d_i = -8$

Here, assumed mean, $a = 110$

$\therefore$ Mean, $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 110 + \frac{(-8)}{100} = 110 - 0.08$

$= 109.92$

Thus, the mean number of seats occupied over the flights is 109.92 or 110 (approx).

7. The weights (in kg) of 50 wrestlers are recorded in the following table:

Weight (in kg)	100 - 110	110 - 120	120 - 130	130 - 140	140 - 150
Number of wrestlesr	4	14	21	8	3

Find the mean weight of the wrestlers. **[CBSE 2013]**

Ans.

Class Interval (Weight in kg)	Class marks (x_i)	Deviation $d_i = (x_i - a)$	Frequency no. of wrestlers (f_i)	$f_i d_i$
100 - 110	105	–20	4	–80
110 - 120	115	–10	14	–140
120 - 130	$a = 125$	0	21	0
130 - 140	135	10	8	80
140 - 150	145	20	3	60
			$\Sigma f_i = 50$	$\Sigma f_i d_i = -80$

Assumed mean, $a = 125$

$\therefore$ Mean $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 125 - \frac{80}{50}$

$= 125 - 1.6$

$= 123.4$

Hence, the mean weight of the wrestler is 123.4 kg.

8. The mileage (km per litre) of 50 cars of the same model was tested by a manufacturer and the details are tabulated as given below:

Mileage	10 - 12	12 - 14	14 - 16	16 - 18
Number of cars	7	12	18	13

Find the mean mileage.

The manufacturer claimed that the mileage of the model was 16 km/litre. Do you agree with this claim ?

Ans.

Class Interval (Mileage km/L)	Class marks (x_i)	Frequency no. of cars (f_i)	$f_i x_i$
10 - 12	11	7	77
12 - 14	13	12	156

Class Interval (Mileage km/L)	Class marks (x_i)	Frequency no. of cars (f_i)	f_ix_i
14 - 16	15	18	270
16 - 18	17	13	221
		$\Sigma f_i = 50$	$\Sigma f_ix_i = 724$

Here, $\Sigma f_i = 50$

$\Sigma f_ix_i = 724$

$\therefore$ Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{724}{50} = 14.48$ km/L

Hence, the mean mileage of the cars is 14.48 km/L.

No, the manufacturer's statement is wrong that the mileage is 16 km/L i.e., 1.52 km/L more than the average mileage.

9. **The following is the distribution of weights (in kg) of 40 people:**

Weight (in kg)	Number of people
40 - 45	4
45 - 50	4
50 - 55	13
55 - 60	5
60 - 65	6
65 - 70	5
70 - 75	2
75 - 80	1

Construct a cumulative frequency distribution (of the less than type) table for the data above. **[CBSE 2015]**

Ans.

Cumulative frequency distribution (less than type) table is:

Weight (in kg)	Number of people (Cumulative frequency)
Less than 45	4
Less than 50	4 + 4 = 8
Less than 55	8 + 13 = 21
Less than 60	21 + 5 = 26
Less than 65	26 + 6 = 32
Less than 70	32 + 5 = 37
Less than 75	37 + 2 = 39
Less than 80	39 + 1 = 40

10. **The following table shows the cumulative frequency distribution of the marks of 800 students in an examination:** **[CBSE 2017]**

Marks	Number of studetns
Below 10	10
Below 20	50
Below 30	130
Below 40	270
Below 50	440
Below 60	570
Below 70	670
Below 80	740
Below 90	780
Below 100	800

Construct a frequency distribution table for the data above.

Ans.

The table of frequency distribution for the given data is

Class Intervals Marks	Frequency Number of Students
0 - 10	10
10 - 20	50 – 10 = 40
20 - 30	130 – 50 = 80
30 - 40	270 – 130 = 140
40 - 50	440 – 270 = 170
50 - 60	570 – 440 = 130
60 - 70	670 – 570 = 100
70 - 80	740 – 670 = 70
80 - 90	780 – 740 = 40
90 - 100	800 – 780 = 20

11. **Form a frequency distribution table from the following data:** **[CBSE 2015]**

Class Interval Marks	Frequency Number of Candidates
More than or equal to 80	4
More than or equal to 70	6
More than or equal to 60	11
More than or equal to 50	17
More than or equal to 40	23
More than or equal to 30	27
More than or equal to 20	30
More than or equal to 10	32
More than or equal to 0	34

Ans.

Class Interval Marks	Frequency Number of Candidates
0 - 10	34 – 32 = 2
10 - 20	32 – 30 = 2
20 - 30	30 – 27 = 3
30 - 40	27 – 23 = 4
40 - 50	23 – 17 = 6
50 - 60	17 – 11 = 6
60 - 70	11 – 6 = 5
70 - 80	6 – 4 = 2
80 - 90	4

12. Find the unknown entries a, b, c, d, e, f in the following distribution of the heights of the students in a class:

Height (in cm)	Frequency	Cumulative Frequency
150 - 155	12	a
155 - 160	b	25
160 - 165	10	c
165 - 170	d	43
170 - 175	e	48
175 - 180	2	f
Total	50	

Ans.

Height (in cm)	Frequency (f_i)	Cumulative Frequency (given)	Cumulative frequency (Calculated)
150 - 155	12	a	12
155 - 160	b	25	$12 + b$
160 - 165	10	c	$22 + b$
165 - 170	d	43	$22 + b + d$
170 - 175	e	48	$22 + b + d + e$
175 - 180	2	f	$24 + b + d + e$
Total	**50**		

Comparing the last two columns,

i.e., cumulative frequency calculated and cumulative frequency given, we get:

$$a = 12 \Rightarrow a = 12$$

$$12 + b = 25$$

$$\Rightarrow b = 25 - 12 = 13$$

$$22 + b = c$$

$$\Rightarrow c = 22 + 13 = 35$$

$$22 + b + d = 43$$

$$\Rightarrow 22 + 13 + d = 43$$

$$\Rightarrow d = 43 - 35$$

$$d = 8$$

$$22 + b + d + e = 48$$

$$\Rightarrow 43 + e = 48$$

$$e = 5$$

$$24 + b + d + e = f$$

$$\Rightarrow 24 + 13 + 8 + 5 = f$$

$$\Rightarrow f = 50$$

Hence, a = 12, b = 13, c = 35, d = 8, e = 5, f = 50.

13. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	Number of patients
10 - 20	60
20 - 30	42
30 - 40	55
40 - 50	70
50 - 60	53
60 - 70	20

Form:

(i) Less than type cumulative frequency distribution,

(ii) More than type cumulative frequency distribution. **[CBSE 2018, 13]**

Ans.

(i) For less than type cumulative frequecy, it is clear that patients with age less than 30 years will include from 10 - 20 & 20 - 30 i.e., 60 + 42 = 102 and so on.

(ii) For more than type cumulative frequency curve, we observe that all 300 patients who take medical treatment have their age more than or equal to 10.

Since, there are 60 patients who take medical treatment in the interval 10 - 20, there are 300 – 60 = 240 patients who take medical treatment whose age is more than or equal to 20.

(i) Less than type

Age (in years)	No. of patients
Less than 10	0
Less than 20	60 + 0 = 60
Less than 30	42 + 60 = 102
Less than 40	55 + 102 = 157
Less than 50	70 + 157 = 227
Less than 60	53 + 227 = 280
Less than 70	20 + 280 = 300

(ii) More than type

Age (in years)	No. of patients
More than or equal to 10	300
More than or equal to 20	300 – 60 = 240
More than or equal to 30	240 – 42 = 198
More than or equal to 40	198 – 55 = 143
More than or equal to 50	143 – 70 = 73
More than or equal to 60	73 – 53 = 20
More than or equal to 70	20 – 20 = 0

14. Given below is a cumulative frequency distribution table showing the marks secured by 50 students of a class:

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the frequency distribution table for the data. [CBSE 2017]

Ans.

Here, we observe that 17 students have scored marks below 20.

⇒ Class interval 0 - 20

frequency = 17.

22 Students have scored marks below 40 i.e., 22 students included 0 - 20 & 20 - 40 both class intervals.

⇒ Class interval 20 - 40

frequency = 22 – 17 = 5

continuing the same manner, we get the complete frequency distribution table.

Class Interval (Marks)	Frequency (No. of Students)
0 - 20	17 – 0 = 17
20 - 40	22 – 17 = 5
40 - 60	29 – 22 = 7
60 - 80	37 – 29 = 8
80 - 100	50 – 37 = 13

15. Weekly income (in ₹) of 600 families is tabulated below:

Weekly income (in ₹)	Number of families
0 - 1000	250
1000 - 2000	190
2000 - 3000	100
3000 - 4000	40
400 - 5000	15
5000 - 6000	5
Total	**600**

Compute the median income. [CBSE 2015, 13]

Ans.

For calculating the median grouped data, we first form its cumulative frequency table.

Weekly Income (in ₹)	Number of families (f_i)	Cumulative Frequency c.f.
0 - 1000	250	250
1000 - 2000	190	440
2000 - 3000	100	540
3000 - 4000	40	580
4000 - 5000	15	595
5000 - 6000	5	600

Number of observations, N = 600 (even).

Here, 'N' is even.

∴ Median will be the average of $\left(\frac{N}{2}\right)^{th}$ and $\left(\frac{N}{2}+1\right)^{th}$ observations.

Median of 600 observations = mean of (300^{th} and 301^{th} observation)

i.e., median class is (1000 - 2000)

Here, lower limit, l = 1000

frequency, f = 190

Cumulative frequency of preceding median class cf = 250

Class size, $h = 1000$.

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right) \times h}{f}$$

$$= 1000 + \frac{(300 - 250)}{190} \times 1000$$

$$= 1000 + \frac{50}{190} \times 1000 = 1000 + \frac{5000}{19}$$

$= 1000 + 263.15 = 1263.15$

Hence, the median income of family is ₹ 1263.15 per week.

16. The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows:

Speed (km/h)	Number of players
85 - 100	11
100 - 115	9
115 - 130	8
130 - 145	5

Calculate the median bowling speed.

Ans.

To calcualte the median, we first construct the cumulative frequency table.

Class Interval (Speed in km/h)	Frequency No. of players (f_i)	Cumulative Frequency
85 - 100	11	11
100 - 115	9	20
115 - 130	8	28
130 - 145	5	33

N = Number of observations = 33 (odd).

Median of 33 observations = 17^{th} observation which is in the class 100 – 115.

Median class = 100 – 115

lower limit, $l = 100$

frequency, $f = 9$

Cumulative frequency, $c.f. = 11$.

Class width, $h = 15$

$\therefore$ Median

$$= l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h = 100 + \left(\frac{16.5 - 11}{9}\right) \times 15$$

$$= 100 + \frac{5.5 \times 15}{9} = 100 + \frac{82.5}{9} = 100 + 9.167$$

$= 109.17$ km/hr.

17. The monthly income of 100 families are given as below:

Income (in ₹)	Number of families
0 - 5000	8
5000 - 1000	26
10000 - 15000	41
15000 - 20000	16
2000 - 25000	3
25000 - 3000	3
30000 - 35000	2
35000 - 4000	1

Calculate the modal income. [CBSE 2016, 12]

Ans.

For modal income, we have to calculate mode.

The mode of grouped data

$$= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

Highest frequency is 41, which lies in class (10000 – 15000)

i.e., Modal class = 10000 – 15000.

Lower limit of modal class, $l = 10000$

Frequency preceding modal class, $f_0 = 26$

Frequency of modal class, $f_1 = 41$

Frequency of class succeeding modal class, $f_2 = 16$

Class width, $h = 5000$.

$$\therefore \text{ Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 10000 + \left[\frac{41 - 26}{82 - 26 - 16}\right] \times 5000$$

$$= 10000 + \left[\frac{15}{40}\right] \times 5000$$

$$= 10000 + 15 \times 125$$

$$= 10000 + 1875$$

$$= 11875$$

Hence, the modal income is ₹ 11,875 per month.

18. The weight of coffee in 70 packets is shown in the following table:

Weight (in g)	Number of packets
200 - 201	12
201 - 202	26
202 - 203	20
203 - 204	9
204 - 205	2
205 - 206	1

Determine the modal weight.

Ans.

In the given data, the highest frequency is 26, which lies in the interval 201 - 202.

$\therefore$ Modal class = 201 - 202. Class width, $h = 1$

Lower limit of modal class, $l = 201$

Frequency preceding model class, $f_0 = 12$

Frequency of modal class, $f_1 = 26$

Frequency of class succeeding Modal class,

$f_2 = 20$

$$\therefore \text{ Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 201 + \left[\frac{26 - 12}{52 - 12 - 20}\right] \times 1$$

$$= 201 + \left[\frac{14}{20}\right] \times 1 = 201 + 0.7$$

$$= 201.7$$

Hence, the modal weight is 201.7 g.

19. Two dice are thrown at the same time. Find the probability of getting:

(i) the same number on both dice.

(ii) different numbers on both dice.

Ans.

Two dice are thrown at the same time.

(i) Let E be the event of getting the same number on both dice.

$\therefore$ Total no. of all possible outcomes, T(E) = 36

For getting the same no. on both dice,

favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$\therefore$ No. of favourable outcomes, F(E) = 6

$\Rightarrow$ Required probability, P(E) = $\frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$

(ii) Probability of getting different numbers on both dice = 1 – Probability of getting same number.

P'(E) = 1 – Probability of getting same number.

$P'(E) = 1 - P(E) = 1 - \frac{1}{6} = \frac{5}{6}$

Alternate Method:

No. of outcomes for getting a different number

= 36 – No. of outcomes for getting same number

= 36 – 6 = 30

$\therefore$ Required probability = $\frac{30}{36} = \frac{5}{6}$

20. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is:

(i) 7? **(ii) a prime number?**

(iii) 1? **[CBSE 2020, 17]**

Ans.

Two dice are thrown simultaneously.

$\therefore$ Total number of outcomes, T(E) = 36

(i) For getting sum as 7,

favourable outcomes are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$\therefore$ No. of favourable outcomes, F(E) = 6

Required probability, P(E) = $\frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$

(ii) For getting sum as prime number i.e., 2, 3, 5, 7, 11.

favourable outcomes are (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5).

$\therefore$ No of favourable outcomes = 15

$\therefore$ Required probability, P(E) = $\frac{F(E)}{T(E)} = \frac{15}{36} = \frac{5}{12}$

(iii) For getting sum as 1.

favourable outcomes: None.

$\therefore$ F(E) = 0

$\therefore$ P(E) = 0 as it is not possible.

21. Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is:

(i) 6 **(ii) 12** **(iii) 7** **[CBSE 2015]**

Ans.

Total number of outcomes, T(E) = 36.

(i) For getting the product = 6

Favourable outcomes are (1, 6), (6, 1), (2, 3), (3, 2).

No. of favourable outcomes, F(E) = 4

$\therefore$ Required Probability, P(E) = $\frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$.

(ii) For getting the product = 12

Favourable outcomes are (2, 6), (6, 2), (3, 4), (4, 3)

No. of favourable outcomes, F(E) = 4

$\therefore$ Required probability, P(E) = $\frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$.

(iii) For getting the product = 7

Favourable outcomes = none

$\therefore$ F(E) = 0

$\Rightarrow$ P(E) = 0.

22. Two dice are thrown at the same time and the product of the numbers appearing on them is noted. Find the probability that the product is less than 9.

Ans.

Two dice are thrown at the same time.

$\therefore$ Total no. of outcomes, T(E) = 36.

For getting the product of on them less than 9.

Favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (6, 1).

No. of favourable outcomes, F(E) = 16

$\therefore$ Required Probability $P(E) = \frac{F(E)}{T(E)} = \frac{16}{36} = \frac{4}{9}$

Hence, required probability is $\frac{4}{9}$.

23. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Ans.

First dice {1, 2, 3, 4, 5, 6}

Second dice {1, 1, 2, 2, 3, 3}

Total number of outcomes, T(E) = 36.

(i) Let E, be the event of getting sum 2.

Favourable outcomes are (1, 1), (1, 1)

No. of favourable outcomes, $F(E_1) = 2$

$\therefore$ Required probability $P(E_1)$

$$= \frac{F(E_1)}{T(E)} = \frac{2}{36} = \frac{1}{18}$$

Hence, $P(E_1) = \frac{1}{18}$.

(ii) Let E_2 be the event of getting sum 3.

Favourable outcomes are (1, 2), (1, 2), (2, 1), (2, 1)

$\therefore$ No. of favourable outcomes, $F(E_2) = 4$

$\therefore$ Required probability $P(E_2)$

$$= \frac{F(E_2)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

Hence, $P(E_2) = \frac{1}{9}$

(iii) Let E_3 be the event of getting sum 4.

Favourable outcomes are (1, 3), (1, 3), (2, 2), (2, 2), (3, 1), (3, 1)

$\therefore$ No. of favourable outcomes $F(E_3) = 6$

$\therefore$ Required probability $P(E_3)$

$$= \frac{F(E_3)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

Hence, $P(E_3) = \frac{1}{6}$.

(iv) Let E_4 be the event of getting sum 5.

Favourable outcomes are (2, 3) (2, 3) (4, 1) (4, 1) (3, 2) (3, 2)

Number of favourable outcomes$F(E_4) = 6$.

Required probability $P(E_4) = \frac{F(E_4)}{T(E)} = \frac{6}{36} = \frac{1}{6}$.

Hence, $P(E4) = \frac{1}{6}$

(v) Let E_5 be the event of getting sum 6.

Favourable outcomes are (3, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1)

No. of favourable outcomes $F(E_5) = 6$

Required Probability $P(E_5) = \frac{F(E_5)}{T(E)} = \frac{6}{36} = \frac{1}{6}$.

Hence, $P(E5) = \frac{1}{6}$

(vi) Let E_6 be the event of getting sum 7.

Favourable outcomes are (4, 3), (4, 3), (5, 2), (5, 2), (6, 1), (6, 1)

No. of favourable outcomes $F(E_6) = 6$

Required probability $P(E_6) = \frac{F(E_6)}{T(E)} = \frac{6}{36} = \frac{1}{6}$

Hence, $P(E_6) = \frac{1}{6}$.

(vii) Let E_7 be the event of getting sum 8.

Favourable outcomes are (5, 3), (5, 3), (5, 3), (6, 2), (6, 2)

No. of favourable outcomes $F(E_7) = 4$

Required probability $P(E_7) = \frac{F(E_7)}{T(E)} = \frac{4}{36} = \frac{1}{9}$

Hence, $P(E_7) = \frac{1}{9}$.

(viii) Let E_8 be the event of getting sum 9.

Favourable outcomes are (6, 3), (6, 3)

No. of favourable outcomes, $F(E_8) = 2$

$\therefore$ Required Probability $P(E_8)$

$$= \frac{F(E_8)}{T(E)} = \frac{2}{36} = \frac{1}{18}$$

Hence, $P(E8) = \frac{1}{18}$,

Hence, $P(E_1) = \frac{1}{18}$, $P(E_2) = \frac{1}{9}$, $P(E_3) = \frac{1}{6}$,

$P(E_4) = \frac{1}{6}$, $P(E_5) = \frac{1}{6}$, $P(E_6) = \frac{1}{6}$,

$P(E_7) = \frac{1}{9}$, $P(E_8) = \frac{1}{18}$,

24. **A coin is tossed two times. Find the probability of getting at most one head.**
[CBSE 2011]

Ans.

A coin is tossed two times.

Possible outcomes are {(H, H), (H, T), (T, H), (T, T)}

Total no. of outcomes, T(E) = 4

Favourable outcomes of getting at most 1 head

{(T, T), (H, T), (T, H)}

No. of favourable outcomes, F(E) = 3

$\therefore$ Required probability $P(E) = \frac{F(E)}{T(E)} = \frac{3}{4}$

Hence, $P(E) = \frac{3}{4}$

25. **A coin is tossed 3 times. List the possible outcomes. Find the probability of getting:**
(i) all heads (ii) at least 2 heads
[CBSE 2016]

Ans.

A coin is tossed 3 times.

Possible outcomes are {(H, H, H), (H, T, T), (T, H, T), (T, T, H), (H, H, T), (H, T, H), (T, H, H), (T, T, T)}

$\therefore$ Total No. of outcomes, T(E) = 8.

(i) Let E_1, be the event of getting all heads.
Favourable outcomes {(H, H, H)}

No. of favourable outcomes $F(E_1) = 1$

$\therefore$ Required probability $P(E_1) = \frac{F(E_1)}{T(E)} = \frac{1}{8}$

(ii) Let E_2 be the event of getting at least two heads.

Favourable outcomes {(HHT), (HTH), (THH), (HHH)}

No. of favourable outcomes, $F(E_2) = 4$

$\therefore$ Required probability, $P(E_2) = \frac{F(E_2)}{T(E)} = \frac{4}{8} = \frac{1}{2}$

26. **Two dice are thrown at the same time. Determine the probabiity that the difference of the numbers on the two dice is 2.**

Ans.

Two dice are thrown at the same time.

Total no. of outcomes, T(E) = 36.

Let E be the event of getting the numbers whose difference is 2.

Favourable outcomes are

{(1, 3), (2, 4), (3, 5), (4, 6), (3, 1), (4, 2), (5, 3), (6, 4)}

No. of favourable outcomes, F(E) = 8

$\therefore$ Required probability, $P(E) = \frac{F(E)}{T(E)} = \frac{8}{36} = \frac{2}{9}$

Hence, $P(E) = \frac{2}{9}$.

27. **A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random. Find the probability of this ball being a:**
(i) red ball (ii) green ball
(iii) not a blue ball [CBSE 2014, 12]

Ans.

Number of red balls = 10

Number of blue balls = 5

Number of green balls = 7.

$\therefore$ Total no. of balls, T(E) = (10 + 5 + 7) = 22.

(i) Let E1 be the event of getting a red ball.

$\therefore$ No of favourable outcomes , F(E1) = 10.

$\therefore$ Required probablity, $P(E_1)$

$= \frac{F(E_1)}{T(E)} = \frac{10}{22} = \frac{5}{11}$

Hence, $P(E_1) = \frac{5}{11}$.

(ii) Let E_2 be the event of getting a green ball.

No. of favourable outcomes, $F(E_2) = 7$

$\therefore$ Required probability, $P(E_2) = \frac{F(E_2)}{T(E)} = \frac{7}{22}$

Hence, $P(E_2) = \frac{7}{22}$.

(iii) Let E_3 be the event of getting a red ball or green ball (i.e., not a blue ball).

No. of favourable outcomes, $F(E_3) = 17$

$\therefore$ Required probability, $P(E_3) = \frac{F(E_3)}{T(E)} = \frac{17}{22}$

Hence, $P(E_3) = \frac{17}{22}$.

$\therefore$ $P(E_1) = \frac{5}{11}$, $P(E_2) = \frac{7}{22}$ and $P(E_3) = \frac{17}{22}$.

28. **The king, queen and jack of clubs are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is:**
(i) a heart (ii) a king [CBSE 2016]

Ans.

If we remove one king, one queen and one jack of clubs from 52 cards, then the no. of remaining cards left = 49.

$\therefore$ Total no. of outcomes T(E) = 49.

(i) Let E_1 be the event of getting a heart.

No. of favourable outcomes, $F(E_1) = 13$

$\therefore$ Required probability, $P(E_1) = \frac{F(E_1)}{T(E)} = \frac{13}{49}$.

(ii) Let E2 be the event of getting a king

number of favourable outcomes $F(E_2) = 3$

[Since out of 4 kings, one club card is already removed].

∴ Required probability, $P(E_2) = \frac{F(E_2)}{T(E)} = \frac{3}{49}$.

29. Refer to Q.28. What is the probability that the card is:

(i) a club **(ii) 10 of hearts**

Ans.

Total no. of outcomes, T(E) = 49.

(i) Let E_3 be the event of getting a club.

No. of favourable outcomes, $F(E_3) = 13 - 3 = 10$

[∵ 3 club cards were already removed].

∴ Required probability, $P(E_3) = \frac{F(E_3)}{T(E)} = \frac{10}{49}$.

(ii) Let E_4 be the event of getting 10 of hearts.

No. of favourable outcomes, $F(E_4) = 1$

∴ Required Probability, $P(E_4) = \frac{F(E_4)}{T(E)} = \frac{1}{49}$

Hence, $P(E_3) = \frac{10}{49}$ & $P(E_4) = \frac{1}{49}$.

30. All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and keeping similar value for other cards, find the probability that the card has a value

(i) 7 **(ii) greater than 7**

(iii) less than 7 **[CBSE 2015]**

Ans.

Out of 52 playing cards, 4 jacks, 4 queens and 4 kings are removed.

∴ No. of remaining cards = 52 – (3 × 4)

= 52 – 12 = 40.

∴ Total no. of outcomes, T(E) = 40.

(i) Let E_1 be the event of getting a card whose value is 7.

Card value 7 may be of spade, heart, diamond, club

∴ No. of favourable outcomes, $F(E_1) = 4\ (1 \times 4)$

∴ Required probability, $P(E_1)$

$$= \frac{F(E_1)}{T(E)} = \frac{4}{40} = \frac{1}{10}.$$

(ii) Let E_2 be the event of getting a card whose value is greater than 7 i.e., (8, 9 or 10)

∴ No. of favourable outcomes, $F(E_2) = 12\ (3 \times 4)$

∴ Required probability, $P(E_2)$

$$= \frac{F(E_2)}{T(E)} = \frac{12}{40} = \frac{3}{10}.$$

(iii) Let E_3 be the event of getting a card whose value is less than 7 i.e., card whose value is {1, 2, 3, 4, 5 or 6}

∴ No. of favourable outcomes, $F(E_3)$

= 24 (6 × 4)

∴ Required probability, $P(E_3)$

$$= \frac{F(E_3)}{T(E)} = \frac{24}{40} = \frac{3}{5}.$$

Hence,

$P(E_1) = \frac{1}{10}$ $P(E_2) = \frac{3}{10}$ $P(E_3) = \frac{3}{5}$

31. An integer is chosen between 0 and 100. What is the probability that it is

(i) divisible by 7?

(ii) not divisible by 7? **[CBSE 2018]**

Ans.

Number of integers between 0 and 100 = 99.

∴ Total no. of outcomes, T(E) = 99.

(i) Let E_1 be the event of choosing an integer which is divisible by 7 i.e., an integer which is a multiple of 7.

Favourable outcomes are {7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98}

No. of favourable outcomes, $F(E_1) = 14$.

∴ Required probability, $P(E_1) = \frac{F(E_1)}{T(E)} = \frac{14}{99}$

(ii) Let E_2 be the event of choosing an integer which is not divisible by 7.

No. of favourable outcomes, $F(E_2)$

= 99 – 14 = 85

∴ Required probability, $P(E_2) = \frac{F(E_2)}{T(E)}$

$$= \frac{85}{99}$$

Hence, $P(E_1) = \frac{14}{99}$ and $P(E_2) = \frac{85}{99}$.

32. Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has:

(i) an even number

(ii) a square number

Ans.

Total no. of cards from 2 to 101

= (101 – 1) = 100

∴ Total no. of outcomes, T(E) = 100.

(i) Let E_1 be the event of selecting a card with even number i.e, 2, 4, ..., 100.
Out of 100 cards,

No. of even cards = 50

$\therefore$ No. of favourable outcomes, $F(E_1) = 50$

$\therefore$ Required probability, $P(E_1) = \frac{F(E_1)}{T(E)}$

$= \frac{50}{100} = \frac{1}{2}$.

(ii) Let E_2 be the event of selecting a card which is a square number.

Favourable outcomes {4, 9, 16, 25, 36, 49, 64, 81, 100}

$\therefore$ No of favourable outcomes, $F(E_2) = 9$

$\therefore$ Required Probability, $P(E_2) = \frac{F(E_2)}{T(E)}$

$= \frac{9}{100}$

Hence, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{9}{100}$.

33. A letter of English alphabets is chosen at random. Determine the probability that the letter is a consonant. [CBSE 2015]

Ans.

In English alphabets, there are 5 vowels (*a, e, i, o, u*) and 21 consonants {*b, c, d, f, g, h, j, k, l, m, n, p, q, r s, t, v, w, x, y, z*}

Total no. of outcomes , T(E) = 26

Favourable outcomes

i.e., consonant, F(E) = 21

$\therefore$ Required probability, $P(E) = \frac{F(E)}{T(E)} = \frac{21}{26}$

Hence, $P(E) = \frac{21}{26}$

34. There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and the rest do not contain any cash prize. If they are well shuffled and an envelope is picked at random, what is the probability that it contains no cash prize?

Ans.

Total no. of sealed envelopes, T(E) = 1000.

Number of envelopes with cash prize

= 10 + 100 + 200 = 310

$\therefore$ Number of favourable outcomes F(E).

= No. of envelopes with no cash prize.

= 1000 – 310 = 690

$\therefore$ Required probability,

$P(E) = \frac{F(E)}{T(E)} = \frac{690}{1000} = \frac{69}{100}$

Hence, $P(E) = \frac{69}{100}$.

35. Box A contains 25 slips of which 19 are marked ₹ 1 and other are marked ₹ 5 each. Box B contains 50 slips, of which 45 are marked ₹ 1 each and others are marked ₹ 13 each. Slips of both boxes are poured into a third box and resuffled. A slip is drawn at random. What is the probability that it is marked other than ₹ 1?

Ans.

Total no. of slips poured in 3rd box = 25 + 50 = 75

$\therefore$ Total no. of outcomes T(E) = 75

Box A

No. of slips of Rupees 1 = 19

No. of slips of Rupees 5 = 6

Box B

No. of slips of Rupees 1 = 45

No. of slips of Rupees 13 = 5

No. of slips of Rupees 5 and Rupees 13 = 6 + 5 = 11 slips

No. of favourable outcomes F(E) = No. of slips marks other than ₹ 1 = 11

$\therefore$ Required probability, $P(E) = \frac{F(E)}{T(E)} = \frac{11}{75}$

Hence, $P(E) = \frac{11}{75}$.

36. A carton of 24 bulbs contain 6 defective bulbs. One bulb is drawn at random. What is the probability that the bulb is not defective? If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest, what is the probability that the second bulb is defective ?

Ans.

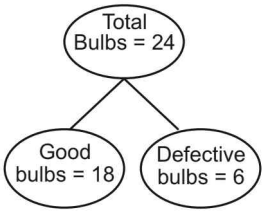
Total no. of bulbs, T(E) = 24

Total bulbs = 24

Good bulbs = 18

Defective bulbs = 6

Let E_1 be the event of selecting a non-defective bulbs i.e., selecting a good bulb.



Number of favourable outcomes, F(E) = 18.

∴ Probability that bulb is not defective, P(E)

$$= \frac{F(E)}{T(E)} = \frac{18}{24} = \frac{3}{4}$$

Suppose, the selected bulb is defected and not replaced, then total no. of bulbs remaining in the carton = 23.

i.e., total remaining bulb T(E') = 23.

out of them, 18 are good bulbs and 5 are defective.

i.e., F(E') = 5

∴ P (selecting second defective bulb)

$$= \frac{F(E')}{T(E')} = \frac{5}{23}.$$

37. A child's game has 8 triangles, of which 3 are blue and the rest are red. It also contains 10 squares, of which 6 are blue and the rest are red. One piece is lost at random. Find the probability that it is a:

(i) triangle

(ii) square

(iii) square of blue colour

(iv) triangle of red colour

Ans.

Total no. of triangles = 8

No. of blue coloured triangles = 3

No. of red coloured triangles = 5

Total no. of squares = 10

No. of blues coloured Square = 6

No. of red coloured Square = 4

Total figures (squares and triangles) = 10 + 8 = 18

i.e., T(E) = 18.

(i) Number of favourable outcomes for lost piece to be a triangle, $F(E_1) = 8$.

∴ Required probability $P(E_1)$

$$= \frac{F(E_1)}{T(E)} = \frac{8}{18} = \frac{4}{9}.$$

(ii) Number of favourable outcomes for the lost piece to be a square, F (E_2) = 10.

∴ Required probability, $P(E_2)$

$$= \frac{F(E_2)}{T(E)} = \frac{10}{18} = \frac{5}{9}$$

(iii) Number of favourable outcomes for the lost peice to be a square of blue colour, $F(E_3) = 6$.

∴ Required probability, $P(E_3)$

$$= \frac{F(E_3)}{T(E)} = \frac{6}{18} = \frac{1}{3}.$$

(iv) Number of favourable outcomes for the lost piece to be a red triangle $F(E_4) = 5$.

∴ Required probability, P(E4) = $\frac{F(E_4)}{T(E)} = \frac{5}{18}$.

38. In a game, the entry fee is ₹ 5. The game consists of tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise, she will lose. For tossing a coin three times, find the probability that she:

(i) loses the entry fee.

(ii) gets double the entry fee.

(iii) just gets her entry fee.

Ans.

Possible outcomes of tossing a coin 3 times {(HHH), (TTT), (HTT), (THT), (TTH), (HHT), (HTH), (THH)}

∴ Total no. of outcomes, T(E) = 8.

(i) Let E_1 be the event of losing the entry fee i.e., getting no head. {(T, T, T)].

∴ No. of favourable outcomes, $F(E_1) = 1$

∴ Required probability, P(E) = $\frac{F(E_1)}{T(E)} = \frac{1}{8}$.

(ii) Let E_2 be the event of getting double entry fee back i.e., getting 3 heads {(H, H, H)}

∴ No. of favourable outcomes, $F(E_2) = 1$

∴ Required probability $P(E_2) = \frac{F(E_2)}{T(E)} = \frac{1}{8}$.

(iii) Let E_3 be the event of getting entry fee back i.e., getting one head or 2 heads {(HTT), (THT), (TTH), (THH), (HTH), (HHT)}

∴ No. of favourable outcomes $F(E_3) = 6$

∴ Required probability $P(E_3) = \frac{F(E_3)}{T(E)} = \frac{6}{8} = \frac{3}{4}$

Hence $P(E_1) = \frac{1}{8}$ $P(E_2) = \frac{1}{8}$ $P(E_3) = \frac{3}{4}$.

39. A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.

(i) How many different scores are possible?

(ii) What is the probability of getting a total of 7?

Ans.

Given, a die has six faces marked as {0, 1, 1, 1, 6, 6}.

Total no. of outcomes T(E) = 36.

(i) Favourable outcomes

{(0, 0), (0, 1), (0, 6), (1, 0), (1, 1), (1,6), (6, 0), (6, 1), (6, 6)}

Total of all combinations of numbers are 0, 1, 2, 6, 7, 12

The difference scores which are possible are 6 scores.

(ii) Favourable outcomes for getting a sum of 7. (1, 6), (1, 6), (1, 6), (1, 6), (1, 6), (1, 6), (6, 1), (6, 1), (6, 1), (6, 1), (6, 1), (6, 1)

$\therefore$ No. of favourable outcomes = 12

$\therefore$ Required probability, $P(E) = \frac{F(E)}{T(E)} = \frac{12}{36} = \frac{1}{3}$.

40. A lot consists of 48 mobile phones, of which 42 are good, 3 have only minor defects and 3 have major defects, Varnika will buy a phone if it is good but the trader will only buy a mobile if it has no major defect. One phone is selected at random from the lot. What is the probability that it is:

(i) acceptable to Varnika?

(ii) acceptable to the trader? [CBSE 2016]

Ans.

Total no. of mobile phones, T(E) = 48.

No. of good phones = 42.

No. of phones with minor defects = 3

No. of phones with major defects = 3.

(i) Let E_1 be the event that the phone is acceptable to Varnika i.e., it is a good phone.

$\therefore$ No. of favourable outcomes $F(E_1) = 42$

$\therefore$ Required probability $P(E_1)$

$$= \frac{F(E_1)}{T(E)} = \frac{42}{48} = \frac{7}{8}.$$

(ii) Let E_2 be the event that the phone is acceptable to the trader i.e., it has no major defects.

= No. of favourable outcomes, $F(E_2)$ = No. of phones which do not have major defects = 48 – 3 = 45

$\therefore$ Required probability, $P(E_2)$

$$= \frac{F(E_2)}{T(E)} = \frac{45}{48} = \frac{15}{16}$$

$P(E_1) = \frac{7}{8}$, $P(E_2) = \frac{15}{16}$.

41. A bag contains 24 balls, of which x are red, '2x are white and 3x are blue. A ball is selected at random. What is the probability that it is:

(i) not red? (ii) white?

Ans.

Total no. of balls, T(E) = 24

No. of red balls = x

No. of white balls = $2x$

No. of blue balls = $3x$

Total balls = 24

$x + 2x + 3x = 24$

$6x = 24 \Rightarrow x = 4$

$\therefore$ No. of red balls = 4

No. of white balls = 2 × 4 = 8

No. of blue balls = 3 × 4 = 12.

(i) Let E_1 be the event of selecting a ball which is not red i.e., can be white or blue

= 8 + 12 = 20

$\therefore$ No. of favourable outcomes, $F(E_1) = 20$

$\Rightarrow$ Required probability, $P(E_1)$

$$= \frac{F(E_1)}{T(E)} = \frac{20}{24} = \frac{5}{6}$$

Let E_2 be the event of selecting a white ball.

$\therefore$ No. of favourable outcomes, $F(E_2) = 8$

$\therefore$ Required probability, $P(E_2)$

$$= \frac{F(E_2)}{T(E)} = \frac{8}{24} = \frac{1}{3}$$

Hence, $P(E_1) = \frac{5}{6}$ and $P(E_2) = \frac{1}{3}$.

42. At a fete, cards bearing numbers 1 to 1000, (one number on one card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that:

(i) the first player wins a prize?

(ii) the second player wins a prize, if the first has won?

Ans.

Total no. of outcomes, T(E) = 1000.

(i) Let E_1 be the event that the first player's wins the prize i.e. selects a card which is a perfect square is square greater than 500.

Favourable outcomes {529, 576, 625, 676, 729, 784, 841, 900, 961}.

$\therefore$ No. of favourable outcomes, $F(E_1) = 9$

Required probability, $P(E_1) = \frac{F(E_1)}{T(E)} = \frac{9}{1000}$.

(ii) Let E_2 be the event that the second player wins the prize i.e., the remaining cards which have a perfect square greater than 500 i.e., 8. No. of favourable outcomes, $F(E_2) = 8$.

Total no. of outcomes as 1 card was already selected by first player $T(E_2) = 999$.

$\therefore$ Required probability, $P(E_2) = \frac{F(E_2)}{T(E_2)} = \frac{8}{999}$

Hence, $P(E_1) = \frac{9}{1000}$ and $P(E_2) = \frac{8}{999}$.

EXERCISE 13.4

1. Find the mean marks of the students for the following distribution:

Marks	Number of Students
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

Ans.

Marks	Class marks (x_i)	Number of students (cumulative frequency)	Frequency (f_i)	(f_ix_i)
0 - 10	5	80	80 – 77 = 3	15
10 - 20	15	77	77 – 72 = 5	75
20 - 30	25	72	72 – 65 = 7	175
30 - 40	35	65	65 – 55 = 10	350
40 - 50	45	55	55 – 43 = 12	540
50 - 60	55	43	43 – 28 = 15	825
60 - 70	65	28	28 – 16 = 12	780
70 - 80	75	16	16 – 10 = 6	450
80 - 90	85	10	10 – 8 = 2	170
90 - 100	95	8	8 – 0 = 8	760
100 - 110	105	0	0	0
			$N = \Sigma f_i = 80$	$\Sigma f_ix_i = 4140$

$\therefore$ Mean $= \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{4140}{80} = 51.75$.

Alternate Method:

Deviation method

Marks Class Interval	(x_i)	Cumulative of frequency c.f.	$d_i = x_i - a$	(f_i)	f_id_i
0 - 10	5	80	–50	80 – 77 = 3	–150
10 - 20	15	77	–40	77 – 72 = 5	–200
20 - 30	25	72	–30	72 – 65 = 7	–210
30 - 40	35	65	–20	65 – 55 = 10	–200
40 - 50	45	55	–10	55 – 43 = 12	–120
50 - 60	$\boxed{a = 55}$	43	0	43 – 28 = 15	0
60 - 70	65	28	10	28 – 16 = 12	120
70 - 80	75	16	20	16 – 10 = 6	120
80 - 90	85	10	30	10 – 8 = 2	60
90 - 100	95	8	40	8 – 0 = 8	320
100 - 110	105	0	50	0	50
				$\Sigma f_i = 80$	$\Sigma f_id_i = -260$

Assumed mean, $a = 55$

$$\Sigma f_i d_i = -260$$

$$\Sigma f_i = 80$$

$$\text{Mean } \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 55 + \frac{-260}{80}$$

$$= 55 - \frac{13}{4}$$

$$= 55 - 3.25$$

$$\bar{x} = 51.75.$$

2. **Determine the mean of the following distribution:**

Marks	Number of Students
Below 10	5
Below 20	9
Below 30	17
Below 40	29
Below 50	45
Below 60	60
Below 70	70
Below 80	78
Below 90	83
Below 100	85

Ans.

Given data is of less than type. First we have to convert it to the continuous type.

From the given table, we observe that 5 students have scored marks below 10. i.e., it lies in the class intervals (0 - 10).

9 students have scored marks below 20 i.e., (9 – 5) = 4 students lie in the class interval (10 - 20)

Thus, following is the complete frequency distribution table.

Marks	Cumulative Frequency (c.f.) (x_i)	Class marks (x_i)	No. of students (f_i)	$u_i = \frac{x_i - a}{h}$ $u_i = \frac{x_i - 45}{h}$	$f_i u_i$
0 - 10	5	5	5 – 0 = 5	–4	–20
10 - 20	9	15	9 – 5 = 4	–3	–12
20 - 30	17	25	17 – 9 = 8	–2	–16
30 - 40	29	35	29 – 17 = 12	–1	–12
40 - 50	$a = 45$	45	45 – 29 = 16	0	0
50 - 60	60	55	60 – 45 = 15	1	15
60 - 70	70	65	70 – 60 = 10	2	20
70 - 80	78	75	78 – 70 = 8	3	24
80 - 90	83	85	83 – 78 = 5	4	20
90 - 100	85	95	85 – 83 = 2	5	10
			$\Sigma f_i = 85 = N$		$\Sigma f_i u_i = 29$

Clearly,

assumed mean, $a = 45$

Class width, $h = 10$

According to step deviation method:

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 45 + \frac{29}{85} \times 10 = 45 + \frac{58}{17}$$

$$= 45 + 3.41 = 48.41.$$

3. **Find the mean age of 100 residents of a town from the following data:**

Age equal and above (in years)	0	10	20	30	40	50	60	70
Number of people	100	90	75	50	25	15	5	0

Ans.

We observe that number of residents whose age is above 0 years = 100

number of residents whose age is above 10 years = 90

i.e., 100 – 90 = 10 residents lie in the interval 0 - 10.

Continuing in this manner, we get a frequency distribution table for the given data.

Class Interval	No. of people (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	f_iu_i
0 - 10	100 – 90 = 10	5	–3	–30
10 - 20	90 – 75 = 15	15	–2	–30
20 - 30	75 – 50 = 25	25	–1	–25
30 - 40	50 – 25 = 25	$\boxed{a = 35}$	0	0
40 - 50	25 – 15 = 10	45	1	10
50 - 60	15 – 5 = 10	55	2	20
60 - 70	5 – 0 = 5	65	3	15
	$N = \Sigma f_i = 100$			$\Sigma f_iu_i = -40$

Here, assumed mean $a = 35$, class width, $h = 10$

Using step deviation method,

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 35 + \frac{(-40)}{100} \times 10$$

$$\text{Mean } (\bar{x}) = 35 - 4$$

$$= 31 \text{ years.}$$

4. The weights of tea in 70 packets are shown in the following table: [CBSE 2014, 12]

Weight (in g)	200 - 201	201 - 202	202 - 203	203 - 204	204 - 205	205 - 206
Number of packets	13	27	18	10	1	1

Find the mean weight of the packets.

Ans.

Weight (in gram)	No. of Packets (f_i)	Class Marks (x_i)	Deviation $d_i = (x_i - a)$	f_id_i
200 - 201	13	200.5	–3	–39
201 - 202	27	201.5	–2	–54
202 - 203	18	202.5	–1	–18
203 - 204	10	$\boxed{a = 203.5}$	0	0
204 - 205	1	204.5	1	1
205 - 206	1	205.5	2	2
	$N = \Sigma f_i = 70$			$\Sigma f_iu_i = -108$

Here, assumed mean, $a = 203.5$

Class width, $h = 1$

Using assumed mean method,

$$\text{Mean, } \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$

$$= 203.5 + \frac{(-108)}{70} \times 1$$

$$= 203.5 - 1.54 = 201.96$$

Hence, required mean weight is 201.96 g.

5. Refer to Q.4 above. Draw the less than type ogive for this data and use it to find the median weight.

Ans.

We know that

Number of packets which weigh less than 200 g = 0

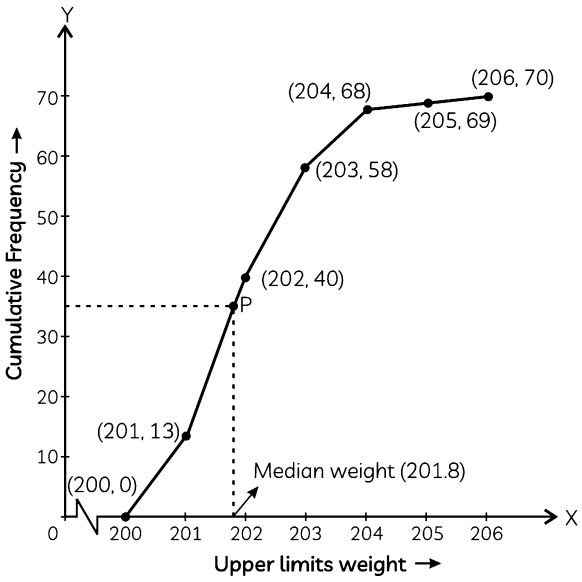
Similarly,

Number of packets which weigh less than 201 k = No. of packet from 0 - 201 g

So, total no. of packets less than 201 = 0 + 13 = 13.

We can say that the cumulative frequency of the class 200 - 201 is 13.

Continuing same way for other classes we will have



Less Than Type

Weight (in gm)	Number of packets
Less than 200	0
Less than 201	0 + 13 = 13
Less than 202	27 + 13 = 40
Less than 203	18 + 40 = 58
Less than 204	10 + 58 = 68
Less than 205	1 + 68 = 69
Less than 206	1 + 69 = 70

Points for less than type ogive graph are (200, 0), (201, 13), (202, 40), (203, 58), (204, 68), (205, 69), (206, 70).

∴ Total number of packet (N) = 70

Now, $\frac{N}{2}$ = 35.

The value = 35 is marked on y-axis and from this point a line parallel to X-axis is drawn. This line meets the curve at a point P. From P draw a perpendicular PT to meet x-axis at T.

T represents the median.

Here median is 201.8.

Hence, the median of given frequency distribution is 201.8.

6. Refer to Q.4 above. Draw the less than type and more than type ogives for the data and use them to find the median weight.

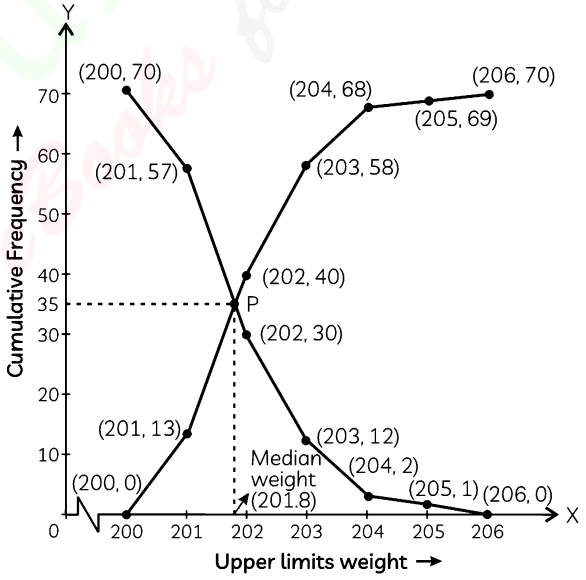
Ans.

For less than type ogive table, please refer to Q. 5.

We can similarly make the table for the number of packets weighing more than or equal to 200, more than or equal to 201 , more than or equal to 202, and so on.

From the table, we observe that all 70 packets has weight more than or equal to 200.

Since, there are 13 packets that lie in the interval 200 - 201, this means that there are (70 – 13) = 57 Packets weighing more than or equal to 201. Continuing in the same manner, we get the following table:



(i) Less than type

Weight (in gram)	Number of Packets
Less than 200	0
Less than 201	0 + 13 = 13
Less than 202	27 + 13 = 40
Less than 203	18 + 40 = 58
Less than 204	10 + 58 = 68
Less than 205	1+ 68 = 69
Less than 206	1 + 69 = 70

(ii) More than type

Weight (in gram)	Number of Packets
More than or equal to 200	70
More than or equal to 201	70 – 13 = 57
More than or equal to 202	57 – 27 = 30
More than or equal to 203	30 – 18 = 12
More than or equal to 204	12 – 10 = 2
More than or equal to 205	2 – 1 = 1
More than or equal to 206	1 – 1 = 0

To draw the less than type ogive, we plot the points corresponding to the ordered pairs given by (upper limit, corresponding cumulative frequency), (200, 0), (201, 13), (202, 40), (203, 58), (204, 68), (205, 69), (206, 70) on the graph paper and join them by free hand.

The curve we get is called a cumulative frequency curve, or an ogive (of the less than type).

To draw the more than type ogive, we plot the points (lower limit, corresponding cumulative frequency), (200, 70), (201, 57), (202, 30), (203, 12), (204, 2), (205, 1), (206, 0) on the graph paper and join them free hand.

The curve we get is called a cumulative frequency curve, or an ogive (of the more than type).

Hence, required median weight = intersection point of X-axis, P = 201.8

7. The table below shows the salaries of 280 people. [CBSE 2010]

Salary [in thousand (₹)]	Number of people
5 - 10	49
10 - 15	133
15 - 20	63
20 - 25	15
25 - 30	6
30 - 35	7
35 - 40	4
40 - 45	2
45 - 50	1

Calculate the median and mode of the data.

Ans.

The cumulative frequency table for the given data is shown below.

Salary (in ₹ thiousand)	Number of people (f_i)	Cumulative Frequency
50 - 10	49	49
10 - 15	133	133 + 49 = 182
15 - 20	63	63 + 182 = 245
20 - 25	15	15 + 245 = 260
25 - 30	6	6 + 260 = 266
30 - 35	7	7 + 266 = 273
35 - 40	4	4 + 273 = 277
40 - 45	2	2 + 277 = 279
45 - 50	1	1 + 279 = 280

$N = \Sigma f_i$
$= 280$

$\therefore \quad \frac{N}{2} = \frac{280}{2} = 140.$

(i) $\frac{N}{2} = 140$, it is less than 182

Hence, we consider 182 cumulative frequency CF.

Thus the median class = 10 - 15

Lower limit, $l = 10$

Frequency, $f = 133$

Cumulative frequency, cf = 49

Class width, $h = 5$

$\therefore \quad \text{median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$

$= 10 + \frac{(140-49)}{133} \times 5 = 10 + \frac{91 \times 5}{133}$

$= 10 + \frac{455}{133} = 10 + 3.42$

$= 13.42$ (in thousands)

$= 13.42 \times 1000 = ₹\ 13{,}420.$

(ii) For calculating the mode of the data. The highest frequency is 133, it lies in the interval 10 - 15, called the modal class.

Lower limit, l = 10

Class width, $h = 5$

Frequency of modal class, $f_1 = 133$

Frequency of class preceeding modal class, $f_0 = 49$

Frequency of class succeeding modal class, $f_2 = 63$

$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 10 + \left(\frac{133-49}{2 \times 133 - 49 - 63}\right) \times 5$

$= 10 + \frac{84 \times 5}{266 - 112} = 10 + \frac{84 \times 5}{154}$

$= 10 + 2.73 = 12.73$ (in thousands)

Hence, the median and modal salary are ₹ 13,420 and ₹ 12,730; respectively.

8. The mean of the following frequency distribution is 50, but the frequencies f_1 and f_2 in classes 20 - 40 and 60 - 80, respectively are not known. Find these frequencies, if the sum of all the frequencies is 120.

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	17	f_1	32	f_2	19

[CBSE 2016, 15, 13, 10]

Ans.

Class	**Frequency (f_i)**	**Class Marks (x_i)**	$u_i = \frac{x_i - a}{h}$	f_iu_i
0 - 20	17	10	–2	–34
20 - 40	f_1	30	–1	$-f_1$
40 - 60	32	$a = 50$	0	0
60 - 80	f_2	70	1	f_2
80 - 100	19	90	2	38
	$\Sigma f_i = 68 + f_1 + f_2$			$\Sigma f_iu_i = 4 + f_2 - f_1$

We have,

$N = \Sigma f_i = 120$ [Given]

$\Sigma f_i = 68 + f_1 + f_2$

$68 + f_1 + f_2 = 120$

$\Rightarrow f_1 + f_2 = 52$ ...(i)

$\Sigma f_iu_i = 4 + f_2 - f_1$

Here, assumed mean, $a = 50$

Class width, $h = 20$

Using step deviation method:

$$\text{mean} = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

Using eqn. (i).

$$50 = 50 + \frac{\left(4 + f_2 - f_1\right)}{120} \times 20 \times 20$$

$\Rightarrow 4 + f_2 - f_1 = 0$

$\Rightarrow f_1 - f_2 = 4$ ...(ii)

Solving equations (i) and (ii), we get

$\Rightarrow f_1 = 28$

Putting the value of f_1 in eqn (i) we get

$f_1 + f_2 = 52$

$28 + f_2 = 52$

$\Rightarrow f_2 = 52 - 28$

$f_2 = 24$

Hence, $f_1 = 28$ & $f_2 = 24$.

9. The median of the following data is 50. Find the values of *p* and *q*, if the sum of all the frequencies is 90.

Marks	**Frequency**
20 - 30	***p***
30 - 40	**15**
40 - 50	**25**
50 - 60	**25**
60 - 70	**q**
70 - 80	**8**
80 - 90	**10**

[CBSE 2019, 17, 15, 13]

Ans.

Marks	**Frequency**	**Cumulative Frequency**
20 - 30	p	p
30 - 40	15	$15 + p$
40 - 50	25	$40 + p$
50 - 60	20	$60 + p$
60 - 70	q	$60 + p + q$
70 - 80	8	$68 + p + q$
80 - 90	10	$78 + p + q$

Given that,

$N = 90$

$\therefore \frac{N}{2} = \frac{90}{2} = 45$, Clearly 45 lies in the interval (50 - 60).

Median = 50

$\therefore$ Median class is (50 - 60)

lower limit, $l = 50$

frequency of median class, $f = 20$

Cumulative frequency of the class preceeding the median class, $cf = 40 + p$

Class width, $h = 10$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$50 = 50 + \left(\frac{45 - (40 + p)}{20}\right) \times 10$$

$$50 = 50 + \left(\frac{5 - p}{2}\right)$$

$$\Rightarrow \quad 5 - p = 0 \quad \Rightarrow p = 5 \qquad \text{...(i)}$$

We know that sum of all the frequencies is 50.

Hence, $78 + p + q = 90$ [Given]

$\Rightarrow 78 + 5 + q = 90$ [Using eqn. (i)]

$\Rightarrow \quad q = 7$

Hence, $p = 5$, $q = 7$.

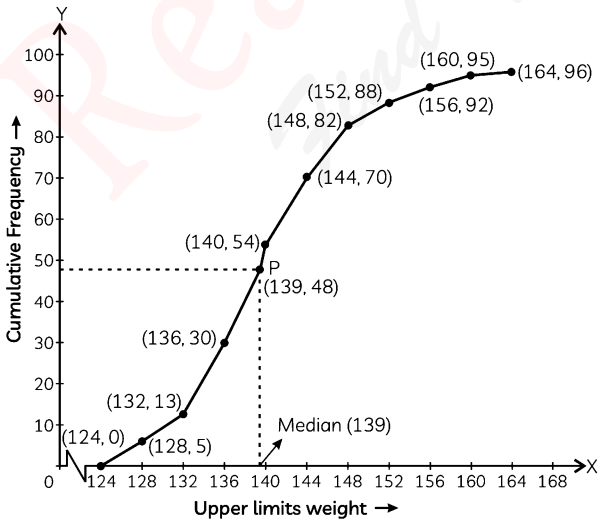
10. The distribution of heights (in cm) of 96 children is given below:

Height (in cm)	Number of children
124 - 128	5
128 - 132	8
132 - 136	17
136 - 140	24
140 - 144	16
144 - 148	12
148 - 152	6
152 - 156	4
156 - 160	3
160 - 164	1

Draw a less than type cumulative frequency curve for this data and use it to compute the median height of the children.

[CBSE 2019, 17]

Ans.



Height (in cm)	Number of children
Less than 124	0
Less than 128	5 + 0 = 5
Less than 132	5 + 8 = 13
Less than 136	13 + 17 = 30
Less than 140	30 + 24 = 54
Less than 144	54 + 16 = 70
Less than 148	70 + 12 = 82
Less than 152	82 + 6 = 88
Less than 156	88 + 4 = 92
Less than 160	92 + 3 = 95
Less than 164	95 + 1 = 96

To draw the less than type ogive, we plot the points corresponding to the ordered pairs given by (upper limit, corresponding cumulative frequency), (124, 0), (128, 5), (132, 13), (136, 30), (140, 54), (144, 70), (148, 82), (152, 88), (156, 92), (160, 95) and (164, 96) and join all these points by free hand to get cumulative frequency curve.

Here, $\frac{N}{2} = \frac{96}{2} = 48$

The value = 48 is marked on y-axis and from this point a line parallel to x-axis is drawn. This line meets the curve at a point P. From P draw a perpendicular PT to meet x-axis at T.

T represents the median.

Here median is 139.

Hence, the median of given frequency distribution is 139.

11. The Size of agricultural holdings in a survey of 200 families is given in the following table:

Size of agricultural holdings (in ha)	Number of families
0 - 5	10
5 - 10	15
10 - 15	30
15 - 20	80
20 - 25	40
25 - 30	20
30 - 35	5

Compute median and mode size of the holdings.

Ans.

Size of agricultural holding (in ha)	Number of families (f_i)	Cumulative Frequency (cf)
0 - 5	10	10 + 0 = 10
5 - 10	15	10 + 15 = 25
10 - 15	30	25 + 30 = 55

Size of agricultural holding (in ha)	Number of families (f_i)	Cumulative Frequency (cf)
15 - 20	80	55 + 80 = 135
20 - 25	40	135 + 40 = 175
25 - 30	20	175 + 20 = 195
30 - 35	5	195 + 5 = 200

(i) Given that N = 200

$\frac{N}{2}$ = 100, Clearly it lies in the interval (15 – 20)

Lower limit, $L = 15$

Class size, $h = 5$

frequency of median class, $f = 80$

Cumulative frequency of preceeding class, $cf = 55$

$$\text{Median} = l + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

$$= 15 + \frac{(100-55)}{80} \times 5 = 15 + \frac{45}{16} = 17.81\, ha$$

$= 17.81\ ha.$

(ii) If we observe the above table we will find that, 80 is the highest frequency.

$\therefore$ Modal class = (15 – 20)

Lower limit, $l = 15$

Size of the class, $h = 5$

Frequency of modal class, $f_1 = 80$

Frequency of class preceeding modal class, $f_0 = 30$

frequency of class preceeding modal class, $f_2 = 40$

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 15 + \left(\frac{80 - 30}{2 \times 80 - 30 - 40}\right) \times 5$$

$$= 15 + \left(\frac{50}{160 - 70}\right) \times 5$$

$$= 15 + \frac{50}{90} \times 5 = 15 + \frac{25}{9}$$

$= 15 + 2.77 = 17.77\ ha.$

12. The annual rainfall record of a city for 66 days is given in the following table.

Rainfall (in cm)	Number of days
0 - 10	22
10 - 20	10
20 - 30	8
30 - 40	15
40 - 55	5
50 - 60	6

Calculate the median rainfall using ogives (of more than type and of less than type).

[CBSE 2020]

Ans.

We observe that the annual rainfall of the city less than 0 is 0. Similarly, less than 10 includes the annual rainfall record of the city from 0 as well as the annual record of a city from 0 - 10.

$\therefore$ Total annual rainfall less than 10 = 0 + 22 = 22 days.

Continuing this way, we will get the remaining less than 20, 30, 40, 50 & 60.

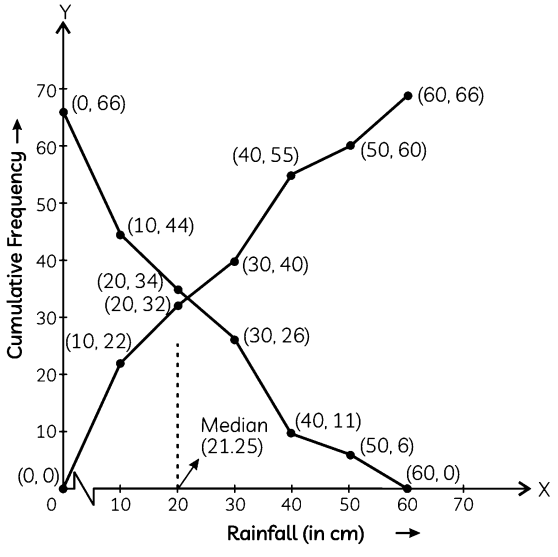
We can similarly make the table for the number of days with rainfall more than or equal to 0, more than or equal to 10 , more than or equal to 20, and so on.

From the table, we observe that for all 66 days rainfall is more than 0 cm .

Since there are 22 days that lie in the interval 0 - 10, this means that there are (66 – 22) = 44 days with rainfall more than or equal to 10

Continuing in the same manner, we get the following table.

Now, we construct a table for less than and more than type.



(i) Less than type		(ii) More than type	
Fainfall (in cm)	**Number of days**	**Rainfall (in cm)**	**Number of days**
Less than 0	0	More than or equal to 0	66
Less than 10	0 + 22 = 22	More than or equal to 10	66 – 22 = 44
Less than 20	22 + 10 = 32	More than or equal to 20	44 – 10 = 34
Less than 30	32 + 8 = 40	More than or equal to 30	34 – 8 = 26
Less than 40	40 + 15 = 55	More than or equal to 40	26 – 15 = 11
Less than 50	55 + 5 = 60	More than or equal to 50	11 – 5 = 6
Less than 60	60 + 6 = 66	More than or equal to 60	6 – 6 = 0

To draw less than type ogive, we plot the points corresponding to the ordered pairs given by (upper limit, corresponding cumulative frequency), (0, 0), (10, 22), (20, 32), (30, 40), (40, 55), (50, 60), (60, 66) on the graph paper and join them by free hand.

To draw more than type ogive, we plot the points (lower limit, corresponding cumulative frequency), (0, 66), (10, 44), (20, 34), (30, 26), (40, 11), (50, 6) and (60, 0) on the graph paper and joint them by free hand.

Total number of days = 66.

Now, $\frac{N}{2} = 33$.

Firstly, we plot a line parallel to X-axis at the intersection point of both ogives, which further intersect at (0, 33) on Y-axis.

Then we draw a line perpendicular to X-axis at the intersection of both ogives, which further intersect at (21.25) on X-axis This gives the required median using ogives.

Hence, median rainfall = 21.25 cm.

13. The following is the frequency distribution of duration for 100 calls made on a mobile phone:

Duration (in seconds)	Number of calls
95 - 125	14
125 - 155	22
155 - 185	28
185 - 215	21
215 - 245	15

Calculate the average duration (in sec) of a call and also find the median from a cumulative frequency curve.

Ans.

Duration (in seconds)	Number of calls (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	f_iu_i
95 - 125	14	110	–2	–28
125 - 155	22	140	–1	–22
155 - 185	28	$\boxed{a = 170}$	0	0
185 - 215	21	200	1	21
215 - 245	15	230	2	30
	$\Sigma f_i = 100$			$\Sigma f_iu_i = 1$

Here, assumed mean, $a = 170$

class width, $h = 30$

By step deviation method,

$$\text{Average } (\bar{x}) = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 170 + \frac{1}{100} \times 30$$

$$= 170 + \frac{3}{10} = 170.3$$

Hence, average duration is 170.3 seconds.

To find the median from a cumulative frequency curve, we prepare less than type or more than type ogive.

We observe that the number of calls with a duration less than 95 seconds is 0.

Similarly, less than 125 seconds includes the number of calls in less than 95 seconds as well as the number of calls from 95 - 125 seconds. So, the total number of calls less than 125 seconds is 0 + 14 = 14. Continuing in this

manner, we will get remaining in less than 155, 185, 215 and 245 seconds.

We construct a table for less than type ogive (cumulative frequency curve).

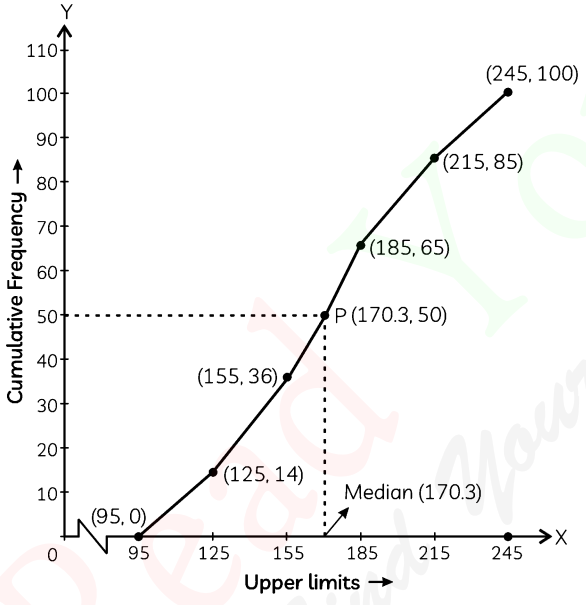
Less than type

Duration (in seconds)	Number of calls
Less than 95	0
Less than 125	0 + 14 = 14
Less than 155	14 + 22 = 36
Less than 185	36 + 28 = 64
Less than 215	64+21 = 85
Less than 245	85 + 15 = 100

To draw less than type ogive, we plot them the points (95, 0), (125, 14), (155, 36), (185, 64), (215, 85), (245, 100) on the graph paper and join them by free hand.

Total number of calls (N) = 100

$\therefore \frac{N}{2} = \frac{100}{2} = 50$



The value 50 is marked on y-axis and from this point a line parallel to x-axis is drawn. This line meets the curve at a point P. From P draw a perpendicular PT to meet x-axis at T.

T represents the median.

Here median is 170.3.

Hence, the median of given frequency distribution is 170.3.

14. 50 students enter a school javelin throw competition. The distance (in metres) thrown are recorded below:

Distance (in m)	Number of students
0 - 20	6
20 - 40	11
40 - 60	17
60 - 80	12
80 - 100	4

(i) Construct a cumulative frequency table.

(ii) Draw a cumulative frequency curve (less than type) and calculate the median distance thrown by using this curve.

(iii) Calculate the median distance by using the formula for median,

(iv) Are the median distance calculated in (ii) and (iii) same? **[CBSE 2017, 13]**

Ans.

(i)

Distance (in m)	Number of students (f_i)	Cumulative Frequency (f_i)
0 - 20	6	6
20 - 40	11	17
40 - 60	17	34
60 - 80	12	46
80 - 100	4	50

(ii)

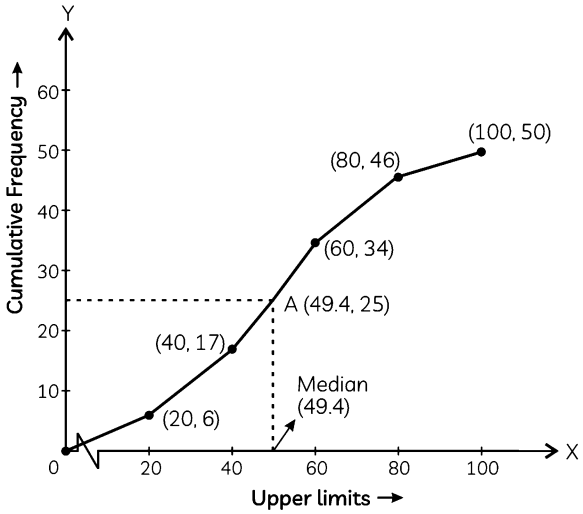
Distance (in m)	Cumulative Frequency
Less than 0	0
Less than 20	6
Less than 40	17
Less than 60	34
Less than 80	46
Less than 100	50

The co-ordinates for drawing an ogive are (0, 0), (20, 6), (40, 17), (60, 34), (80, 46) and (100, 50) and join all these points by free hand.

Here, $N = 50$

$\therefore \frac{N}{2} = \frac{50}{2}$

$= 25$



The value = 25 is marked on y-axis and from this point a line parallel to x-axis is drawn. This line meets the curve at a point P. From P draw a perpendicular PT to meet x-axis at T. T represents the median.

Here median is 49.4.

Hence, the median of given frequency distribution is 49.4

(iii) As $\frac{N}{2} = \frac{50}{2} = 25$

Median class is (40 - 60)

Lower limit, l = 40. class width, h = 20

Frequency of median class = 17.

Cumulative frequency of preceding median class = 17.

$$\therefore \quad \text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$$

$$= 40 + \frac{(25 - 17)}{17} \times 20$$

$$\therefore \quad \text{Median} = 40 + \frac{8 \times 20}{17}$$

$$= 40 + \frac{160}{17} = 40 + 9.41$$

$$= 49.41 \text{ m}.$$

(iv) Yes, median distance calculated in parts (ii) and (iii) are the same.

DIKSHA 2.0

Recommended by NCERT

(Selected top questions)

1. While computing mean of grouped data, we assume that the frequencies are:

(A) **evenly distributed over all the class.**

(B) **centered at the class marks of the class.**

(C) **centered the upper limits of the class.**

(D) **centered the lower limits of the class.**

Ans. (B)

Explanation: In grouped data, when we calculate the mean. We assume that the frequencies are centered at the class marks of the class.

2. A set of numbers consists of three 4s, two 5s, six 6s, eight 8s and seven 10s. What is the mode of this collection of numbers?

(A) **10** (B) **7.5**

(C) **7** (D) **8**

Ans. (D)

Explanation: Here, set of numbers are :

4, 4, 4, 5, 5, 6, 6, 6, 6, 6, 6, 8, 8, 8, 8, 8, 8, 8, 8, 10, 10, 10, 10, 10, 10, 10

So, mode of this collection of numbers is 8.

As it occurs maximum number of times.

3. If a letter is chosen at random from the letter of English alphabet, then the probability that it is a letter of the word 'DELHI' is:

(A) $\frac{1}{5}$ (B) $\frac{1}{26}$

(C) $\frac{5}{26}$ (D) $\frac{21}{26}$

Ans. (C)

Explanation: Total number of alphabets in English = 26 letters

Number of letters in a word 'DELHI' = 5 letters

So, the number of favourable outcomes = 5.

Probability = $\frac{5}{26}$.

4. A dice is thrown twice. The probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw is:

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. (A)

Explanation: Here, let P(A) and P(B) be the probability of the events,

Then, P(A and B) = P(A).P(B)

$$= \frac{3}{6} \times \frac{4}{6}$$

$$= \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1}{3}$$

5. If a number x is chosen at random from the numbers −2, −1, 0, 1, 2. Then, the probability that $x^2 < 2$ is:

(A) $\frac{2}{5}$ (B) $\frac{4}{5}$

(C) $\frac{1}{5}$ (D) $\frac{3}{5}$

Ans. (D)

Explanation: Here, x can take anyone of the five given values.

So, total number of possible outcomes = 5

We observe that $x^2 < 2$ when x takes anyone of the following three values −1, 0 and 1.

So, favourable number of elementary events = 3

Hence, $P(x^2 < 2) = \frac{3}{5}$.

6. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set:

(A) **is increased by 2.**

(B) **is decreased by 2.**

(C) **is two times the original median.**

(D) **remains the same as that of the original set.**

Ans. (D)

Explanation: Remains the same as that of the original set

Since, $n = 9$

then, median term $= \frac{9+1}{2} = 5^{th}$ term

Now, last four observations are increased by 2.

But the median is 5th observation, which remains unchanged.

There will be no change in median.

7. The mean and median of a distribution are both equal to 635.97. Find the mode.

Ans.

Mean = Median = 635.97 [Given]

Mode = 3 median − 2 mean

= 3 mean − 2 mean [Mean = Median]

= mean

= 635.97.

8. If X, M and Z are denoting mean, median and mode of a data and X : M = 9 : 8, then the ratio M : Z is?

Ans.

Mode = 3 median − 2 mean

$Z = 3M - 2X$ ...(i)

$X : M = 9 : 8$

$\frac{X}{M} = \frac{9}{8}$

$X = \frac{9M}{8}$

On Putting the value of X in eq (i), we get

$$Z = 3M - 2 \times \frac{9M}{8}$$

$$= 3M - \frac{9M}{4}$$

$$= \frac{3M}{4}$$

$$\frac{M}{Z} = \frac{4}{3}$$

$M : Z = 4 : 3.$

9. The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of the number of boys to the number of girls who appeared in the examination.

Ans.

Let the number of boys in the school be x and the number of girls in the school be y.

Average score of boys = 71

Total score of boys in the examination of school = $71x$

Average score of girls = 73

Total score of girls in the examination of school = $73y$

Average score of the school in examination = 71.8

$$\frac{\text{total score of boys + total score of girls}}{\text{total score of boys and girls}} = 71.8$$

$$\frac{71x+73y}{x+y} = 71.8$$

$$71x + 73y = 71.8x + 71.8y$$
$$73y - 71.8y = 71.8x - 71x$$
$$1.2y = 0.8x$$
$$\frac{1.2}{0.8} = \frac{x}{y}$$
$$\frac{12}{8} = \frac{x}{y}$$
$$\frac{x}{y} = \frac{3}{2}$$
$$x : y = 3 : 2$$

10. If the mean of x and $\frac{1}{x}$ is M, find the mean of x^3 and $\frac{1}{x^3}$.

Ans.

$$x + \frac{1}{x} = \text{M (Mean)}$$

Now, Mean of x^3 and $\frac{1}{x^3} = \frac{x^3 + \frac{1}{x^3}}{2}$ ...(i)

We know:

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

So, $a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)$ ...(ii)

Putting (ii) in eqn. (i)

$$= \frac{\left(x + \frac{1}{x}\right)^3 - 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right)}{2}$$

$$= \frac{M^3 - 3M}{2}$$

11. The mean of 20 observations is 12. If each observation is increased by 5, then find the new mean.

Ans.

Mean of 20 observations = 12

Here, each observation is increased by 5

So, new mean will also increased by 5

Thus, new mean = 12 + 5 = 17.

12. Two unbiased coins are tossed simultaneously then the probability of getting no head is $\frac{A}{B}$, then $(A + B)^2$ is?

Ans.

If two unbiased coins are tossed simultaneously we obtained possible outcomes:

HH, HT, TH, TT

Hence, Total number of outcomes = 4

No head is obtained if the event TT occurs.

Hence, Number of favourite outcomes = 1

Hence, Required probability = $\frac{1}{4}$

But, given probability = $\frac{A}{B}$

So, A = 1

and B = 4

therefore, $(A + B)^2 = (1 + 4)^2$

$= (5)^2$

$= 25$

So, $(A + B)^2 = 25$.

13. If odds in against of an event be 3 : 8, then the probability of occurence of this event is?

Ans.

Let the event be E.

Number of favourable outcomes to event E are m and total outcomes be n.

According to the question,

$$\frac{m}{n-m} = \frac{3}{8}$$
$$8m = 3n - 3m$$
$$11m = 3n$$
$$\frac{m}{n} = \frac{3}{11}$$

Hence,

$$P(E) = \frac{\text{number of outcomes favourable to E}}{\text{number of total outcomes}}$$
$$= \frac{m}{n}$$
$$= \frac{3}{11}$$

So, probability of occurrence of this event is $\frac{3}{11}$.

14. Two dices were rolled once. Find the probability of getting such numbers on the two dice, whose product is 12.

Ans.

When 2 dices are rolled, total possible outcomes are:

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$\therefore$ Total outcomes = 36

Favourable outcomes = (2, 6) (3, 4) (4, 3) (6, 2) = 4

Therefore, probability $= \frac{4}{36} = \frac{1}{9}$.